MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

DIFFERENTIATION

Examples

1. Differentiate the following functions w.r.t x using first principle.

$f(x) = \tan x$

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2. Differentiate the following functions w.r.t x using first principle.

e^{x^2}

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3. Differentiate the following functions w.r.t x using first principle.

$\sqrt{\sin x}$

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4. If $y = \sin x + e^x$, Then find $\frac{dy}{dx}$.

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5. If $Y = x^2 + \sin^{-1} x + \log_e x$, then find $\frac{dy}{dx}$

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6. If $y = \log_{10} x + \log_e x + \log_{10} 10$, then find $\frac{dy}{dx}$

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7. If $y = x^{-1/2} + \log_5 x + \frac{\sin x}{\cos x} + 2^x$, then find $\frac{dy}{dx}$

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8. If $y = m^2 \sec^{-1} x$, then find $\frac{dy}{dx}$.

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9. If $y = \log_e x^3 + 3\sin^{-1} x + kx^2$, then find $\frac{dy}{dx}$

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10. If $y = e^x \sin x$, then find $\frac{dy}{dx}$

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11. If $y = e^x \tan x + x . \log_e x$, then find $\frac{dy}{dx}$

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12. Let f,g and h be differentiable function. If $f(0)=1, g(0)=2, h(0)=3$ and the derivatives of their pair wise products at x = 0 are $(fg)'(0)=6, (gh)'(0)=4$ and $(hf)'(0)=5$ then the value of $((fgh)'(0))/2$ is

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13. If $y=(1+x)(1+x^2)(1+x^4)(1+x^{2n})$, then find $\frac{dy}{dx} at x = 0.$

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14. If $y=\frac{x}{x^2+1}$, then find $\frac{dy}{dx}$

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15. if $y=\frac{x\sin x}{\log_e x}$, then find $\frac{dy}{dx}$

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16. If $y=\frac{3a^2x-x^3}{a^3-3ax^2}$ then find $\frac{dy}{dx}$

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17. If $y=\frac{e^x-\tan x}{x^n+\cot x}$, then find $\frac{dy}{dx}$

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18. If $y=\frac{\log_e x}{x}+e^x\sin x+\log_5 x$ then find $\frac{dy}{dx}$.

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19. If $y=\frac{x^4+x^2+1}{x^2+x+1}$ then $\frac{dy}{dx}=ax+b$, find a and b

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20. Let $y=\frac{\sec x+\tan x-1}{\tan x-\sec x+1}$.If $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=a+\sqrt{b}$, then value of $a+b$ is equal to

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21. If $y=\frac{\tan^{-1}x-\cot^{-1}x}{\tan^{-1}x+\cot^{-1}x}$, find $\left(\frac{dy}{dx}\right]\Big)_{x=-1}$ 0 (b) 1 (c) $2/\pi$ (d) -1

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22. If $y=e^{(\tan^{-1}x)thenf\in d}$(dy)/(dx)`

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23. $y=\log_e\left(\tan^{-1}\sqrt{1+x^2}\right)$ then $\frac{dy}{dx}$ is

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24. Find $\frac{dy}{dx}$, when $y = e^{ax}\cos(bx + c)$

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25. Differentiate the following w. r. t. x. (i) $\sqrt{\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}$ (ii) $\log_e\left(\frac{x + \sqrt{x^2 - a^2}}{x - \sqrt{x^2 - a^2}}\right)$

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26. If $y = x\cos y + y\cos x$, then find $\frac{dy}{dx}$

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27. If $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1, then \frac{dy}{dx} =$

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28. If $\sqrt{1-x^6}+\sqrt{1-x^6}=a\left(x^3-y^3\right)$, then prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

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29. Let $y=\sqrt{x+\sqrt{x+\sqrt{x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

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30. If $y=\sec^{-1}\left(\sqrt{1+x^2}\right)$, when $-1<x<1$, then find $\frac{dy}{dx}$

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31. $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}} find \frac{dy}{dx}$

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32. Prove that $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}=\frac{x}{2}, x<\pi.\right.$

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33. If $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then find $\frac{dy}{dx}$ when $-1 \leq x \leq 1.$

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34. Differentiate the following functions with respect to x :

$\tan^{-1}\left(\frac{a+x}{1-ax}\right)$ (ii) `tan^(-1)((acosx-bsinx)/(bcosx+asinx)),\ -pi/2-1`

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35. if $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$ then find $\frac{dy}{dx}$

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36. Find $\frac{dy}{dx}$ for `y=tan^(-1)sqrt((a-x)/(a+x),)-a

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37. Sketch the graph for $y = \sin^{-1}(\sin x)$.

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38. Sketch the graph for $y = \cos^{-1}(\cos x)$.

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39. Sketch the graphs for $y = \sin\left(\sin^{-1} x\right)$

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40. Sketch the graphs for $y = \cos\left(\cos^{-1} x\right)$

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41. Sketch the graphs for $y = \tan(\tan^{-1} x)$

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42. Draw the graph of $y = \text{cosec}(\text{cosec}^{-1} x)$ or $y = \sec(\sec^{-1} x)$.

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43. Sketch the graphs for $y = \sec(\sec^{-1} x)$

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44. Sketch the graphs for $y = \cot(\cot^{-1} x)$ and hence find $\frac{dy}{dx}$

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45. If $x = e^{-t^2}, y = \tan^{-1}(2t+1)$, then $\frac{dy}{dx} =$

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46. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$.

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47. If $x = \sqrt{a^{\sin} \hat{}\, ((-1)t)}, y = \sqrt{a^{\cos} \hat{}\, ((-1)t)},$ show that $\frac{dy}{dx} = -\frac{y}{x}$

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48. If $x = a\left(\cos t + \frac{1}{2}\log\tan^2 t\right)$ and $y = a\sin t$ then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

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49. If $y = x^{\sin x}$, then find $\frac{dy}{dx}$

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50. $x^y \cdot y^x = 1$. Find $\frac{dy}{dx}$

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51. If $(\tan^{-1} x)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

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53. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is

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54. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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55. If $x \in \left(\frac{1}{\sqrt{2}}1\right)$. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) w.r.t. \cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

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56. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x - \frac{1}{2}$.

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57.

If $y=f(x^3), z=g(x^5), f'(x)=\tan x,$ and $g'(x)=\sec x,$ then find t

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58. Find the derivative of f(tan x) w.r.t. g (sec x) at $x=\frac{\pi}{4}$, where f'(1)=2 and $g'(\sqrt{2})=4.$

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59. If $y=x^3\log_e x,$ then find y'' and y'''.

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60. If $y=\sin(\sin x)$, prove that $\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+y\cos^2 x=0$.

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61. If $x = a(t - \sin t), y = a(1 - \cos t)$ then find $\frac{d^2y}{dx^2}$.

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62. 112. If f(x)= $\begin{bmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{bmatrix}$ find $f'(x)$

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63. $\begin{vmatrix} x^2 + a^2 & ab & ac \\ ab & x^2 + b^2 & bc \\ ac & bc & x^2 + c^2 \end{vmatrix} =$

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64. Let $f(x) = |x^3 \sin x \cos x6 - 10pp^2p^3|, wherep$ is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is p (b) $p - p^3$ $p + p^3$ (d) independent of p

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65. if y= sin mx, them the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} \text{ Where } y_n = \frac{d^n y}{dx^n} \text{ is}$$

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66. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$ then find constant term and coefficient of x

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68. If $y = f(x) = x^3 + x^5$ and g is the inverse of f find $g'(2)$

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69. Let $f(x) = \exp(x^3 + x^2 + x)$ for any real number and let $g(x)$ be the inverse function of $f(x)$ then $g'(e^3)$

A. $\frac{1}{6e^3}$

B. $\frac{1}{6}$

C. $\frac{1}{34e^{19}}$

D. 6

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70. Let f be a twice differentiable function such that $f''(x) = -f(x), and f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2.$ Find $h(10)$ if $h(5) = 11$

A. 0

B. 9

C. 11

D. None of these

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71. if $y = \frac{(a-x)\sqrt{a-x}-(b-x)\sqrt{x-b}}{\sqrt{a-x+\sqrt{x-b}}}$, then $\frac{dy}{dx}$ wherever it is defined is

A. $\frac{x-(a-b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x+(a+b)}{\sqrt{(a-x)(x-b)}}$

C. $\frac{2x-(a+b)}{\sqrt{(a-x)(x-b)}}$

D. None of these

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72. If $x^2 + y^2 = R^2 (where R > 0) and k = \frac{y^n}{(1 + y^2)^3}$ then find k in terms of R alone.

A. $\frac{1}{R}$

B. $-\frac{1}{R}$

C. R

D. $\frac{1}{2R}$

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73. Let $f(x) = x + \sin x$. Suppose g denotes the inverse function of f. The value of $g'\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ has the value equal to

A. (a)$2 + \sqrt{2}$

B. (b)$\sqrt{2} - 2$

C. (c)$2 - \sqrt{2}$

D. (d)$2\sqrt{2}$

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74. If $e^{f(x)} = \log x$ and g(x) is the inverse function of f(x), then $g'(x)$ is

A. e^{e^x}

B. e^x

C. $e^{e^x + x}$

D. None of these

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75. Given: $f(x) = 4x^3 - 6x^2 \cos 2a + 3x \sin 2a \sin 6a + \sqrt{\ln(2a - a^2)}$

then

A. $af'\left(\frac{1}{2}\right) < 0$

B. $af'\left(\frac{1}{2}\right) \leq 0$

C. $af'\left(\frac{1}{2}\right) > 0$

D. None of these

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76. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that f''(x) -2f'(x)-15f(x)=0 for all x. Then the product ab is

A. 15

B. -15

C. 10

D. 16

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77. Which of the following is not true $\log(1+x) < x$ for $x > 0$

A. $e^y = xy' + 1$

B. $y' = -\frac{1}{(x-1)}$

C. $y' + e^y = 0$

D. $y' = e^y$

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78. If $x^p y^q = (x+y)^{(p+q)}$ then $\frac{dy}{dx} = ?$

A. independent of p

B. independent of q

C. dependent both p and q

D. $\frac{y}{x}$

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79. Two functions f & g have first & second derivatives at x=0 & satisfy the relations,

$f(0) = \frac{2}{g(0)}, f'(0) = 2g'(0) = 4g(0), g(0) = 5\text{ f}(0) = 6f(0) = 3$ then-

A. If $h(x) = \frac{f(x)}{g(x)}$, then $h'(0) = 15$

B. If $k(x) = f(x).\, g(x).\, \sin x$ then $k'(0)d = 2$

C. $\lim_{x \to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$

D. None of above

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80. $f(x) = \left|x^2 - 3x\right|x\left| + 2\right|$. Then which of the following is/are true ?

A. $f'(x) = 2x - 3$ for $x \in (0, 1) \cup (2, \infty)$

B. $f'(x) = 2x + 3$ for $x \in (-\infty, -2) \cup (-10)$

C. $f'(x) = -2x - 3$ for $x \in (-2, -1)$

D. None of the above

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81. Consider $f(x) = \frac{x}{x^2 - 1}$ and $g(x) = f''(x)$ Statement I Graph of $g(x)$ is cancave up for $x > 1$. Statement II $\frac{d^n}{dx^n} f(x) = \frac{(-1)^n n!}{2} \left\{ \frac{1}{(x-1)^{n+1}} + \frac{1}{(x+1)^{n+1}} \right\} n \in N$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is correct.

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82. Statement 1: If differentiable function $f(x)$ satisfies the relation $f(x)+f(x-2)=0\,\forall x \in R,$ and if $\left(\frac{d}{dx}f(x)\right)_{x=a}=b, then \left(\frac{d}{dx}f(x)\right)_{x=a+4000}=b$. Statement 2: $f(x)$ is a periodic function with period 4.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is correct.

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83. Let $f(x) = x[x]$, where [*] denotes the greatest integer function, when x is not an integer then find the value of $f'(x)$

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84. f(x) is a polynomial of degree

A. 2

B. 3

C. 4

D. 5

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85. The moment when A is at $(0, 0)$ and B is at $(1, 2)$. The derivative $\frac{dy}{dx}$ of line AB is

A. $\frac{1}{9}$

B. $\frac{1}{7}$

C. $\frac{1}{5}$

D. None of these

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86. The moment when A is at $(1, 2)$ and $Bisat(0, 0)$. The derivative $\frac{dx_B}{dx_A}$, is

A. 16

B. 8

C. 9

D. 2

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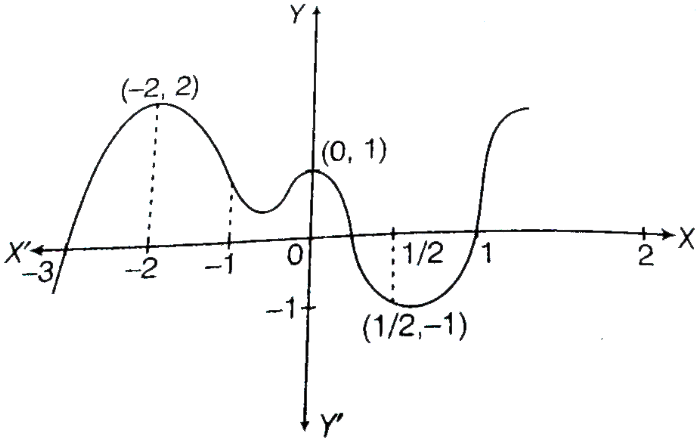
87. If $p''(x)$ has real roots α, β, γ. Then , $[\alpha]+[\beta]+[\gamma]$ is



A. -2

B. -3

C. -1

D. 0

Answer: -2

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88. Match the Column I with Column II and mark the correct option from the given below.

	Column I	Column II	
(i)	If $f'(x)=\sqrt{3x^2+6}$ and $y=f(x^3)$ then at $x=1$, $\frac{dy}{dx}$ is	p.	-2
(ii)	If f is a differentiable function such that $f(xy)=f(x)+f(y): x, y\in R$, then $f(e)+f(1/e)$ is	q.	-1
(iii)	If f is a twice differentiable function such that $f''(x)=-f(x)$ and $f'(x)=g(x)$. If $h(x)=[f(x)]^2+[g(x)]^2$ and $h(5)=9$, then $h(10)$ is	r.	0
(iv)	$y=\tan^{-1}(\cot x)+\cot^{-1}(\tan x)$, $\frac{\pi}{2}<x<\pi$, then $\frac{dy}{dx}$ is	s.	9

A.

i	ii	iii	iv
s	r	s	p

B.

i	ii	iii	iv
p	q	r	s

C.

	i	ii	iii	iv
()	q	p	r	r

D.

i	ii	iii	iv
s	p	q	q

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89. Match the Column I with Column II and mark the correct option from the given below.

	Column I		Column II
(i)	If $y = \cos^{-1}(\cos x)$, then y' at $x = 5$ is equal to	p.	-1
(ii)	For the function $f(x) = \log_e \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, if $\frac{dy}{dx} = \sec x + p$, then p is equal to	q.	0
(iii)	The derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ at $x = -1$ is	r.	1/2
(iv)	The derivative of $\frac{\log\|x\|}{x}$ at $x = -1$ is	s.	1

A.

i	ii	iii	iv
p	q	r	s

B.

i	ii	iii	iv
q	p	r	s

C.

i	ii	iii	iv
s	r	q	p

D.

i	ii	iii	iv
r	s	p	q

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90. If $y=\sqrt{(x-\sin x)+\sqrt{x-\sin x}+\ldots}$, then $\left|\left|\frac{dx}{dy}\right|^2_{x=\frac{\pi}{2}}-2\pi\right|=$ ....

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91. Let $f(x)=\int_{-2}^{x}e^{(1+t)^2}dt$ and $g(x)=f(h(x))$,where $h(x)$ is defined for all $x\in R$. If $g'(2)=e^4$ and $h'(2)=1$ then absolute value of sum of all possible values of $h(2)$ is ___

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92. If $f(x)=\cos\left\{\frac{\pi}{2}[x]-x^3\right\}, 1<x<2$ and $[x]=$ the greatest integer $\leq x$, then find $f'\left(\sqrt[3]{\frac{\pi}{2}}\right)$

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93. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2} x \cos x^3 \cos ecx^2$ $\frac{2}{3} \sin x^3 \sec x^2 \tan x$ (d) none of these

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94.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3) \text{ for all } x \in R.$

The value of f(1) is

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95. Let

$f(x) = x^2 + x g'(1) + g''(2) \text{ and } g(x) = x^2 + x f'(2) + f''(3).$

Then

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96. If $y = \frac{\sin x}{1 + \frac{\cos x}{1 + \frac{\sin x}{1 + \frac{\cos x}{1 + \ddots \to \infty}}}}$, prove that $\frac{dy}{dx} = \frac{(1+y)\cos x + y \sin x}{1 + 2y + \cos x - \sin x}$

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97. If $3\sqrt{3\sqrt{x - \frac{1}{3\sqrt{x}}}} = 2, then 3\sqrt{x}\left(\frac{1}{3\sqrt{x}}\right)$=__________

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98. If

$y = \tan^{-1}\left(\frac{1}{x^2 + x + 1}\right) + \tan^{-1}\left(\frac{1}{x^2 + 3x + 3}\right) + \tan^{-1}\left(\frac{1}{x^2 + 5x + 7}\right)$

to nterms, show that $\frac{dy}{dx} = \frac{1}{(x+n)^2 + 1} - \frac{1}{x^2 + 1}$

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99. if $f(\theta) = \cos\theta_1 . \cos\theta_2 . \cos\theta_3 . \dots . \cos\theta_n,$ show that

$\{\tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \dots . + \tan\theta_n\} = -\left\{\frac{f'(\theta)}{f(\theta)}\right\},$

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100. Find the sum of $\sin x + 3\sin 3x + 5\sin 5x + \dots\dots\dots. + (2k-1)\sin(2k-1)x.$

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101. Find the sum of the series $Summition_{r=1}^{n} rx^{r-1}$ using calculus .

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102. Show that 1/(x+1)+2/(x^2+1)+4/x^4+1)+.....+2^n/(x^2^n+1)= /(x-1)-2^(n+1)/(x^2(n+1)$-1)$

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103.

$f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x,$ then $\frac{d}{dx}\{f_n(x)\}$ is

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104. If $y^3 - y = 2x$, prove that $\frac{d^2y}{dx^2} = -\frac{24y}{(3y^2 - 1)^3}$.

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105. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ then $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = ky$, then find the value of k.

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106. Let

$y = 1 + \frac{a_1}{x - a_1} + \frac{a_2 x}{(x - a_1)(x - a_2)} + \frac{a_3 x^2}{(x - a_1)(x - a_2)(x - a_3)} + \dots$

Find $\frac{dy}{dx}$

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107. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then find $\frac{dy}{dx}$

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108. let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in AP, then show that $f'(a_1), f'(a_2), f'(a_3)$ are in AP.

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109. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = xf(x)$, then find $\frac{dy}{dx}$ at $x = 1$

.

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EXAMPLE

1. Differentiate the following functions w.r.t x using first principle.

$f(x) = e^{2x}$

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2. If $(a+bx)e^{\frac{y}{x}} = x$, Prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

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3. Show that the function $y = f(x)$ defined by the parametric equations $x = e^t \sin(t), y = e^t . \cos(t),$ satisfies the relation $y' '(x+y)^2 = 2(xy' - y)$

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SOLVED EXAMPLES

1. Let $f(x)$ be real valued differentiable function not identically zero such that $f\left(x+y^{2n+1}\right. = f(x) + \{f(y)\}^{2n+1}, n \in N$ and x, y are any real numbers and $f'(0) \leq 0$. Find the values of $f(5)$ and $f'(10)$

A. 3, 2

B. 0, 1

C. 1, 5

D. 5, 1

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2. Let $f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$ and $f(0) = b$. Find $f''(x)$ (where$y$ is independent of x), when $f(x)$ is differentiable.

A. 0

B. 1

C. a

D. None of these

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3. The functions $u = e^x . \sin x$ and $v = e^x . \cos x$ satisfy the equation

A. a) $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$

B. b) $v\frac{du}{dx} + u\frac{dv}{dx} = u^2 + v^2$

C. c) $\frac{du}{dx} + \frac{dv}{dx} = 2v$

D. d) $\frac{du}{dx} + \frac{dv}{dx} = 2u$

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4. If f(x)= 18x+11 then find the value of $f''(x)$, is

A. $1/18$

B. $1/3$

C. $1/9$

D. $1/6$

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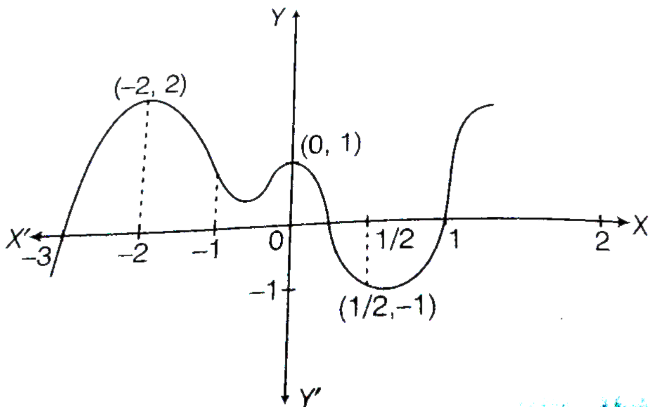
5. The minimum number of real roots of equation $(p''(x))^2 + p'(x).\, p'''(x) = 0$



A. 5

B. 7

C. 6

D. 4

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6. Find a, b, c, and d, where $f(x) = (ax+b)\cos x + (cx+d)\sin x$ and $f'(x) = x\cos x$ is identity in x.

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7. If for all x, y the function f is defined by $f(x)+f(y)+f(x).\ f(y) =$ and $f(x) > 0$. Then show $f'(x) = 0$ when $f'(x) = 0$ is differentiable.

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Exercise (More Than One Correct Option Type Questions)

1. If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$, then $\frac{dy}{dx}$ equals.

A. $2\sec x(\sec x - \tan x)$

B. $-2\sec x(\sec x - \tan x)^2$

C. $2\sec x(\sec x - \tan x)^2$

D. $-2\sec x(\sec x + \tan x)^2$

Answer: B

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2. If $y = \frac{x^4 + x^2 + 1}{x^2 + x + 1}$ then $\frac{dy}{dx} = ax + b$, find a and b

A. $a = 2, b = 1$

B. $a = -2, b = 1$

C. $a = 2, b = -1$

D. $a = -2, b = -1$

Answer: C

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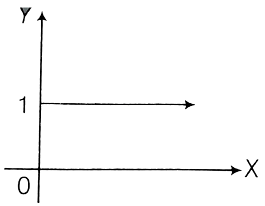
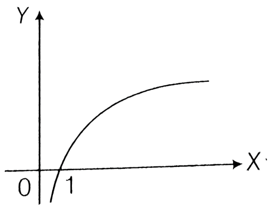
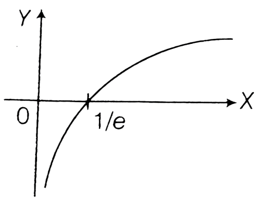
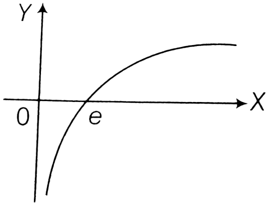
3. Which of the following could be the sketch graph of $y = \frac{d(x \ln x)}{dx}$

A. 

B. 

C. 

D. 

Answer: C

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4. Let $f(x) = x + 3\ln(x-2) \& g(x) = x + 5\ln(x-1),$ then the set of x satisfying the inequality `f^(prime)(x)

A. $(2, 7/2)$

B. $(1, 2) \cup (7/2, \infty)$

C. $(2, \infty)$

D. $7/2, \infty$

Answer: D

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5. If $\cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a, then \frac{dy}{dx} =$

A. $-\frac{x}{y}$

B. $-\frac{y}{x}$

C. $\frac{y}{x}$

D. $\frac{x}{y}$

Answer: C

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6. If $f(x) = |x|^{|\sin x|}$,then $f'\left(\frac{\pi}{4}\right)$ equals

A. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

B. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

C. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

D. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

Answer: A

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7. $y = \frac{x}{a + \frac{x}{b + \frac{x}{a + \frac{x}{b + \ldots \infty}}}}, \frac{dy}{dx} = \frac{b}{a(b + 2y)}$

A. $\frac{a}{ab + 2ay}$

B. $\frac{b}{ab + 2ay}$

C. $\frac{a}{ab + 2by}$

D. $\frac{b}{ab + 2ay}$

Answer: D

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8. If $y = x^{x^2}$, then $\frac{dy}{dx}$ equals

A. $2 \log x . x^2$

B. $(2\log x+1).\, x^{x^2}$

C. $(2\log x+1).\, x^{x^2}+1$

D. $x^{x^2+1}.\, (\log(ex^2))$

Answer: D

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9. If $x\sqrt{1+y}+y\sqrt{1+x}=0,\quad \text{prove that}\ \frac{dy}{dx}=-\frac{1}{(x+1)^2}.$

A. $\frac{1}{(1+x)^2}$

B. $\frac{-1}{(1+x)^2}$

C. $\frac{-1}{(1+x)}+\frac{1}{(1+x)^2}$

D. None of these

Answer: B

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10. If $x^2e^y + 2xye^x + 13 = 0$ then $\frac{dy}{dx} =$

A. $\frac{-2xe^{y-x} + 2y(x-1)}{x(xe^{y-x}+2)}$

B. $\left[\frac{2xe^{y-x} + 2y(x-1)}{x(xe^{y-x}+2)}\right]$

C. $\frac{2xe^{y-x} + 2y(x-1)}{x(xe^{y-x}+2)}$

D. None of these

Answer: B

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11. If $x = e^{y+e^{y+e^{y+\dots\infty}}}$, $x > 0$, then $\frac{dy}{dx}$ is equal to

A. $\frac{x}{1+x}$

B. $\frac{1+x}{x}$

C. $\frac{1-x}{x}$

D. $\frac{1}{x}$

Answer: C

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12. Let g be the inverse function of f and $f'(x) = \frac{x^{10}}{1+x^2}$. If $g(2) = a$ then $g'(2)$ is equal to

A. $\frac{5}{2^{10}}$

B. $\frac{1+a^2}{a^{10}}$

C. $\frac{a^{10}}{1+a^2}$

D. $\frac{1+a^{10}}{a^2}$

Answer: B

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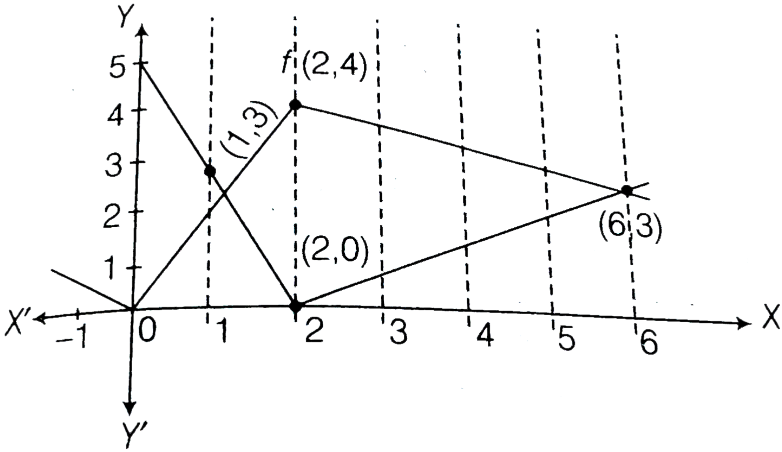
13. If f and g are the function whose graphs are as shown, let $u(x) = f(g(x)), w(x) = g(g(x))$



Then the value of $u'(1) + w'(1)$ is

A. $-\frac{1}{2}$

B. $-\frac{3}{2}$

C. $-\frac{5}{4}$

D. does not exist

Answer: B

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14. $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all real x and $f(5) = 2 = f'(5) then f^2(10) + g^2(10)$ is

A. 2

B. 4

C. 8

D. None of these

Answer: C

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15. If $y = (f_0 f_0 f)(x)$ and $f(0) = 0, f'(0) = 2$ then $y'(0)$ is equal to

A. 6

B. 7

C. 8

D. 9

Answer: C

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16. If $y^2 = P(x)$ is a polynomial of degree 3, then $2\left(\frac{d}{dx}\right)\left(y^2\frac{d^2y}{dx^2}\right)$ is equal to $P^x + P'(x)$ (b) $P^x \dot{P}^x$ $P(x)\dot{P}^x$ (d) a constant

A. $p'''(x).\,p'(x)$

B. $p''(x).\,p'''(x)$

C. $p(x).\,p'''(x)$

D. None of these

Answer: C

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17. If $y = f(x)$ and $x = g(y)$ are inverse functions of each other, then

A. $g''(y) = \frac{1}{f''(x)}$

B. $g''(y) = \frac{f''(x)}{(f''(x))^3}$

C. $g''(y) = \frac{f'(x)}{(f'(x))^3}$

D. None of these

Answer: B

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18. If y is a function of x then $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$. If x is a function of y then the equation becomes

A. $\frac{d^2x}{dy^2} - x\frac{dx}{dy} = 0$

B. $\frac{d^2x}{dy^2} + y\left(\frac{dx}{dy}\right)^2 = 0$

C. $\frac{d^2x}{dy^2} - y\left(\frac{dx}{dy}\right)^2 = 0$

D. $\frac{d^2x}{dy^2} - x\left(\frac{dx}{dy}\right)^2 = 0$

Answer: C

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19. Leg $g(x) = \ln(f(x))$, whre $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then, for $N = 1, 2, 3,$

$g^{N+\frac{1}{2}} - g^{\frac{1}{2}} =$

A. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

Answer: A

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20. If the functions $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$, the value of g'(1) is ....

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21. Let $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$ then the value of $\frac{d}{d(\tan\theta)}f(\theta)$ is

A. 1

B. 2

C. 3

D. 4

Answer: A

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22. If $y=\log_{\sin x}(\tan x)$, then $\frac{dy}{dx}$ at $x=\frac{1}{4}$ is equal to

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. None of these

Answer: C

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23. If $y = \sum_{r=1}^{x} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{1+x^2}$

B. $\frac{1}{1+(1+x)^2}$

C. 0

D. None of these

Answer: B

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24. If $y = \left(\frac{\sin^{-1}(\sin\alpha\sin x)}{1-\cos\alpha\sin x}\right)$, then $y'(0)$ is equal to

A. 1

B. $2\tan\alpha$

C. $\left(\frac{1}{2}\right)\tan\alpha$

D. $\sin\alpha$

Answer: D

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25. If $f(x)=\cot^{-1}\left(\frac{x^{x}-x^{-x}}{2}\right)$ then f'(1) equals

A. -1

B. 1

C. $\log_{e}2$

D. $-\log_{e}2$

Answer: A

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26. The function $f(x)=e^x+x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)\cdot$ The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{1n2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. None of these

Answer: B

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27. If $f''(x)=-f(x)$ and $g(x)=f'(x)$ and $F(x)=\left(f\left(\frac{x}{2}\right)\right)^2+\left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5)=5,$ then $F(10)$ is (a)5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

Answer: A

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28. about to only mathematics

A. $n^2(y^2 - 4)$

B. $n^2(4 - y^2)$

C. $n^2(y^2 + 4)$

D. None of these

Answer: C

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29. If $x = f(t)\cos t - f'(t)\sin t$ and $y = f(t)\sin t + f'(t)\cos t$, then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = f(t) - f(t)$ (b) $\{f(t) - f(t)\}^2$ (c) $\{f(t) + f(t)\}^2$

(d) none of these

A. $f(t) - f''(t)$

B. $[f(t) - f''(t)]^2$

C. $[f(t) + f''(t)]^2$

D. None of these

Answer: C

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30. If $y = at^2 + 2bt - c$ and $t = ax^2 + 2bx + c$, then $\frac{d^3y}{dx^3}$ equals

A. $24a^2(at + b)$

B. $24a(ax + b)^2$

C. $24a(at + b)^2$

D. $24a^2(ax + 0b)$

Answer: D

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31. Differential coefficient of $\left(x^{\frac{l+m}{m-n}}\right)^{1/(n-l)} \cdot \left(x^{\frac{m+n}{n-l}}\right)^{1/(l-m)} \cdot \left(x^{\frac{n+l}{l-m}}\right)^{1/(m-n)}$ wrt x is

A. 1

B. 0

C. -1

D. x^{lmn}

Answer: B

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32. if $y=(A+Bx)e^{mx}+(m-1)^{-1}e^{x}$ then $\frac{d^2y}{dx^2}-2m\frac{dy}{dx}+m^2y$ is equal to:

A. e^{x}

B. e^{mx}

C. e^{-mx}

D. $e^{(1-m)x}$

Answer: A

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33. If $f(x)=\frac{\cos^{-1}1}{\sqrt{13}}(2\cos x-3\sin x)+\frac{\sin^{-1}1}{\sqrt{13}}x(2\cos x+3\sin x)w\dot{r}\dot{t}\sqrt{1+x^2}$, then find $\frac{df(x)}{dx}$ at $x=\frac{3}{4}$.

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{10}{3}$

D. 0

Answer: C

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34. If $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2x - 4}}$ then the value of 10 $f'(102^{+})$, is

A. -1

B. 0

C. 1

D. does not exist

Answer: C

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35. Let $y = 1n(1+\cos x)^2$. Then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{\frac{y}{2}}}$ equal (b) $\frac{2}{1+\cos x}$ $\frac{4}{1+\cos x}$ (d) $\frac{-4}{(1+\cos \mathrm{x})^2}$

A. 0

B. $\frac{2}{1+\cos x}$

C. $\frac{4}{(1+\cos x)}$

D. $\frac{-4}{(1+\cos x)^2}$

Answer: A

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36. If $f(x) = \frac{a+\sqrt{a^2-x^2}+x}{\sqrt{a^2-x^2}+a-x}$ where $a > 0$ then $f'(0)$ has the value equal to

A. $\sqrt{a}$

B. a

C. $\frac{1}{\sqrt{a}}$

D. $\frac{1}{a}$

Answer: D

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37. Let $\frac{u}{x} and v(x)$ be differentiable functions such that $\frac{u(x)}{v(x)} = 7. I \frac{f(u'(x))}{v'(x)} = p and \left(\frac{u(x)}{v(x)}\right) = q, then \frac{p+q}{p-q}$ has the value of to 1 (b) 0 (c) 7 (d) -7

A. 1

B. 0

C. 7

D. -7

Answer: A

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38. If f(x) $= |\log_e|x||$, then f'(x) equals

A. $\frac{1}{|x|}, x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

Answer: B

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39. If $f(x)$ is given by

$f(x) = (\cos x + i \sin x)(\cos 3x + i \sin 3x)$.........

.......$[\cos(2n-1)x + i \sin(2n-1)x]$,

then f''(x) is equal to

A. $n^3 f(x)$

B. $-n^4 f(x)$

C. $-n^2 f(x)$

D. $n^4 f(x)$

Answer: B

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40. Let $f(x) = x^n$, n being a non-negative integer, The value of n for which the equality $f'(x+y) = f'(x) + f'(y)$ is valid for all $x, y > 0,$ is

A. 0,1

B. 2

C. 2,4

D. None of these

Answer: B

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41. If $f(x) = \sin\left\{\frac{\pi}{3}[x] - x^2\right\}$ for $2 < x < 3$ and [x] denotes the greatest integer less than or equal to x, then $f'(\sqrt{\pi/3})$ is equal to

A. $\sqrt{\pi/3}$

B. $-\sqrt{\pi/3}$

C. $-\sqrt{\pi}$

D. None of these

Answer: B

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42. The function $u = e^x \sin x; v = e^x \cos x$ satisfy the equation a. $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ b. $\frac{d^2u}{dx^2} = 2v$ c. $\frac{d^2v}{dx^2} = -2u$ d. $\frac{du}{dx} + \frac{dv}{dx} = 2v$

A. $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$

B. $v\frac{d^2u}{dx^2} = 2v$

C. $\frac{d^2v}{dx^2} = -2u$

D. All of these

Answer: D

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43. If $f(x) = \log_{x^2}(\log x)$,then f '(x)at x= e is

A. e

B. $-e$

C. e^2

D. e^{-1}

Answer: D

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44. Let f be a differentiable function satisfying

$[f(x)]^n = f(nx) \text{ for all } x \in R.$

Then, $f'(x)f(nx)$

A. $f(x)$

B. 0

C. $f(x)f'(nx)$

D. None of these

Answer: C

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45. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then|f'(-3)|$ equals_______.

A. 4

B. 2

C. -2

D. 0

Answer: C

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46.

If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$, then prove that $\frac{dy}{dx}=\frac{y^2-}{2y^3-2}$

A. $\frac{y+x}{y^2-2x}$

B. $\frac{y^3-x}{2y^2-2xy-1}$

C. $\frac{y^3+x}{2y^2-x}$

D. None of these

Answer: D

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47. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 0

D. None of these

Answer: D

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48. Let

$f(x) = x^2 + xg^2(1) + g''(2)$ and $g(x) = f(1).x^2 + xf'(x) + f''(x)$,

then find $f(x)$ and $g(x)$.

A. $f'(1) = 4 + f'(2)$

B. $g'(2) = 8 + g'(10$

C. $g''(2) + f''(3) = 4$

D. All of the above

Answer: D

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49. if $f(x) = x^n$ then the value of

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + - - - + \frac{(-1)^n f''^{--n} \times (1)}{n!}$$

A. 1

B. 2^n

C. 2^{n-1}

D. 0

Answer: D

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50. If $y + \log(1 + x) = 0$ which of the following is true?

A. $e^{y} = xy' - 1$

B. $y' = -\frac{1}{(x+1)}$

C. $y' + e^{y} = 0$

D. $y' = e^{y}$

Answer: B::C

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51. If $y = 2^{3^{x}}$, then y' equals

A. $3^{x}. \log 3. \log 2$

B. $y. (. \log 2y). \log 3. \log 2$

C. $2^{3^{x}}.3^{x}. \log 6$

D. $2^{3^{x}}.3^{x}. \log 3. \log 2$

Answer: B::D

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52. If g is the inverse of f and $f(x) = x^2 + 3x - 3, (x > 0)$. then $g'(1)$ equals

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53. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0$ and $y(1) = 1$, then

A. $y'(1) = \frac{4}{3}$

B. $y''(1) = -\frac{1}{3}$

C. $y''(1) = -8\frac{22}{27}$

D. $y'(1) = \frac{2}{3}$

Answer: A::C

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54. Let $y = \sqrt{x + \sqrt{x + \sqrt{x + \infty}}}$, $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2y-1}$ (b) $\frac{x}{x+2y}$ (c) $\frac{1}{\sqrt{1+4x}}$ (d) $\frac{y}{2x+y}$

A. $\frac{1}{2y-1}$

B. $\frac{x}{x+2y}$

C. $\frac{1}{\sqrt{1-4x}}$

D. $\frac{y}{2x+y}$

Answer: A::D

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55. If $y = x^{(\ln x)^{\ln(\ln x)}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{y}{x}\left(\ln x^{\ln x - 1} + 2\ln + (\ln x)\right)$

B. $\frac{y}{x}(\ln x)^{\ln(\ln x)}(\ln(\ln x) + 1)$

C. $\frac{y}{x \ln x}\left((nx)^2 + 2 \ln x (\ln x)\right)$

D. $\frac{y}{x} \cdot \frac{\ln y}{\ln x}(2 \ln(\ln x) + 1)$

Answer: D

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56. Which of the following functions are not derivable at $x = 0$?

A. $f(x) = \sin^{-1} 2x\sqrt{1 - x^2}$

B. $g(x) = \sin^{-1}\left(\frac{2^x + 1}{1 + 4^x}\right)$

C. $h(x) = \sin^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$

D. $k(x) = \sin^{-1}(\cos x)$

Answer: B::C::D

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57. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x-1}}}{\sqrt{x-1} - 1}x$. Then

A. $f'(10) = 1$

B. $f'(3/2) = -1$

C. domain of $f(x)$ is $x \leq 1$

D. None of these

Answer: A::B

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58. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ is equal to

A. $-\frac{2^y}{2^x}$

B. $\frac{1}{1-2^x}$

C. $1 - 2^y$

D. $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

Answer: A::B::C::D

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59. For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds?

A. If $f(x) > 0$ for all real $x \not\Rightarrow f'(x) > 0$

B. If $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

C. If $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

D. If $f'(x) > 0$ for all real $x \not\Rightarrow f(x) > 0$

Answer: A::C

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60. If $\sqrt{y+x} + \sqrt{y-x} = c$, where $c \neq 0$, then $\frac{dy}{dx}$ has the value equal to

A. $\frac{2x}{e^2}$

B. $\frac{x}{y+\sqrt{y^2-x^2}}$

C. $\frac{y-\sqrt{y^2-x^2}}{x}$

D. $\frac{c^2}{2y}$

Answer: C

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61. If $y=\tan x \tan 2x \tan 3x, (\sin 12x \neq 0)$ then $\frac{dy}{dx}$ has the value equal to

A.

$3\sec^2 3x \tan x \tan 2x + \sec^2 x \tan 2x \tan 3x + 2\sec^2 2x \tan 3x \tan x$

B. $2y(\cos ec2x + 2\cos ec4x + 3\cos ec6x)$

C. $3\sec^2 3x - 2\sec^2 2x - \sec^2 x$

D. $\sec^2 x + 2\sec^2 2x + 3\sec^2 3x$

Answer: A::B::C

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Exercise (Statement I And Ii Type Questions)

1. Consider $f(x) = \frac{x}{x^2 - 1}$

Statement I Graph of $f(x)$ is concave up for $x > 1$.

Statement II If $f(x)$ is concave up then $f''(x) > 0$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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2. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then Statement I The value of $f(2) = \sin^{-1}\left(\frac{4}{5}\right)$. Statement II $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = -2$, for $x < 1$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: C

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3. Statement I if $f(0) = a, f'(0) = b, g(0) = 0, (fog)'(0) = c$ then $g'(0) = \frac{c}{b}$. Statement II $(f(g(x)))' = f'(g(x)).g'(x)$, for all n

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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4. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is constinous, $g(0) = 0$, $g'(0) = 0, g''(0) = 0$ and $f(x) = g(x)\sin x$.

Statement I $\lim_{x \to 0} (g(x)\cot x - g(0)\cos ecx) = f''(0)$

Statement II $f'(0) = g'(0)$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: B

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5. Statement I If $y = \sin^{-1}(3x - 4x^3)$, then $\frac{dy}{dx} = \frac{3}{\sqrt{1 - x^2}}$ only when $\frac{-1}{2} \leq x < \frac{1}{2}/.$

Statement II $\sin^{-1}(3x - 4x^3)$

$$= \begin{cases} -\pi - 3\sin^{-1}x & -1 \le x \le -\frac{1}{2} \\ 3\sin^{-1}x & -\frac{1}{2} \le x \le \frac{1}{2} \\ \pi - 3\sin^{-1}x & \frac{1}{2} \le x \le 1 \end{cases}$$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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6. If $y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$, then

Statement I $\frac{dy}{dx} = \frac{2}{1 + x^2}$ for $x \in R$

Statement II $\cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) = \begin{cases} 2\tan^{-1}x & x \ge 0 \\ -2\tan^{-1}x & x < 0 \end{cases}$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: D

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7. If $y = x + [x],$ then

Statement I $\frac{dy}{dx} = 1$ for all $x \in R$

Statement II

$\frac{d([x])}{dx} = \{(\quad 0, \qquad x \notin \text{Integer}), (\text{does not exist} \qquad x. \in \text{integer}$

):}

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: D

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8. Statement I If $f(x)$ is a continuous function defined from R to Q and $f(5) = 3$, then differential coefficient of $f(x) w.r.t. x$ will be 0

Statement II Differentation of constant functions is always zero.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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9. Statement I Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right) w.r.t. \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0 < x < 1$.

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $-1 \leq x \leq 1$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: C

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10. Suppose the function $f(x)$ satisfies the relation $f(x+y^3)=f(x)+f(y^3)\,\dot{\forall} x, y \in R$ and is differentiable for all x. Statement 1: If $f'(2)=a, then f'(-2)=a$ Statement 2: $f(x)$ is an odd function.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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Exercise (Passage Based Questions)

1. Let $\frac{f(x+y)-f(x)}{2}=\frac{f(y)-1}{2}+xy$, for all $x, y \in R, f(x)$ is differentiable and $f'(0)=1$. Domain of $\log(f(x))$, is

A. R^{+}

B. $R-\{0\}$

C. R

D. R^{-}

Answer: C

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2. Let $\dfrac{f(x+y)-f(x)}{2}=\dfrac{f(y)-1}{2}+xy$, for all $x, y \in R, f(x)$ is differentiable and $f'(0)=1.$

Range of $y=\log_{3/4}(f(x))$ is

A. $(-\infty, 1)$

B. $[3/4, \infty$

C. $(-\infty, \infty)$

D. R

Answer: A

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3. Let $\dfrac{f(x+y)-f(x)}{2}=\dfrac{f(y)-1}{2}+xy$, for all $x, y \in R, f(x)$ is differentiable and $f'(0)=1.$ Let $g(x)$ be a derivable function at $x=0$ and follows the function rule $g\left(\dfrac{x+y}{k}\right)=\dfrac{g(x)+g(y)}{k}, k \in R, k \neq 0, 2$ and $g'(0)-\lambda g'(0) \neq 0.$

If the graphs of $y = f(x)$ and $y = g(x)$ intersect in coincident points then λ can take values

A. -3

B. 1

C. -1

D. 4

Answer: C

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4. Find the derivative of log(sin(logx)).

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5. Left hand derivative and right hand derivative of a function $f(x)$ at a point $x = a$ are defined as

$$f'(a^-) = \lim_{h \to 0^+} \frac{f(a) - f(a-h)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

$$\text{and } f'(a^+) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(a) - f(a+h)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(a) - f(x)}{a - x}$$ respectively.

Let f be a twice differentiable function. We also know that derivative of a even function is odd function and derivative of an odd function is even function.

If f is even function, which of the following is right hand derivative of f' at $x = a$?

A. (a) $\lim_{h \to 0^-} \frac{f'(a) + f'(-a+h)}{-h}$

B. (b) $\lim_{h \to 0^+} \frac{f'(a) + f'(-a-h)}{h}$

C. (c) $\lim_{h \to 0^-} \frac{-f'(-a) + f'(-a-h)}{-h}$

D. (d) $\lim_{h \to 0^+} \frac{f'(a) + f'(-a+h)}{-h}$

Answer: A

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6. Find $\frac{dy}{dx}$ if $ax + by + c = 0$

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7. If $f(x) = \sin^{-1}(3x - 4x^3)$. Then answer the following

The value of f'(0), is

A. -3

B. 3

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: B

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8. If $f(x) = \sin^{-1}(3x - 4x^3)$. Then answer the following

The value of $f'\left(\frac{1}{\sqrt{2}}\right)$, is

A. -3

B. 3

C. $-3\sqrt{2}$

D. $3\sqrt{2}$

Answer: C

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9. Let the derivative of $f(x)$ be defined as $D^* f(x) = \lim_{h \to 0} \frac{f^2 x + h - f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$.

If $u = f(x), v = g(x)$, then the value of $D^* (u.\ v)$ is

A. (a)$(D^* u)v + (D^* v)u$

B. (b)$u^2(D^* v) + v^2(D^* u)$

C. (c)$D^{*}u + D^{*}v$

D. (d)$uvD^{*}(u+v)$

Answer: B

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10. Let the derivative of $f(x)$ be defined as $D^{*}f(x) = \lim_{h \to 0} \frac{f^{2}x + h - f^{2}(x)}{h}$, where $f^{2}(x) = \{f(x)\}^{2}$.

If $u = f(x), v = g(x)$, then the value of $D^{*}\left(\frac{u}{v}\right)$ is.

A. $\frac{u^{2}(D^{*}v) - v^{2}(D^{*}u)}{v^{4}}$

B. $\frac{u(D^{*}v) - v(D^{*}u)}{v^{2}}$

C. $\frac{v^{2}(D^{*}u) - u^{2}(D^{*}v)}{v^{4}}$

D. $\frac{v(D^{*}u) - u(D^{*}v)}{v^{2}}$

Answer: C

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11. A curve is represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then The relation between the parameter 't' and the angle a between the tangent to the given curve andthe x-axis is given by, 't' equals

A. $\frac{\pi}{2} - \alpha$

B. $\frac{\pi}{4} + \alpha$

C. $\alpha - \frac{\pi}{4}$

D. $\frac{\pi}{4} - \alpha$

Answer: C

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12. A curve is represented paramtrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then

The value of $\frac{d^2y}{dx^2}$ at the point where $t = 0$ is

A. 1

B. 2

C. -2

D. 3

Answer: B

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13. A curve is represented parametrically by the equations $x = e^1 \cos t$ and $y = e^1 \sin t$, where t is a parameter. Then, If $F(t) = \int (x+y)dt$, then the value of $F\left(\frac{\pi}{2}\right) - F(0)$ is

A. 1

B. -1

C. $e^{\pi/2}$

D. 0

Answer: C

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14. Equation

$x^n - 1 = 0, n > 1, n \in N, \quad \text{has roots} \quad 1, a_1, a_2, \ldots, a_{n-1}.$

The value of $(1 - a_1)(1 - a_2)\ldots(1 - a_{n-1})$ is

A. $\frac{n^2}{2}$

B. n

C. $(-1)^n n$

D. None of the above

Answer: B

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15. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_n,$. The value of $\sum_{r=2}^{n} \frac{1}{2 - a_r},$ is

A. $\frac{2^{n-1}(n-2)+1}{2^n - 1}$

B. $\frac{2^n(n-2)+1}{2^n - 1}$

C. $\frac{2^{n-1}(n-1)-1}{2^n - 1}$

D. None of the above

Answer: A

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16. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_n,$.

The value of $\sum_{r=2}^{n} \frac{1}{2 - a_r},$ is

A. $\frac{n}{4}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n-1}{2}$

D. None of these

Answer: D

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Differentiation Exercise 5:

1. Match the entries between the following two columns.

Column I	Column II
(A) $y = f(x)$ be given by $x = t^5 - 5t^3 - 20t + 7$ and $y = 4t^3 - 3t^2 - 18t + 3$, then $-5 \times \frac{dy}{dx}$ $t = 1$	(p) 0
(B) $P(x)$ be a polynomial of degree 4 with $P(2) = -1$, $P'(2) = 0$, $P''(2) = 2$, $P'''(2) = -12$ and $P''''(2) = 24$, then $P''(3)$ is equal to	(q) -2
(C) $y = \frac{1}{x}$, then $\dfrac{\frac{dy}{\sqrt{1+y^4}}}{\frac{dx}{\sqrt{1+x^4}}}$	(r) 2
(D) $f\left(\frac{2x+3y}{5}\right) = \frac{2f(x)+3f(y)}{5}$ and $f'(0) = p$ and $f(0) = q$, then $f''(0)$	(s) -1

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Exercise (Subjective Type Questions)

1.

$f: R \to R$, $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R$.

The value of f(1) is

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2. Let $f(x)=\sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)$, then the value of $\frac{d(\tan f(x))}{d(\tan^{-1}x)}$, when $x=\frac{1}{2}$, is..........

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3. If $f(x)=\begin{vmatrix}(x-1)^4 & (x-1)^3 & 1\\ (x-b)^4 & (x-b)^3 & 1\\ (x-c)^4 & (x-c)^3 & 1\end{vmatrix}$ then $f'(x)=\lambda.\begin{vmatrix}(x-a)^4 & (x-a)^2 \\ (x-b)^4 & (x-b)^2 \\ (x-c)^4 & (x-c)^2 \end{vmatrix}$

. The value of λ is

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4. Let $p(x)$ be a polynomial of degree 4 such that $P(1)=P(3)=P(5)=P'(7)=0$. If the real number $a\neq 1,3,5$ is such that $P(a) = 0$ can be expressed as $a = \frac{p}{q}$, where p and q are relatively prime, then `(p-8q) is..........

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5. If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t_2}$, then $\left(\frac{dy}{dx}\right)_{(1.1)}$ is............

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6. If $x^2 + y^2 + z^2 - 2xyz = 1$, then the value of $\frac{dx}{\sqrt{1-x^2}} + \frac{dy}{\sqrt{1-y^2}} + \frac{dz}{\sqrt{1-z^2}}$ is equal to...........

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7. If y is twice differentiable function of x, then the expression $(1 - x^2).\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y$ by means of the transformation $x = \sin t$ in terms of t is $\frac{d^2y}{dt^2} + \lambda y$. Thus λ is....

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8. The derivative of $f(x) = \cos^{-1}\left(\frac{1}{\sqrt{3}}(2\cos x - 3\sin x)\right) + \left\{\sin^{-1}\left(\frac{1}{\sqrt{3}}(2\cos x + 3\sin x)\right)\right.$ w.r.t. $\sqrt{1 + x^2}$ at $x = \frac{1}{\sqrt{3}}$ is.........

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9. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$ and $f''(x) - 2f'(x) - 15f(x) = 0$ for all x, then the value of $|a + b|$ is equal to......

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10. Suppose $A = \frac{dy}{dx}$ of $x^2 + y^2 = 4$ at $(\sqrt{2}, \sqrt{2})$, $B = \frac{dy}{dx}$ of $\sin y + \sin x = \sin x . \sin y$ at (π, π) and $C = \frac{dy}{dx}$ of $2e^{xy} + e^x . e^y - e^x - e^y = e^{xy+1}$ at $(1, 1)$, then $(A - B - C)$ has the value equal to.......

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11. A function is reprersented parametrically by the equations $x = \frac{1+t}{t^3}; y = \frac{3}{2t^2} + \frac{2}{t}\dot{T}henthevalueo\frac{f(dy)}{dx} - x\left(\frac{dy}{dx}\right)^3$ is________

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12. Suppose the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at x=1` has the value equal to (a) 19 (b) 9 (c) 17 (d) 14

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13. If $x + y = 3e^2$ then $D(x^y)$ vanishes when x equals to

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14. Let $h(x)$ be differentiable for all x and let $f(x) = (kx + e^x)h(x)$, where k is some constant. If $h(0) = 5, h'(0) = -2, and f'(0) = 18,$ then the value of k is

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Exercise (Questions Asked In Previous 13 Years Exam)

1. For $x \in R, f(x) = |\log_e 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at $x = 0$

B. $g'(0) = \cos(\log 2)$

C. $g'(0) = -\cos(\log 2)$

D. g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

Answer: b

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2. Let $f: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$.

Then, h'(1) equals.

A. $g'(2) = \frac{1}{15}$

B. $h'(1) = 666$

C. $h(0) = 16$

D. $h(g(3)) = 36$

Answer: b,c

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3.

If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals

A. $\frac{3}{1+9x^3}$

B. $\dfrac{9}{1+9x^3}$

C. $\dfrac{3x\sqrt{x}}{1-9x^3}$

D. $\dfrac{3x}{1-9x^3}$

Answer: c

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4. Let $g(x)=\log_e f(x)$ where $f(x)$ is twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=f(x)$, for $N=1,2,3,\ldots,$ then $g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right) equals$

A. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

Answer: a

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5. $\frac{d^2x}{dy^2}$ equals

A. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

B. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

C. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

D. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Answer: d

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6. If f''(x) =- f(x) and g(x) = f'(x) and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) =5, then F(10) is

A. 0

B. 5

C. 10

D. 25

Answer: b

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7. If y is a function of x and $\log(x+y)-2xy=0,$ then the value of $y'(0)$ is

(a)1 (b) -1 (c) 2 (d) 0

A. 1

B. -1

C. 2

D. 0

Answer: a

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8. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then prove that $\frac{dy}{dx}=\frac{1}{x^3y}$

A. $yy'' - 2(y')^2+1=0$

B. $yy'' + (y')^2+1=0$

C. $yy'' + (y')^2-1=0$

D.

Answer: b

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Exercise For Session 1

1. Differentiate the following with respect of $x: e^{a\log a}+e^{a\log x}+e^{a\log a}$

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2. Evaluate: (i) $\int \sin^{-1}(\cos x)\,dx,\ \ 0 \leq x \leq \pi$ (ii) $\int \tan^{-1}\left\{\sqrt{\left(\frac{1-\cos 2x}{1+\cos 2x}\right)}\right\}dx,\ \ 0 \leq x \leq \pi/2$

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3. Differentiate the following with respect of $x: (\log)_3 x + 3(\log)_e x + 2\tan x$

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4. Differentiate $|x| + a_0x^n + a_1x^{n-1} + a_2x^{n-1} + \ldots + a_{n-1}x + a_n$

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5. y = $\sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, x > 0. Find dy/dx

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6. Differentiate w.r.t x : $x^n \log_a xe^x$

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7. Differentiate: $\frac{2^x \cot x}{\sqrt{x}}$

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8. Differentiate $\frac{\sin x - x\cos x}{x\sin x + \cos x}$ with respect to $x\cdot$

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9. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots\ldots\ldots$

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10. Find the values of 'x' for which the rate of change of $\frac{x^4}{4} + \frac{x^3}{3} - x$ is more than $\frac{x^4}{4}$

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Exercise For Session 2

1. Differentiate the following w.r.t.x. $(x^2 + x + 1)^4$

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2. Differentiate the following w.r.t.x. $\sqrt{x^2 + x + 1}$

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3. Differentiate the following w.r.t.x. $\sin^3 x$

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4. Differentiate the following w.r.t.x. $\frac{1}{\sqrt{a^2 - x^2}}$

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5. Differentiate the following w.r.t.x. $e^x \sin x$

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6. Differentiate the following w.r.t.x. $\sin^{-1}\left(\frac{a + b\cos x}{b + a\cos x}\right), b > a > 1$

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7. Differentiate the following w.r.t.x. e^{e^x}

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8. Differentiate the following w.r.t.x. $\log\left(x+\sqrt{a^2+x^2}\right)$

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9. Differentiate w.r.t. 'x' : f(x) = $\log\left(\frac{a+b\sin x}{a-b\sin x}\right)$

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10. Differentiate the following w.r.t.x. $\log\sqrt{\frac{1+\sin x}{1-\sin x}}$

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11. Differentiate the following w.r.t.x. $\frac{e^x+\log x}{\sin 3x}$

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12. Differentiate the following w.r.t.x. $\sin\left(m\sin^{-1}x\right), |x|<1$

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13. Differentiate the following w.r.t.x. $a^{(\sin^{-1}x)^2}, |x| < 1$

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14. Differentiate the following w.r.t.x. $e^{\cos^{-1}\left(\sqrt{1-x^2}\right)}, |x| < 1$

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15. Differentiate the following w.r.t.x. $\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}, |x| < 1$

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16. Differentiate the following w.r.t.x. $\log_{10}x + \log_x 10 + \log_x x + \log_{10}10$

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17. Differentiate the following w.r.t.x. $5^{3-x^2}+\left(3-x^2\right)^5$

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18. Differentiate the following w.r.t.x. $\dfrac{\sqrt{a^2+x^2}+\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}-\sqrt{a^2-x^2}}$

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19. Differentiate the following w.r.t.x. $\sqrt{4+\sqrt{4+\sqrt{4+x^2}}}$

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20. Differentiate the following w.r.t.x. The differentiation coneffiecient of $f(\log_e x)w.r.t.x,$ where $f(x)=\log_e x,$ is

A. $\dfrac{x}{\log_e x}$

B. $\dfrac{1}{x}\log_e x$

C. $\frac{1}{x \log_e x}$

D. None of these

Answer: C

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21. If f(x) $= |\log_e |x||,$ then f'(x) equals

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. $-\frac{1}{x}$

D. None of these

Answer: B

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22. If $f(x) = \sin x, g(x) = x^2$ and $h(x) = \log x$. IF $F(x) = h(f(g(x)))$, then $F'(x)$ is

A. $2x\cot x^2$

B. $2\cos ec^3 x$

C. $-2\cos ec^2 x$

D. None of these

Answer: A

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23. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$ then find $f'\left(\frac{\pi}{4}\right)$

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. None of these

Answer: A

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24. If $y = f\left(\frac{3x+4}{5x+6}\right)$ and $f'(x) = \tan x^2$, then $\frac{dy}{dx}$ is equal to

A. $-2\tan\left(\frac{3x+4}{5x+6}\right)^2 \cdot \frac{1}{(5x+6)^2}$

B. $f\left(\frac{3\tan x^2+3}{5\tan x^2+6}\right)\tan x^2$

C. $2x\tan\left(\frac{3x-4}{5x-6}\right)$

D. $\tan x^2$

Answer: A

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25. If y = |cos x| + |sin x|,then $\frac{dy}{dx}$ at $x\frac{2\pi}{3}$ is

A. $\frac{1}{2}\left(\sqrt{3}+1\right)$

B. $2(\sqrt{3}-1)$

C. $\frac{1}{2}(\sqrt{3}-1)$

D. None of these

Answer: C

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26. If $f'(x) = \sin x + \sin 4x.\cos x,$ then $f'(2x^2)$ is

A. $4x\{\cos(2x^2) - \sin 8x^2.\sin 2x^2\}$

B. $4x\{\cos(2x^2) + \sin 8x^2.\cos 2x^2\}$

C. $\{\cos(2x^2) - \sin 8x^2.\sin 2x^2\}$

D. None of these

Answer: D

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27. If $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2), then \frac{dy}{dx}$ at x = 1 is

A. 1

B. -1

C. -2

D. 2

Answer: D

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Exercise For Session 3

1. If $\log(x^2 + y^2) = 2\tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx} = \frac{x+y}{x-y}$

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2. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

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3. If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

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4. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then prove that $\frac{dy}{dx}=\frac{1}{x^3y}$

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5. If $\sin(xy)+\cos(xy)=0$, then $\frac{dy}{dx}$ is

A. $\frac{y}{x}$

B. $-\frac{y}{x}$

C. $-\frac{x}{y}$

D. $\frac{x}{y}$

Answer: B

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6. If $ax^2 + 2hxy + by^2 = 0$ then $\frac{dy}{dx}$ is

A. $\frac{y}{x}$

B. $\frac{x}{y}$

C. $-\frac{x}{x}$

D. None of these

Answer: A

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7. If $x^2e^y + 2xye^x + 13 = 0$ then $\frac{dy}{dx} =$

A. $\frac{-2xe^{y-x} - 2y(x-1)}{x(xe^{y-x}+2)}$

B. $\frac{2xe^{x-y} - 2y(x-1)}{x(xe^{y-x}+2)}$

C. $\frac{2xe^{x-y} + 2y(x-1)}{x(xe^{y-x}+2)}$

D. None of these

Answer: A

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8. If $\log(x+y) = 2xy$, then $y'(0)$ is

A. 1

B. -1

C. 2

D. 0

Answer: A

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9. If $x\log_e y + y\log_e x = 5$, then $\frac{dy}{dx}$ is

A. $-\frac{y}{x}\left(\frac{x\log y + y}{x + y\log x}\right)$

B. $-\frac{x}{y}\left(\frac{x\log y + y}{x + y\log x}\right)$

C. $-\frac{y}{x}\left(\frac{x\log y - y}{x + y\log x}\right)$

D. None of these

Answer: A

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Exercise For Session 3:

1. If $x\sqrt{1+y} + y\sqrt{1+x} = 0, \quad \text{prove that } \frac{dy}{dx} = -\frac{1}{(x+1)^2}.$

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Exercise For Session 4

1. If $y = \tan^{-1}\left(\frac{1 - \cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

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2. If $y = \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$, then $\frac{dy}{dx}$ is

A. $\begin{cases} \frac{2}{1+x^2}, & x > 0 \\ -\frac{2}{1+x^2}, & x < 0 \end{cases}$

B. $\begin{cases} \frac{2}{1+x^2}, & x > 0 \\ \text{does not exist}, & x = 0 \\ \frac{-2}{1+x^2}, & x < 0 \end{cases}$

C. $\begin{cases} \frac{2}{1+x^2}, & x < 0 \\ \text{does not exist}, & x = 0 \\ \frac{-2}{1+x^2}, & x > 0 \end{cases}$

D. None of these

Answer: C

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3. If $y = \tan^{-1}\frac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)}$, find $\frac{dy}{dx}$.

A. $\left\{\left(\frac{1}{2}, \quad \cos\frac{x}{2} > \sin\frac{x}{2}\right), \left(-\frac{1}{2}, \quad \cos\frac{x}{2} < \sin\frac{x}{2}\right), (\text{doesnotexist}, x =$

B. $\left\{\left(-\frac{1}{2}, \quad \cos\frac{x}{2} > \sin\frac{x}{2}\right), \left(\frac{1}{2}, \quad \cos\frac{x}{2} < \sin\frac{x}{2}\right), (\text{doesnotexist}, x$

C. $\begin{cases} -\frac{1}{2}, & \cos\frac{x}{2} \geq \sin\frac{x}{2} \\ \frac{1}{2}, & \cos\frac{x}{2} < \sin\frac{x}{2} \end{cases}$

D. None of these

Answer: B

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4. If $y = \cot^{-1}(\cot x)$, then $\frac{dy}{dx}$ is

A. $1, x \in R$

B. $1, x \in R - \{n\pi\}$

C. $\{(1,, x \in R - \{n\pi\}), (\text{does not exist}, x \in \{n\pi\}, n \in \text{integer})$

D. None of these

Answer: C

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5. Sketch for the curve $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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6. Draw the graph of $y = \frac{1-x^2}{1+x^2}$ and hence draw the graph of $y = \cos^{-1}.\ \frac{1-x^2}{1+x^2}.$

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7. Draw the graph of $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

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8. Draw the graph of $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$.

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9. Defiine $y = \sin^{-1}(3x - 4x^3)$ in terms of $\sin^{-1}$ x and also draw its graph.

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10. Defiine $y = \sin^{-1}(3x - 4x^3)$ in terms of $\sin^{-1}$ x and also draw its graph.

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Exercise For Session 5

1. If $x = 2\cos\theta - \cos 2\theta \text{and} y = 2\sin\theta - \sin 2\theta \text{P r o v et h a t}$ $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$.

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2. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then move that $\frac{dy}{dx} = -\frac{y\log x}{x\log y}$.

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3. If $x = \cos t$ and $y = \sin t$, prove : $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$ at $t = \frac{2\pi}{3}$

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4. $x = t + \frac{1}{t}, y = t - \frac{1}{t}$

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5. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and y=$\tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1.$ Prove that dy/dx=-1

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6. If $x = a\sec^3\theta andy = a\tan^3\theta, f \in d\frac{dy}{dx}\mathrm{ah}\eta = \frac{\pi}{3}.$

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7. Let $y = x^3 - 8x + 7andx = f(t)\dot{I}\frac{dy}{dt} = 2andx = 3att = 0,$ then find the value of $\frac{dx}{dt}att = 0.$

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Exercise For Session 6

1. Differentiate the following w.r.t.x.

x^x

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2. Differentiate the following w.r.t.x.

$x^{\sqrt{x}}$

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3. Differentiate the following w.r.t.x.

x^{x^x}

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4. Differentiate the following w.r.t.x. x^{x^2}

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5. Differentiate the following w.r.t.x.

$x^x \sqrt{x}$

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6. Differentiate the following w.r.t.x.

$(\cos x)^x$

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7. Differentiate the following w.r.t.x.

$(\sin x)^{\cos x}$

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8. Differentiate the following w.r.t.x.

$x^{\cos^{-1}x}$

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9. Differentiate the following w.r.t.x.

$\cos(x^x)$

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10. Differentiate the following w.r.t.x.

$\log(x^x + \cos ec^2 x)$

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11. If $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$, find $\frac{dy}{dx}$

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12. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

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13. If $x^y + y^x = 2$, find $\frac{dy}{dx}$.

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14. If $(\cos x)^y = (\sin y)^x$, then find $\frac{dy}{dx}$.

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15. If $x\sin(a+y) + \sin a.\cos(a+y) = 0$. Prove that : $\frac{dy}{dx} = \left(\frac{\sin^2(a+y)}{\sin a}\right)$

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16. If $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+\ldots\to\infty}}}$, prove that $\frac{dy}{dx}=\frac{\sin x}{1-2y}$

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17. If $y=(\tan x)^{(\tan x)^{(\tan x)\ldots\infty}}$, then find $\frac{dy}{dx}$ at $x=\frac{\pi}{4}$

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18. If $y=e^{x^{e^x}}+x^{e^{e^x}}+e^{x^{x^e}}$, prove that $\frac{dy}{dx}=e^{x^{e^x}}.x^{e^x}\left\{\frac{e^x}{x}+e^x.\log x\right\}+x^{e^{e^x}}.e^{e^x}\left\{\frac{1}{x}+e^x\log x\right\}+e^{x^{x^e}}.x^{x^e}.x^e$

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Exercise For Session 7

1. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x \in (-1, 1)$

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2. Find the derivative of $\tan^{-1}\frac{2x}{1-x^2}$ w.r.t. $\sin^{-1}\frac{2x}{1+x^2}, |x| < 1.$

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3. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

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4. Find the differential coefficient of thefollowing functions $x^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$

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5. Differentiate $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right)$ with respect to $\sqrt{1-a^2x^2}$, if $-\frac{1}{\sqrt{2}} < ax < \frac{1}{\sqrt{2}}$.

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6. Differentiate $\log\sin x$ w.r.t. $\sqrt{\cos x}$

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7. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ with respect to $\cos^{-1}x^2$

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8. Differnetiate $x^x w.r.t.\ x\log x.$

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9. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.r.t. \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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10. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.r.t. \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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11. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.r.t. \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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Exercise For Session 8

1. $y = x^x$, prove that $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

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2. If $y = A\cos(\log x) + B\sin(\log x)$ then prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

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3. If $y = x\log\{\frac{x}{(a+bx)}]$, then show that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2.$

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4. If $y = \log\left\{x + \sqrt{x^2 + a^2}\right\}$, prove that: $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

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5. $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$ then prove that $\left(x^2+1\right)y_2+xy_1=2$

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6. If $x=at^2,\ y=2at$, then $\frac{d^2y}{dx^2}=-\frac{1}{t^2}$ (b) $\frac{1}{2at^3}$ (c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2at^3}$

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7. $x=a\cos^3\theta,\ y=a\sin^3\theta$ then find $\frac{d^2y}{dx^2}$

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8. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that $\left(1-x^2\right)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx}=0$

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9. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then prove that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

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10. If $y = \frac{ax+b}{cx+d}$, then prove that $2y_1y_3 = 3(y_2)^2$

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11. If $x = f(t)$ and $y = \phi(t)$, prove that $\frac{d^2y}{dx^2} = \frac{f_1\phi_2 - f_2\phi_1}{f_1^3}$ where suffixes denote differentiation $w.r.t.t.$

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12. If $x = \sin t, y = \sin Kt$ then show that $(1-x^2)y_2 - xy_1 + K^2y = 0$

.

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13. If $x^2 + y^2 = 1, then$ $yy^{-2}(y')^2 + 1 = 0$ $y^{+}(y')^2 + 1 = 0$ $yy^{+}(y')^{-2} - 1 = 0$ $yy^{+}2(y')^2 + 1 = 0$

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14. Let f(x) be polynomial function of defree 2 such that $f(x) > 0$ for all $x \in R.$ If $g(x) = f(x) + f'(x) + f''(x)$ for all x, then

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Exercise For Session 9

1. If f(x) g(x) and h(x) are three polynomials of degree 2 and Δ = $\begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ then $\Delta(x)$ is a polynomial of degree (dashes denote the differentiation).

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2. If $f, g, and\ h$ are differentiable functions of x and $d(x) = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2 f)'' & (x^2 g)'' & (x^2 h)'' \end{vmatrix}$ prove that $d'(x) = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3 f'')' & (x^3 g'')' & (x^3 h'')' \end{vmatrix}$

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3. Find $\frac{dy}{dx}$ at $x = -1$,when $(\sin y)^{\sin\left(\left(\frac{\pi}{2}\right)x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$

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Exercise For Session 10

1. The function $f(x) = e^x + x$, being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

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2. Let $g(x)$ be the inverse of an invertible function $f(x)$, which is differentiable for all real x. Then $g^{f(x)}$ equals. $-\frac{f^x}{(f'(x))^3}$ (b) $\frac{f'(x)f^x - (f'(x))^3}{f'(x)}$ $\frac{f'(x)f^x - (f'(x))^2}{(f'(x))^2}$ (d) none of these

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3. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$. Then $g'(f(x))$ equal. $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

A. $f'(c)$

B. $\dfrac{1}{f'(c)}$

C. $f(c)$

D. None of these

Answer: B

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4. If f(x)=x + tan x and f si the inverse of g, then g'(x) equals

A. $\dfrac{1}{[g(x) - x]^2}$

B. $\dfrac{1}{2 - [g(x) - x]^2}$

C. $\dfrac{1}{2 + [g(x) - x]^2}$

D. None of these

Answer: C

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