MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

ELLIPSE

Examples

1. If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$ such that $SP = 8,$ then

find the length of SQ is (a) 2 (b) 1 (c) $\frac{8}{9}$ (d) $\frac{16}{9}$

Watch Video Solution

2. about to only mathematics

Watch Video Solution

3. If the distance between the directrices is thrice the distance between the foci, then find eccentricity of the ellipse.

Watch Video Solution

4. If $P(x, y)$ is any point on the ellipse $16x^2 + 25y^2 = 400$ and $f_1 = (3, 0) F_2 = (-3, 0)$, then find the value of $PF_1 + PF_2$.

Watch Video Solution

5. Find the eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from

the center of the ellipse is $\sqrt{5}$

Watch Video Solution

6. An ellipse having foci at (3, 3) and $(-4, 4)$ and passing through the origin has eccentricity equal to (a) $\frac{3}{7}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{3}{5}$

Watch Video Solution

7. Find the lengths of major and minor axes,the coordinate of foci, vertices and the

eccentricity of the ellipse $3x^2 + 2y^2 = 6$. Also the equation of the directries.

Watch Video Solution

8. Find the equation of the ellipse whose focus is $(-1, 1)$, the corresponding directrix is $x - y + 3 = 0$, and eccentricity is $\frac{1}{2}$. Also find its center, the second focus, the equation of the second directrix, and the length of latus rectum.

Watch Video Solution

9. If the line $lx + my + n = 0$ cuts the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ at points whose eccentric angles differ by $\frac{\pi}{2}$, then find the value of $\frac{a^2l^2 + b^2m^2}{n^2}$.

Watch Video Solution

10. If the chord, joining two points whose eccentric angles are α and β, cuts the major axis ofthe ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a distance

c from the centre, then $\tan\alpha/2.\tan\beta/2$ is equal to

Watch Video Solution

11. If the angle between the straight lines joining foci and the ends of minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{\pi}{2}$ then the eccentricity is

Watch Video Solution

12. find the equation of the ellipse refer refer to it Centre whose major axis is equal to distance between the foci and latus rectum is 10.

Watch Video Solution

13. The ratio of any triangle PQR inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and that of triangle formed by the corresponding points on the auxilliary circle is $\frac{b}{a}$.

Watch Video Solution

14. If the extremities of a line segment of length l moves in two fixed perpendicular straight lines, then the locus of the point which divides this line segment in the ratio 1 : 2 is-

Watch Video Solution

15. Find the lengths of and the equations to the focal radii drawn to the point $\left(4\sqrt{3}, 5\right)$ of

the ellipse $25x^2 + 16y^2 = 1600$

Watch Video Solution

16. Find the position of the point (4,-3) relative to the ellipse $5x^2 + 7y^2 = 140.$

Watch Video Solution

17. Number of integral values of $'\alpha'$ for which the point $\left(7, -\frac{5}{4}\alpha, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

Watch Video Solution

18. If straight line $lx + my + n = 0$ is a tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2l^2 + b^2m^2 = n^2$.

Watch Video Solution

19. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$.

Watch Video Solution

20. The values of λ for which the line y=x+λ touches the ellipse $9x^2 + 16y^2 = 144$, are

Watch Video Solution

21. If the line $3x + 4y = \sqrt{7}$ touches the ellipse $3x^2 + 4y^2 = 1,$ then the point of contact is

Watch Video Solution

22. Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$ which are perpendicular to the line $y + 2x = 4$.

Watch Video Solution

23. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Watch Video Solution

24. Find the point on the ellipse $16x^2 + 11y^2 = 256$ where the common tangent to ti and the circle $x^2 + y^2 - 2x = 15$ toch.

Watch Video Solution

25. Find the maximum area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which touches the line $y = 3x + 2.$

Watch Video Solution

26. Find the slope of a common tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and a concentric circle of radius r.

Watch Video Solution

27. Prove that the product of the perpendicular from the foci on any tangent to an ellipse is equal to the square of the semi-minor axis.

Watch Video Solution

28. The locus of the middle point of the portion of a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ included between axes is the curve

Watch Video Solution

29. Show that the tangents at the extremities of the latus rectum of an ellipse intersect on the corresponding directrix.

Watch Video Solution

30. The normal at an end of a latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through an end of the minor axis if (A) $e^4+e^2=1$ (B) $e^3+e^2=1$ (C) $e^2+e=1$ (D) $e^3+e=1$

Watch Video Solution

31. The line $lx+my+n=0$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. then prove that

$$\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)^2}{n^2}$$

Watch Video Solution

32. A normal inclined at 45° to the axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{\left(a^2-b^2\right)^2}{2\left(a^2+b^2\right)}$ sq units

Watch Video Solution

33. Any ordinate MP of an ellipse meets the auxillary circle in Q. Ptove that the locus of the point of intersection of the normals at P and Q is the circle $x^2 + y^2 = (a + b)^2$.

Watch Video Solution

34. Prove that the chord of contact of tangents drawn from the point (h,k) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at the centre, if $\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$

Watch Video Solution

35. Show that the locus of the middle points of chord of an ellipse which paas through a fixed point, is another ellipse

Watch Video Solution

36. Show that the tangents at the ends of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2.$

Watch Video Solution

37. Tangents at right angle are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that the focus of the middle points of the chord of contact is the curve $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2+y^2}{a^2+b^2}$.

Watch Video Solution

38. A ray emanating from the point (-3,0) is incindent on the ellipse $16x^2+25y^2=400$ at

the point p with ordinate 4. Find the equation of the reflected ray after first reflection.

Watch Video Solution

39. For the ellipse $4(x-2y+1)^2+9(2x+y+2)^2=180$, lengths of major and minor axes are respectively

Watch Video Solution

40. Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then find the product $(AB).(CD)$

A. 52

B. 65

C. 78

D. None of these

Answer: A::B::C

Watch Video Solution

41. Let P_i and P_i' be the feet of the perpendiculars drawn from the foci $SandS'$ on a tangent T_i to an ellipse whose length of semi-major axis is 20. If $\sum_{i=0}^{10}(SP_i)\left(S'\Pi'\right)=2560,$ then the value of eccentricity is (a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

Answer: B::C

Watch Video Solution

42. The coordinates of the vertices $BandC$ of a triangle ABC are (2, 0) and (8, 0), respectively. Vertex A is moving in such a way that $4\frac{\tan B}{2}\frac{\tan C}{2} = 1$. Then find the locus of A

A. $\frac{x-5^2}{25}+\frac{y^2}{16}=1$

B. $\frac{(x-5)^2}{16}+\frac{y^2}{9}=1$

C. $\frac{(x-5)^2}{25}+\frac{y^2}{9}=1$

D. $\frac{(x-5)^2}{16}+\frac{y^2}{25}=1$

Answer: A::B

Watch Video Solution

43. A ray emanating from the point (0,6) is incident on the ellipse $25x^2+16y^2=1600$ at

the point P with ordinate S. After reflection, ray cuts the Y-axis at B. The length of PB is

A. 5

B. 7

C. 12

D. 13

Answer: A::B::C::D

 Watch Video Solution

44. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ at four distinct points and $a = b^2 - 5b + 7$, then b does not lie in $[4, 5]$ (b) $(-\infty, 2) \cup (3, \infty)$ $(-\infty, 0)$ (d) $[2, 3]$

A. (1,4)

B. $(-\infty, 2) \cup (3, \infty)$

C. $(2, 3)$

D. None of these

Answer: B

Watch Video Solution

45. The normal at a variable point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse at $QandR\cdot$ Then the locus of the midpoint of QR is a conic with eccentricity e' such that (a)e' is independent of e (b) $e'=1$ (c)$e'=e$ (d) $e'=\frac{1}{e}$

A. e' is indipendant of e

B. e'=1

C. e'=e

D. e'=1/e

Answer:

Watch Video Solution

46. If the curves $\frac{x^2}{4} + y^2 = 1$ and $\frac{x^2}{a^2} + y^2 = 1$ for a suitable value of a cut on four concyclic points, the equation of the circle passing through these four points is

A. $x^2 + y^2 = 8$

B. $x^2 + y^2 = 4$

C. $x^2 = y^2 = 2$

D. $x^2 + y^2 = 1$

Answer: A::B

Watch Video Solution

47. If P is the length of perpendicluar drawn from the origin to any normal to the ellipse

$\frac{x^2}{25} + \frac{y^2}{16} = 1$, then the maximum value of p is

A. 5

B. 4

C. 2

D. 1

Answer: D

Watch Video Solution

48. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$. If $f(x)$ is a positive decr4easing function, then the set of values of k for which the major axis is the x-axis is $(-3,2)$. the set of values of k for which the major axis is the y-axis is $(-\infty,2)$. the set of values of k for which the major axis is the y-axis is $(-\infty,-3)\cup(2,\infty)$ the set of values of k for which the major axis is the y-axis is $(-3,-\infty,)$

A. $k\in(-2,3)$

B. $k \in (-3, 2)$

C. $k \in (-\infty, -3) \cup (2, \infty)$

D. $k \in (-\infty, -2) \cup (3, \infty)$

Answer: B::C

Watch Video Solution

49. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then the maximum value of ab is 4 (b) 2 (c) 1 (d) none of these

A. 1

B. 2

C. 4

D. 8

Answer: D

Watch Video Solution

50. Extremities of the latus rectum of the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ having a major

axis 2a lies on

A. $x^2 = a(a - y)$

B. `x=a(a+y)

C. $y^2 = a(a + x)$

D. y^(2)=a(a-x)`

Answer: A::B

Watch Video Solution

51. The locus of the image of the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}-1,(a>b),$ with respect to any of the tangents to the ellipse is

$(x+4)^2+y^2=100$ (b) $(x+2)^2+y^2=50$

$(x-4)^2+y^2=100$ (d) $(x+2)^2+y^2=50$

A. $(x+4)^2+y^2=100$

B. (x+2)^(2)+y^(2)=50`

C. $(x-4)^2+y^2=100$

D. $\left(x-20^2+y^2=50\right.$

Answer: A::B::D

Watch Video Solution

52. A tangent to the ellipse $4x^2 + 9y^2 = 36$ is cut by the tangent at the extremities of the major axis at T and T^1, the circle on TT^1 as diameter passes through the point

A. $(-\sqrt{5}, 0)$

B. $(\sqrt{5}, 0)$

C. $(\sqrt{3}, 0)$

D. $(-\sqrt{3}, 0)$

Answer: A::B::D

Watch Video Solution

53. Consider the ellipse $\frac{x^2}{\tan^2\alpha} + \frac{y^2}{\sec^2\alpha} = 1$ where $\alpha \in \left(0, \frac{\pi}{2}\right)$. Which of the following quantities would vary as α varies?

A. (a)degree of flatness

B. (b)ordinate of the vertex

C. (c)coordinate of the foci

D. (d)length of latusrectum

Answer: A::C::D

Watch Video Solution

54. Let $A(\theta)$ and $B(\phi)$be the extrenities of a chord of an emplise. If the slope of AB is equal to the slope of the tangent at a point C(alpha) on the ellipse, then alpha is equal to

A. $\frac{\theta+\phi}{2}$

B. $\frac{\theta - \phi}{2}$

C. $\frac{\theta + \phi}{2} + \pi$

D. $\frac{\theta + \phi}{2} - \pi$

Answer: A::C

Watch Video Solution

55. A series of concentric ellipses $E_1, E_2, E_3 \ldots, E_n$ are drawn such that E touches the extremities of the major axis of E_{n-1}, and the foci of E_n coincide with the

extremities of minor axis of E_{n-1} If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is

A. (a) $\frac{3-\sqrt{5}}{2}$

B. (b) $\frac{\sqrt{5}-1}{2}$

C. (c) $\frac{2-\sqrt{3}}{2}$

D. (d) $\frac{\sqrt{3}-1}{2}$

Answer: A::B

Watch Video Solution

56. A series of concentric ellipse $E_1, E_2, E_3, \ldots, E_n$ is constructed as follows: Ellipse E_n touches the extremities of the major axis of E_{n-1} and have its focii at the extremities of the minor axis of E_{n-1}. If eccentricity of ellipse E_n is e_n, then the locus of $\left(e_n^2, e_{n-1}^2\right)$ is

A. a parabola

B. an ellipse

C. a hyperbola

D. a rectangular hyperbola

Answer: A::B::C

Watch Video Solution

57. A series of concentric ellipse $E_1, E_2, E_3, \ldots, E_n$ is constructed as follows: Ellipse E_n touches the extremities of the major axis of E_{n-1} and have its focii at the extremities of the minor axis of E_{n-1} If equation of ellipse E_1 is $\frac{x^2}{9} + \frac{y^2}{16} = 1$, then equation pf ellipse E_3 is

A. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

B. $x^2 + y^{49} = 1$

C. $\frac{x^2}{25} + \frac{y^2}{41} = 1$

D. $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Answer: A::B::D

Watch Video Solution

58. Consider an ellipse $E, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point O andhaving AB and CD

as its major and minor axes, respectively. If S_1 is one of the focus of the ellipse, the radius of the incircle of triangle OCS_1 is 1 unit, and $OS_1 = 6$ units, then the value of $\frac{a-b}{2}$ is_____

A. 63π

B. 64π

C. 65π

D. 66π

Answer: C

Watch Video Solution

59. An ellipse $E, \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centred at point O has AB and CD as its major and minor axes, respectively. Let S_1 be one of the foci of the ellipse, the radius of the incircle of traingle OCS_1 be 1 unit, adn $OS_1 = 6$ units

The perimeter of ΔOCS_1 is

A. (a)10

B. (b)15

C. (c)20

D. (d)25

Answer: A::B::C

Watch Video Solution

60. Consider an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$

units. Q. The equation of the director circle of (E) is

A. (a) $x^2 + y^2 = 48.5$

B. (b) $x^2 + y^2 = 97$

C. (c) $x^2 + y^2 = \sqrt{48.5}$

D. (d) $x^2 + y^2 = \sqrt{97}$

Answer: A::B::D

Watch Video Solution

61. If the normals at the four points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then the value of $\left(\sum_{i=1}^{4} x_i\right)\left(\sum_{i=1}^{4} \frac{1}{x_i}\right)$

Watch Video Solution

62. If $x, y \in R$, satisfies the equation $\frac{(x-4)^2}{4} + \frac{y^2}{9} = 1$, then the difference between the largest and the smallest valus of the expression $\frac{x^2}{4} + \frac{y^2}{9}$ is_____

Watch Video Solution

63. Statement 1 Feet of prependiculars drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on the circle $x^2 + y^2 = 16$

Statement 2 If prependiculars are from foci of an ellipse to its any tangent, the feet of these perpendicular lie on director circle of the ellipse.

Watch Video Solution

64. Statement 1 the condition on a and b for which two distinct chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ passing through (a,-b) are bisected by the line x+y=b is $a^2+6ab-7b^2>0.$

Statement 2 Equation of chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose mid-point (x_1,y_1) is $T=S_1$

Watch Video Solution

65. The line lx+my=n is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Watch Video Solution

66. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

Watch Video Solution

67. Triangles are formed by pairs of tangent dreawn from any point on the ellipse $a^2x^2 + b^2y6(2) = \left(a^2 + b^2 \text{ ˆ } (2)\right.$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the chord of contact. Show that the orthocentre of each such triangles lies triangle lies on the ellipse.

Watch Video Solution

68. Show that the tangents drawn at those points of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = (a + b),$

where it is cut by any tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, intersect at right angles.

Watch Video Solution

69. If $x\log_e(\log_e x)-x^2+y^2=4$ then $\left(\frac{dy}{dx}\right)_{atx=e}$ is equal to (A) $\frac{2e+1}{\sqrt{4+e^2}}$ (B) $\frac{e}{2\sqrt{4+e^2}}$ (C) $\frac{2e+1}{2(4+e^2)}$ (D) $\frac{2e-1}{2\sqrt{4+e^2}}$

Watch Video Solution

70. If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (b>a)$ at the point whose eccentric angles are θ and $\frac{\pi}{2}+\theta$, then prove that $\frac{2\cot\omega}{\sin 2\theta}=\frac{e^2}{\sqrt{1-e^2}}$

Watch Video Solution

71. Let ABC be an equilateral triangle inscribed in the circle $x^2+y^2=a^2$. Suppose pendiculars from A, B, C to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,(a>b)$ meets the ellipse

respectivelily at P, Q, R so that P, Q , R lies on same side of major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P Q nad R are concurrent.

Watch Video Solution

72. show that the area of the triangle inscribed in the circle $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meet the ellipse respictively at P,Q,R so that P,Q,R lie on the same side of the major axis as A,B,C respictively. Prove that the normal to the

ellipse drawn at the points P,Q and R are concurrent.

Watch Video Solution

73. Two concentric ellipses are such that the foci of one are on the other and their major axes are equal. Let $eande'$ be their eccentricities. Then. the quadrilateral formed by joining the foci of the two ellipses is a parallelogram the angle θ between their axes is given by $\theta = \cos^{-1}\sqrt{\frac{1}{e^2} + \frac{1}{e'^2} = \frac{1}{e^2 e'^2}}$ If

$e^2 + e'^2 = 1$, then the angle between the axes of the two ellipses is 90^0 none of these

Watch Video Solution

74. If the normals at the four points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then the value of $\left(\sum_{i=1}^{4} x_i\right)\left(\sum_{i=1}^{4} \frac{1}{x_i}\right)$

Watch Video Solution

Example

1. Find the locus of the points of the intersection of tangents to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which make an angle theta.

Watch Video Solution

2. Prove that the locus of the middle points of normal chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

the curve

$$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\left(\frac{a^6}{x^2}+\frac{b^6}{y^2}\right)=\left(A^2-B^2\right)^2$$

Watch Video Solution

3. Match the following

Column I	Column II
A. Let $f(x) = \begin{cases} a^x, & x < 2 \\ 8, & x = 2 \\ \dfrac{b(x^2 - b^2)}{(x-2)}, & x > 2 \end{cases}$ If f is continuous at $x = 2$, then the locus of the pair of perpendicular tangents to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $x^2 + y^2 = r^2$, then r^2 is divisible by	(p) 3
B. If the ellipse $\dfrac{(x-h)^2}{M} + \dfrac{(y-k)^2}{N} = 1$ has major axis on the line $y = 2$, minor-axis on the line $x = -1$, major axis has length 10 and minor axis has length 4. Then, $h + k + M + N$ is divisible by	(q) 4
C. If PQ is a focal chord of ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$, which passes through $S(3, 0)$ and $PS = 2$, then length of PQ is divisible by	(r) 5
D. A tangent to the ellipse $\dfrac{x^2}{27} + \dfrac{y^2}{48} = 1$ having slope $\left(-\dfrac{4}{3}\right)$ cuts the x and y-axis at the points A and B respectively. If O is the origin, then area of ΔOAB is divisible by	(s) 6

Watch Video Solution

4. From any point on the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$. Tangents are drawn to the conic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that the normals at the points of contact meet on the conic $a^2x^2 + b^2y^2 = \frac{1}{4}(a^2 - b^2)^2$.

Watch Video Solution

Exercise For Session 1

1. If the length of the major axis of an ellipse in 3 times the length of minor axis , then its eccentricity is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{2\sqrt{2}}{3}$

D. $\frac{2\sqrt{2}}{5}$

Answer: C

Watch Video Solution

2. The equation $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse , if

A. $a<4$

B. $a>4$

C. $4<a<10$

D. $a>10$

Answer: A

Watch Video Solution

3. Find the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latus rectum is half of its major axis.

A. 1/sqrt3

B. 1/sqrt2

C. sqrt3/2

D. $\sqrt{\left(\frac{2}{3}\right)}$

Answer: B

Watch Video Solution

4. If the eccentricity of an ellipse is $\frac{1}{\sqrt{2}}$, then its latusrectum is equal to its

A. minor axis

B. semi minor axis

C. major axis

D. semi major axis

Answer: D

Watch Video Solution

5. If the distance between the foci of an ellipse is equal to length of minor axis, then its eccentricity is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Watch Video Solution

6. The eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the centre of the ellipse is 2, is

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{2}$

D. $\frac{5\pi}{3}$

Answer: A::B

Watch Video Solution

7. If $\tan\theta_1 . \tan\theta_2 = \frac{a^2}{b^2}$ then the chord Joining two points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at (A) focus (B) centre (C) end of the major axis (D) end of the major axis

A. focus

B. center

C. end of major axis

D. end of minor axis

Answer: B

Watch Video Solution

8. If the eccentricities of the two ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and equal , then the value $\frac{a}{b}$, is

A. $\frac{5}{13}$

B. $\frac{6}{13}$

C. $\frac{13}{5}$

D. $\frac{13}{6}$

Answer: C

Watch Video Solution

9. The ratio of the area of triangle inscribed in ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse.

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Watch Video Solution

10. If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$ such that SP=16, then the length SQ is

A. $\frac{2}{9}$

B. $\frac{4}{9}$

C. $\frac{8}{9}$

D. $\frac{16}{9}$

Answer: D

Watch Video Solution

11. Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci at S and S'. If A be the area of triangle PSS' then the maximum value of A, is

A. 12 sq units

B. 24 sq units

C. 36 sq units

D. 48 sq units

Answer: A

Watch Video Solution

12. $S_1 and S_2$ are the foci of an ellipse of major axis of length 10 units, and P is any point on the ellipse such that the perimeter of triangle

PS_1 is 15. Then the eccentricity of the ellipse is

0.5 (b) 0.25 (c) 0.28 (d) 0.75

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{7}{25}$

D. $\frac{3}{4}$

Answer: A

Watch Video Solution

13. Find the latus rectum, eccentricity, coordinates of the foci and the length of axes of the ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$.

Watch Video Solution

14. The distance between the foci of an ellipse is 10 and its latus rectum is 15, find its equation referred to its axes as axes of coordinates.

Watch Video Solution

15. Find the equation of the ellipse whose axes are parallel to the coordinate axes having its centre at the point $(2, -3)$ one focus at $(3, -3)$ and vertex at $(4, -3)$.

Watch Video Solution

16. Find the equation of the ellipse whose foci are $(2, 3), (-2, 3)$ and whose semi-minor axes is $\sqrt{5}$.

Watch Video Solution

17. Show that the equation $(10x-5)^2+(10y-5)^2=(3x+4y-1)^2$ represents an ellipse, find the eccentricity of the ellipse.

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{\sqrt{2}}{3}$

Answer: B

Watch Video Solution

18. The locus of extremities of the latus rectum of the family of ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is

Watch Video Solution

Exercise For Session 2

1. The number of values of c such that the straight line $y = 4x + c$ touches the curve

$\frac{x^2}{4} + \frac{y^2}{1} = 1$ is (a) 0 (b) 1 (c) 2 (d) infinite

A. 0

B. 1

C. 2

D. infinite

Answer: C

 Watch Video Solution

2. If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts off intercepts of length h and k on the axes, then $\frac{a^2}{h^2}+\frac{b^2}{k^2}=$ (A) 0 (B) 1 (C) -1 (D) Non of these

A. -1

B. 0

C. 1

D. None of these

Answer: C

Watch Video Solution

3. The equations of the tangents to the ellipse $3x^2 + y^2 = 3$ making equal intercepts on the axes are

A. $y = \pm x \pm 2$

B. $y = \pm x \pm 4$

C. $y = \pm x \pm \sqrt{30}$

D. $y = \pm x \pm \sqrt{35}$

Answer: A

Watch Video Solution

Watch Video Solution

4. If $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then find the eccentric angle θ of point of contact.

A. 0

B. 45°

C. 60°

D. 90°

Answer: B

Watch Video Solution

5. The number of values of $\phi \in [0, 2\pi]$ for which the line $2x \cos \phi + 3y \sin \phi = 6$ touches the ellipse $4x^2 + 9y^2 = 36$ is a) four b) two c)one d) infinite

A. 1

B. 2

C. 4

D. infinite

Answer: D

Watch Video Solution

6. The common tangent of $x^2 + y^2 = 4$ and $2x^2 + y^2 = 2$ is

A. x+y+4=0

B. x-y+7=0

C. 2x+3y+8=0

D. None of these

Answer: D

Watch Video Solution

7. If the normal at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes at G and g respectively, then find the ratio $PG:Pg$.

(a) $a:b$ (b) $a^2:b^2$ (c) $b:a$ (d) $b^2:a^2$

A. a:b

B. $a^2:b^2$

C. b:a

D. $b^2 : a^2$

Answer: D

Watch Video Solution

8. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from the point $P(0, 6)$ is one (b) two (c) three (d) four

A. one

B. two

C. three

D. four

Answer: C

Watch Video Solution

9. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is 4 (b) 2 (c) 1 (d) none of these

A. 4

B. 2

C. 1

D. none of these

Answer: A

Watch Video Solution

10. about to only mathematics

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

Answer: B

Watch Video Solution

11. The line $5x - 3y = 8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, If 'theta' be eccentric angle of the foot of this normal then 'theta' is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

Watch Video Solution

12. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on

the ellipse $4x^2 + 5y^2 = 20$, then

$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$t = -\frac{2}{\sqrt{5}}$ (d) $t = -\frac{1}{\sqrt{5}}$

Watch Video Solution

13. If the normal at any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the major and minor axes in L and M respectively and if C is the centre of the ellipse , then $a^2\, CL^2 + b^2\, CM^2$ is equal to

(A) $(a-b)$ (B) $\left(a^2-b^2\right)^2$ (C) $(a+b)$ (D) $\left(a^2+b^2\right)$

Watch Video Solution

14. about to only mathematics

Watch Video Solution

15. The tangent and normal at any point P of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cut its major axis in point Q and R respectively. If QR=a prove that

the eccentric angle of the point P is given by

$$e^2 \cos^2 \phi + \cos \phi - 1 = 0$$

Watch Video Solution

Exercise For Session 3

1. Find the angle between the pair of tangents from the point (1,2) to the ellipse $3x^2 + 2y^2 = 5.$

A. $\tan^{-1}\left(\frac{12}{5}\right)$

B. $\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)$

C. $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

D. $\tan^{-1}(\sqrt{5})$

Answer: C

Watch Video Solution

2. If the chords of contact of tangents from two poinst (x_1, y_1) and (x_2, y_2) to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then find the value of $\frac{x_1 x_2}{y_1 y_2}$.

A. $\frac{a^2}{b^2}$

B. $-\frac{b^2}{a^2}$

C. $-\frac{a^4}{b^4}$

D. $\frac{b^4}{a^4}$

Answer: C

Watch Video Solution

3. If the tangents from the point $(\lambda, 3)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ are at right angles then λ is

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: B

Watch Video Solution

4. The eccentric angle of one end of a diameter of $x^2 + 3y^2 = 3$ is $\frac{\pi}{6}$, then the eccentric angle of the other end will be

A. $\frac{5\pi}{6}$

B. $-5\frac{\pi}{6}$

C. $-2\frac{\pi}{3}$

D. $2\frac{\pi}{3}$

Answer: B

Watch Video Solution

5. The locus of the point of intersection of the tangent at the endpoints of the focal chord of the ellipse `(x^2)/(a^2)+(y^2)/(b^2)=1(b

A. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

B. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

C. $x^2+y^2=a^2+b^2$

D. $x^2-y^2=a^2+b^2$

Answer: A

Watch Video Solution

6. the centre of the ellipse $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$, is

A. (0,0)

B. (1,0)

C. (0,1)

D. (1,1)

Answer: D

Watch Video Solution

7. The locus of the point of intersection of two prependicular tangents of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is

A. $x^2 + y^2 = 4$

B. $x^2 + y^2 = 9$

C. $x^2 + y^2 = 13$

D. $x^2 + y^2 = 5$

Answer: C

Watch Video Solution

8. The area of the parallelogram inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose diaonals are the conjugate diameters of the ellipse is given by

A. (a)2ab

B. (b)3ab

C. (c)4ab

D. (d)5ab

Answer: A

Watch Video Solution

9. Find the locus of the vertices of equilateral triangle circumscribing the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Watch Video Solution

10. A tangent to the ellipse $x^2+4y^2=4$ meets the ellipse $x^2+2y^2=6$ at P and Q. The angle between the tangents at P and Q of the ellipse $x^2+2y^2=6$ is

Watch Video Solution

11. The locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which pass through the positive end of major axis, is.

Watch Video Solution

12. Ifchord ofcontact ofthe tangents drawn from the point $(\alpha,\beta)$to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,touches the circle

$x^2 + y^2 = c^2$, then the locus of the point (α , β) is

Watch Video Solution

13. Find the centre and eccentricity of the ellipse $4(x - 2y + 1)^2 + 9(2x + y + 2)^2 = 5.$

Watch Video Solution

14. A ray emanating from the point $(0, \sqrt{5})$ is incident on the ellipse $9x^2 + 4y^2 = 36$ at the

point P with abscissa 2. find the equation of the reflected ray after first reflection.

Watch Video Solution

Exercise Single Option Correct Type Questions

1. Given f is increasing, the equation $\frac{x^2}{f(2a)} + \frac{y^2}{f(a^2-3)} = 1$ represents an ellipse with X-axis as major axis if

A. [-1,3]

B. [1,3]

C. (-1,3)

D. (0,5)

Answer: C

Watch Video Solution

2. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)}$=1 represents an ellipse with major axis as Y-axis and f is a decreasing function,then

A. $\alpha \in (1\infty, 1)$

B. $\alpha \in (5, \infty)$

C. $\alpha \in (1, 4)$

D. $\alpha \in (-1, 5)$

Answer: D

Watch Video Solution

3. The curve represents by the equation $\frac{x^2}{\sin\sqrt{2} - \cos\sqrt{3}} + \frac{y^2}{\sin\sqrt{3} - \cos\sqrt{2}} = 1$ is

A. (a) an ellipse with foci on X-axis

B. (b) an ellipse on focii Y-axis

C. (c) a hyperbola with foci on X-axis

D. (d) an hyperbola with foci on Y-axis

Answer: A

Watch Video Solution

4. The maximum distance of the centre of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ from the chord of

contact of mutually perpendicular tangents of the ellipse is

A. (a) 144/5

B. (b) 16/5

C. (c) $\frac{9}{5}$

D. (d) None of these

Answer: B

Watch Video Solution

5. S and $T$are the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and B is an end of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is e then find value of $4e$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Watch Video Solution

6. A circle of radius $\frac{5}{\sqrt{2}}$ is concentric with the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, then the acute angle made by the common tangent with the line $\sqrt{3}x-y+6=0$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer: D

Watch Video Solution

7. Consider the particle travelling clockwise on the elliptical path $\frac{x^2}{100} + \frac{y^2}{25} = 1$ The particle leaves the orbit at the point $(-8, 3)$ and travels in a straight line tangent to the ellpse. At what point will the particle cross the y-axis?

A. $\left(0, -\frac{25}{3}\right)$

B. $\left(0, \frac{7}{3}\right)$

C. $\left(0, \frac{25}{3}\right)$

D. $(0, 9)$

Answer: c

Watch Video Solution

8. C is the centre of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and A and B are two points on the ellipse such that $\angle ACB = 90^\circ$, then $\frac{1}{(CA)^2} + \frac{1}{(CB)^2} =$

A. (a) $\frac{7}{12}$

B. (b) $\frac{12}{7}$

C. (c) $\frac{25}{144}$

D. (d) $\frac{144}{25}$

Answer: C

Watch Video Solution

9. Let (α,β) be a point from which two perpendicular tangents can be drawn to the ellipse $4x^2 + 5y^2 = 20$. If $F = 4\alpha + 3\beta$, then

A. (a) $-15 \leq F \leq 15$

B. (b) $F \geq 0$

C. (c) $-5 \leq F \leq 20$

D. (d) $F \leq -5\sqrt{5}$ or $F \geq 5\sqrt{5}$

Answer: A

Watch Video Solution

10. If $a = \left[t^2 - 3t + 4\right]$ and $b = [3 + 5t]$, where [.] donates the greatest integer

function, then the latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $t=\frac{3}{2}$ is

A. 20

B. 10

C. $\frac{1}{5}$

D. $\frac{1}{10}$

Answer: C

Watch Video Solution

11. If the line $x + 2y + 4 = 0$ cutting the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angies are 30° and 60° subtends right angle at the origin then its equation is

A. $\frac{x^2}{8} + \frac{y^2}{4} = 1$

B. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

C. $\frac{x^2}{4} + \frac{y^2}{16} = 1$

D. None of the above

Answer: B

Watch Video Solution

Watch Video Solution

12. An arc of a bridge is semi-elliptical with the major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal, then prove that the best approximation of the height of the acr 2 m from the center of the base is $\frac{8}{3}m$.

A. $\frac{11}{4}m$

B. $\frac{8}{3}m$

C. $\frac{7}{2}m$

D. $2m$

Answer: B

Watch Video Solution

13. A tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at any point P meets the line $x = 0$ at a point Q. Let R passes through a fixed point. The fixed point is (3, 0) (b) (5, 0) (c) (0, 0) (d) (4, 0)

A. (3,0)

B. (4,0)

C. (5,0)

D. (0,0)

Answer: D

Watch Video Solution

14. If tangents are drawn from any point on the circle $x^2 + y^2 = 25$ the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ then the angle between the tangents is

A. `π/4

B. π/2

C. π/3

D. π/6

Answer: C

Watch Video Solution

15. the equation of the chord of contact of the pair of tangents drawn to the ellipse $4x^2 + 9y^2 = 36$ from the point (m, n) where

$m\dot{n} = m + n, m, n$ being nonzero positive integers, is $2x + 9y = 18$ (b) $2x + 2y = 1$ $4x + 9y = 18$ (d) none of these

A. $2x + 9y = 18$

B. $2x + 2y = 1$

C. $4x + 9y = 18$

D. $4x + 2y = 1$

Answer: C

Watch Video Solution

16. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ and $\frac{x^4}{4} + \frac{y^2}{b^2} = 1$. Then the value of b and the other common tangent are given by : (A) $b = \sqrt{3}$ (B) $x + 2y + 4 = 0$ (C) $b = 3$ (D) $x - 2y - 4 = 0$

A. $b = \sqrt{3}, x + 2y + 4 = 0$

B. $b = 3, x + 2y + 4 = 0$

C. $b = \sqrt{3}, x + 2y - 4 = 0$

D. $b = \sqrt{3}, x - 2y - 4 = 0$

Answer: A

Watch Video Solution

17. Find a point on the curve $x^2 + 2y^2 = 6$ whose distance from the line x+y=7, is minimum.

A. $(\sqrt{2}, \sqrt{2})$

B. $(0, \sqrt{3})$

C. $(2, 1)$

D. $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

Answer: C

Watch Video Solution

18. From a point on the axis of x common tangents are drawn to the parabola the ellipse y^2=4x and the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$. If these tangents from an equilateral trianlge with their chord of contact w.r.t parabola, then set of exhaustive values of a is

A. $(0, 3)$

B. $\left(0, \frac{3}{2}\right)$

C. $\left(\frac{3}{2}, 3\right)$

D. $\left(1, \frac{3}{2}\right)$

Answer: C

Watch Video Solution

19. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with vertices having eccentric angles $alpna, \beta, \gamma$, respectively is (x_1, y_1) then $\sum \cos\alpha \cos\beta + \sum \sin\alpha \sin\beta =$

A. $\frac{9h^2}{a^2} + \frac{9k^2}{b^2} + \frac{3}{2}$

B. $9h^2 - 9k^2 + a^2b^2$

C. $\frac{9h^2}{a^2} + \frac{9k^2}{b^2} + 3$

D. $\frac{9h^2}{2a^2} + \frac{9k^2}{2b^2} - \frac{3}{2}$

Answer: D

Watch Video Solution

20. A parabola is drawn whose focus is one of the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where

a>b) and whose directrix passes through the other focus and perpendicular to the major axes of the ellipse. Then the eccentricity of the ellipse for which the length of latus-rectum of the ellipse and the parabola are same is

A. $\sqrt{2}-1$

B. $\sqrt{3}-1$

C. $2\sqrt{2}-2$

D. $3\sqrt{3}-5$

Answer: A

Watch Video Solution

21. If the maximum distance of any point on the ellipse $x^2 + 2y^2 + 2xy = 1$ from its center is r, then r is equal to $3 + \sqrt{3}$ (b) $2 + \sqrt{2}$ $\frac{\sqrt{2}}{\sqrt{3 - \sqrt{5}}}$ (d) $\sqrt{2 - \sqrt{2}}$

A. a) $\frac{\sqrt{6} + 1}{2}$

B. b) $\frac{\sqrt{5} + 1}{2}$

C. c) $\frac{\sqrt{3} + 1}{2}$

D. d) $\frac{\sqrt{2} + 2}{2}$

Answer: B

Watch Video Solution

22. The length of the common chord of the ellipse $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$ and the circle $(x-1)^2+(y-2)^2=1$ is (A) 2 (B) $\sqrt{3}$ (C) 4 (D) none of these

A. zero

B. one

C. three

D. eight

Answer: A

Watch Video Solution

23. The eccentricity of ellipse $ax^2 + by^2 + 2gx + 2fy + c = 0$ if its axis is parallel to x-axis is

A. (a) $\sqrt{\left(\frac{b-a}{b}\right)}$

B. (b) $\sqrt{\left(\frac{a+b}{b}\right)}$

C. (c) $\sqrt{\left(\frac{a+b}{a}\right)}$

D. (d)None of these

Answer: A

Watch Video Solution

24. A circle has the same center as an ellipse and passes through the foci $F_1 and F_2$ of the ellipse, such that the two curves intersect at four points. Let P be any one of their point of intersection. If the major axis of the ellipse is

17 and the area of triangle PF_1F_2 is 30, then the distance between the foci is

A. (a)13

B. (b)11

C. (c)9

D. (d)7

Answer: A

Watch Video Solution

25. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angles $\frac{\pi}{4}$ is

A. $\frac{(a^2-b^2)ab}{a^2+b^2}$

B. $\left(\frac{a^2-b^2}{a^2+b^2}\right)$

C. $\frac{(a^2+b^2)ab}{(a^2-b^2)}$

D. $\frac{(a^2+b^2)}{(a^2-b^2)}$

Answer: A

Watch Video Solution

Watch Video Solution

26. An ellipse is inscribed in a circle and a point within the circle is chosen at random. If the probability that this point lies outside the ellipse is $\frac{2}{3}$ then the eccentricity of the ellipse is: (A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{8}{9}$ (D) $\frac{2}{3}$

A. $2\frac{\sqrt{2}}{3}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{8}{9}$

D. $\frac{2}{3}$

Answer: A

 Watch Video Solution

27. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

A. parabola

B. ellipse

C. hyperbola

D. circle

Answer: D

Watch Video Solution

28. about to only mathematics

A. $\frac{1}{10}$

B. $\frac{\sqrt{8161}}{10}$

C. $\frac{\sqrt{8061}}{10}$

D. None of the above

Answer: D

Watch Video Solution

29. The equation of the locus of the middle point of the portion of the tangent to the ellipse $x/16 + \frac{y^2}{9} = 1$ included between the co-ordinate axes is the curve

A. $9x^2 + 16y^2 = 4x^2y^2$

B. $16x^2 + 9y^2 = 4x^2y^2$

C. $3x^2 + 4y^2 = 4x^2y^2$

D. $9x^2 + 16y^2 = x^2y^2$

Answer: A

Watch Video Solution

30. The tangent at a point $P(a\cos\varphi, b\sin\varphi)$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets its auxiliary circle at two points, the chord joining which subtends a right angle at the center. Find the eccentricity of the ellipse.

A. $e^2\left(1+\cos^2\theta\right)=1$

B. $e^2\left(\cos ec^2\theta+1\right)=1$

C. $e^2\left(1+\sin^2\theta\right)=1$

D. $e^2\left(1+\tan^2\theta\right)=1$

Answer: C

Watch Video Solution

Exercise More Than One Correct Option Type Questions

1. The locus of extremities of the latus rectum of the family of ellipse $b^2x^2+a^2y^2=a^2b^2$ is

A. $x^2 - ay = a^2$

B. $x^2 - ay = b^2$

C. $x^2 + ay = a^2$

D. $x^2 + ay = b^2$

Answer: A::C

Watch Video Solution

2. The distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the center is 2. Then the

eccentric angle of the point is $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{7\pi}{4}$

Answer: A::B::C::D

Watch Video Solution

3. If the equation of family of ellipse is $x^2 \sec^2\theta + y^2 \cos ec^2\theta = 1, where \frac{\pi}{4} < \theta < \frac{\pi}{2}$, then the locus of extremities of the latusrectum is

A. $2y^2(1+x^2) = (1-x^2)^2$

B. $2x^2\left(1+y^2 = (1-y^2\right)^2$

C. $2y(1-x^2) = 1+x^2$

D. $2y^2(1+x^2) = 1+x^4-2x^2$

Answer: B::D

Watch Video Solution

Watch video solution

4. Let $F1, F_2$ be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at ponit P.then

A. (a)$PNbi\sec ts\angle F_1PF_2$

B. (b)$PTbi\sec ts\angle F_1PF_2$

C. (c)$PTbi\sec ts\angle(180\circ - \angle F_1PF_2)$

D. (d)None of above

Answer: A::C

Watch Video Solution

5. $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$ will represent ellipse if r lies in the interval (a).(-∞ ,2) (b). (3,∞) (c). (5,∞) (d).(1,∞)

A. $(- \infty, - 2)$

B. $(1, \infty)$

C. $(3, \infty)$

D. $(5, \infty)$

Answer: A::D

Watch Video Solution

6. A laturectum of an ellipse is a line

A. passing through a focus

B. passing through the najor axis

C. perpendicular to the major axis

D. parallel to the major axis

Answer: A::B::C

Watch Video Solution

7. An ellipse passes through the point $(4, -1)$ and touches the line $x + 4y - 10 = 0$. Find its equation if its axes coincide with the coordinate axes.

A. $x^2 + 64y^2 = 80$

B. $x^2 + 4y^2 = 20$

C. $x^2 + 20y^2 = 100$

D. $x^2 + 8y^2 = 40$

Answer: A::B

Watch Video Solution

8. If P is any point lying on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$,then

A. $SP + S'P = 2a, \quad \text{if} \quad a > b$

B. $SP + S'P = 2b, \quad \text{if} \quad b > a$

C. $\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\phi}{2}\right) = \frac{1-e}{e+1}$

D.

Answer: A::B::C

Watch Video Solution

9. If (5, 12) and (24, 7) are the foci of an ellipse passing through the origin, then find the eccentricity of the ellipse.

A. $\frac{\sqrt{386}}{38}$

B. $\frac{\sqrt{386}}{12}$

C. $\frac{\sqrt{386}}{13}$

D. $\frac{\sqrt{386}}{25}$

Answer: A::B

Watch Video Solution

10. about to only mathematics

A. $x - 2y = 0$

B. $2x - y = 0$

C. $x + 2y = 0$

D. $2x + y = 0$

Answer: A::C

Watch Video Solution

11. In the ellipse

$$25x^2 + 9y^2 - 150x - 90y + 225 = 0$$

A. foci are at $(3, 1), (3, 9)$

B. $e = \frac{4}{5}$

C. center is $(5, 3)$

D. major axis axis is 6

Answer: A::B

Watch Video Solution

12. If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point 0 sanormal to the circle $x^2 + y^2 - 8x - 4y = 0$ then θ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 0

D. $-\frac{\pi}{4}$

Answer: A::C

Watch Video Solution

13. The product of eccentricities of two conics is unity, one of them can be a/an

A. parabola

B. ellipse

C. hyperbola

D. circle

Answer: A::B::C

 Watch Video Solution

14. The parametric $\angle\alpha$ where$-\pi < \alpha \leq \pi$ of the point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at which the tangent drawn cuts the intercept of minimum length on the coordinates axes, is/are

A. $\tan - \sqrt{\frac{b}{a}}$

B. $-\tan(-1)\sqrt{\frac{b}{a}}$

C. $\pi - \tan^{-1}\sqrt{\frac{b}{a}}$

D. $\pi + \tan^{-1}\sqrt{\frac{b}{a}}$

Answer: A::B::C

Watch Video Solution

15. If latus recturn of the ellipse $x^2 \tan^2 \alpha + y^2 \sec^2 \alpha = 1$ is $\frac{1}{2}$ then $\alpha(0 < \alpha < \pi)$ is equal to

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{5\pi}{12}$

D. $\frac{\pi}{2}$

Answer: A::C

Watch Video Solution

Exercise Passage Based Questions

1. A conic is represented by

$C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$

Q. The centre of conic C is

A. (0,0)

B. (1,0)

C. (0,1)

D. (1,1)

Answer: D

Watch Video Solution

2. A conic is represented by

$C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$

Q. The lenghts of axis of conic C is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{2}{3}$

D. $\frac{2}{\sqrt{5}}$

Answer: B

Watch Video Solution

3. A conic is represented by

$C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$

Q. The centre of conic C is

A. $2, 2\sqrt{2}$

B. $4, 2\sqrt{3}$

C. $6, 2\sqrt{5}$

D. $5, 2\sqrt{6}$

Answer: A

Watch Video Solution

4. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the

prependicular segeent from the focii on tangent at point P is

A. 20

B. 45

C. 40

D. 90

Answer: A

Watch Video Solution

5. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the prependicular segeent from the focii on tangent at point P is

A. $\left(5, \frac{5}{3}\right)$

B. $3, \left(\frac{4}{3}\right)$

C. $\left(5, \left(\frac{10}{3}\right)\right.$

D. $3, \left(\frac{8}{3}\right)$

Answer: C

Watch Video Solution

6. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the prependicular segeent from the focii on tangent at point P is

A. $e_1 = \frac{3}{\sqrt{5}}$

B. $e_1 = \frac{\sqrt{5}}{3}$

C. $e_1 = \frac{3}{\sqrt{10}}$

D. $e_1 = \frac{\sqrt{10}}{3}$

Answer: B

Watch Video Solution

7. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ interset at four distinct points A,B,C, and D. Their common tangents form a peaallelogram A'B'C'D'.

if A'B'C'D' is a square, then r is equal to

Watch Video Solution

8. Curves $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A,B,C and D. Their common tangents from a parallelogram PQRS. Q. If ABCD is sqyare, then the value of $25r^2$ is

A. 12

B. 15

C. 20

D. 25

Answer: D

Watch Video Solution

9. Curves $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A,B,C and D. Their common tangents from a parallelogram PQRS. Q. If ABCD is sqyare, then the value of $25r^2$ is

A. $1 : 4$

B. $1:2$

C. $3:4$

D. $9:16$

Answer: B

Watch Video Solution

10. An ellipse whose distance between foci S and S' is 4 units is inscribed in the ΔABC touching thesides AB, AC and $BCatP, Q$ and R. If centre of

ellipse is at origin and major axis along x-axis

$SP + S'P = 6$, then

A. $9x^2 + 5y^2 = 45$

B. $4x^2 + 9y^2 = 46$

C. $5x^2 + 9y^2 = 45$

D. $9x^2 + 4y^2 = 36$

Answer: C

Watch Video Solution

11. An ellipse whose distance between foci S and S' is 4 units is inscribed in the $\triangle ABC$ touching the sides AB,AC and BC at P,Q and R, respectively. If centre of ellipse is at origin and major axis along X-axis, SP+S'P=6` Q. Equation of the ellipse is

A. $\left(x^2+y^2-14\right)^2=4\left(5x^2+9y^2-45\right)$

B. $\left(x^2+y^2-14\right)^2=4\left(5x^2+9y^2-54\right)$

C. $\left(x^2+y^2-14\right)^2=4\left(9x^2+5y^2-45\right)$

D. $\left(x^2+y^2-14\right)^2=4\left(9x^2+5y^2-54\right)$

Answer: A

Watch Video Solution

12. An ellipse whose distance between foci S and S' is 4 units is inscribed in the $\triangle ABC$ touching the sides AB,AC and BC at P,Q and R, respectively. If centre of ellipse is at origin and major axis along X-axis, SP+S'P=6` Q. Equation of the ellipse is

A. $5x^2 + 9y^2 = 15$

B. $5x^2 + 9y^2 = 60$

C. $9x^2 + 5y^2 = 14$

D. $9x^2 + 5y^2 = 144$

Answer: B

Watch Video Solution

13. The line$2px + y\sqrt{1 - p^2} = 1(|p| < 1)$ for different values of p, touches a fixed ellipse whose exes are the coordinate axes. Q. The eccentricity of the ellipse is

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{5}}$

Answer: C

Watch Video Solution

14. For all real p, the line $2px + y\sqrt{1-p^2} = 1$ touches a fixed ellipse whose axex are the

coordinate axes

The foci of the ellipse are

A. $\left(\pm \frac{\sqrt{3}}{2}, 0 \right)$

B. $\left(0, \pm \frac{\sqrt{3}}{2} \right)$

C. $\left(\pm \sqrt{3}, 0 \right)$

D. $\left(0, \pm \sqrt{3} \right)$

Answer: B

Watch Video Solution

15. The line$2px + y\sqrt{1 - p^2} = 1(|p| < 1)$ for different values of p, touches a fixed ellipse whose exes are the coordinate axes. Q. The locus of the point of intersection of prependicular tangents of the ellipse is

A. $x^2 + y^2 = 2$

B. $x^2 + y^2 = \frac{3}{2}$

C. $x^2 + y^2 = \frac{5}{4}$

D. $x^2 + y^2 = \frac{1}{2}$

Answer: C

Watch Video Solution

Exercise Single Integer Answer Type Questions

1. Two concentric ellipse be such that the foci of one be on the other and if 3/5 and 4/5 be their eccentricities. If θ is the angle between their axes, then the values of $2\left(1+\sin^2\theta+\sin^4\theta\right)$ must be

Watch Video Solution

2. Rectangle ABCD has area 200.An ellipse with area 200π passes through A and C and has foci at B and D. Find the perimeter of the rectangle.

Watch Video Solution

3. The number of points on the ellipse $\frac{x^2}{50}+\frac{y^2}{20}=1$ from which a pair of perpendicular tangents is drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is 0 (b) 2 (c) 1 (d) 4

Watch Video Solution

4. The length of the sides of the square which can be made by four perpendicular tangents to the ellipse $\frac{x^2}{7}+\frac{2y^2}{11}=1$, is

Watch Video Solution

5. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point $P(0,6)$ is one (b) two (c) three (d) four

Watch Video Solution

6. If p is the length of the perpendicular from a focus upon the tangent at any point P of the the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and r is the distance of P from the focus , then $\frac{2a}{r}-\frac{b^2}{p^2}$ is equal to

Watch Video Solution

7. An ellipse passing through the origin has its foci (3, 4) and (6, 8). The length of its semi-minor axis is b. Then the value of $\frac{b}{\sqrt{2}}$ is____

Watch Video Solution

8. The maximum value of 5λ for which four normals can be drawn to ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ through a point (λ,0) is

Watch Video Solution

9. An ellipse with major and minor axis $6\sqrt{3}$ and 6 respectively, slides along the coordinates axes and always remains confined in the first quardrant. If the length if arc decribed by center of ellipse is $\frac{\pi\lambda}{6}$ then the value of lambda is

Watch Video Solution

Ellipse Exercise 5 Matching Type Questions

1. Match the following

Column I	Column II
(A) For the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with vertices A and A', tangents drawn at the point P in the first quadrant meets the $y-$axis at Q and the chord $A'P$ meets the y-axis at M. If O is the origin, then OQ^2-MQ^2 is a	(p) Natural number
(B) If $y=x$ and $3y+2x=0$ are the equations of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and e be the eccentricity, then $4\,(1+e^2+e^4+\ldots+\infty)$ is a	(q) Composite number
(C) If the variable line $y=kx+2h$ is tangent to an ellipse $2x^2+3y^2=6$, then the locus of $P(h,k)$ is a conic C whose eccentricity is e, thus $3e^2$ is a	(r) Prime number
(D) If extremities of the latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>1)$ having positive ordinates lie on the parabola $x^2=-2(y-2)$, then a is a	(s) Perfect number

Watch Video Solution

2. Match the following

Column I	Column II
(A) For the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ with vertices A and A', tangents drawn at the point P in the first quadrant meets the $y-$axis at Q and the chord $A'P$ meets the y-axis at M. If O is the origin, then OQ^2-MQ^2 is a	(p) Natural number
(B) If $y=x$ and $3y+2x=0$ are the equations of a pair of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and e be the eccentricity, then $4\,(1+e^2+e^4+\ldots+\infty)$ is a	(q) Composite number
(C) If the variable line $y=kx+2h$ is tangent to an ellipse $2x^2+3y^2=6$, then the locus of $P(h,k)$ is a conic C whose eccentricity is e, thus $3e^2$ is a	(r) Prime number
(D) If extremities of the latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>1)$ having positive ordinates lie on the parabola $x^2=-2(y-2)$, then a is a	(s) Perfect number

Watch Video Solution

3. Match the following

Column I	Column II
(A) The minimum and maximum distances of a point (2,6) from the ellipse $9x^2 + 8y^2 - 36x - 16y - 28 = 0$ are L and G, then	(p) $L + G = 16$
(B) The minimum and maximum distances of a point (1,2) from the ellipse $4x^2 + 9y^2 + 8x - 36y + 4 = 0$ are L and G, then	(q) $L + G = 6$
(C) The minimum and maximum distances of a point $\left(\frac{9}{5}, \frac{12}{5}\right)$ from the ellipse $4(3x + 4y)^2 + 9(4x - 3y)^2 = 900$ are L and G, then	(r) $G - L = 8$
(D) The minimum and maximum distances of a point (0,4) from the ellipse $25x^2 + 9y^2 = 225$ are L and G, then	(s) $G - L = 6$

Watch Video Solution

Exercise Statement I And Ii Type Questions

1. Let the equation of ellipse be $\frac{x^2}{a^2+1}=\frac{y^2}{a^2+2}=1$ Statement 1 If eccentricity of the ellipse be $\frac{1}{\sqrt{6}}$, then length of latusrectum is $\frac{10}{\sqrt{6}}$. Statement 2 Length of latusrectum=$\frac{2\left(a^2+1\right)}{\sqrt{a^2+2}}$

Watch Video Solution

2. Statement 1 : The area of the ellipse $2x^2+3y^2=6$ is more than the area of the circle $x^2+y^2-2x+4y+4=0$. Statement

2 : The length f the semi-major axis of an ellipse is more that the radius of the circle.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: B

Watch Video Solution

3. Statement 1 The equation of the director circle to the ellipse $4x^2 + 9y^2 = 36 is x^2 + y^2 = 13$

Statement 2 The locus of the point of intersection of perpendicular tangents to an ellipse is called the director circle.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

 Watch Video Solution

4. In an ellipse, the sum of the distances between foci is always less than the sum of focal distances of any point on it. Statement 2 : The eccentricity of any ellipse is less than 1.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation

for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

Watch Video Solution

5. Statement 1 The sum of the focal distances of a point on the ellipse $4x^2 + 5y^2 - 16x - 30y41 = 0is2\sqrt{5}.$

Statement 2 The equation

$4x^2 + 5y^2 - 16x - 30y + 41 = 0$ can be expressed as $4(x-2)^2 + 5(y-3)^2 = 20$.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: B

Watch Video Solution

6. Statement 1 : The locus of the center of a variable circle touching two circle $(x-1)^2+(y-2)^2=25$ and $(x-2)^2+(y-1)^2=16$ is an ellipse.

Statement 2 : If a circle $S_2=0$ lies completely inside the circle $S_1=0$, then the locus of the center of a variable circle $S=0$ that touches both the circles is an ellipse.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: C

Watch Video Solution

7. Statement 1 The tangent and normal at any point P on a ellipse bisect the external and internal angles between the focal distance of P.

Watch Video Solution

8. Statement 1 : In a triangle ABC , if base BC is fixed and the perimeter of the triangle is constant, then vertex A moves on an ellipse.

Statement 2 : If the sum of the distances of a point P from two fixed points is constant, then the locus of P is a real ellipse.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

Watch Video Solution

Ellipse Exercise 7 Subjective Type Questions

1. If the normal at any point P on the ellipse cuts the major and mirror axes in G and g respectively and C be the centre of the ellipse, then

Watch Video Solution

2. If theta is the difference of the eccentric angles of two points on an ellipse, the tangents at which are at right angles. Prove that $ab\sin\theta = d_1 d_2$, where d_1, d_2 are the semi diameters parallel to the tangents at the points and a,b are the semi-axes of the ellipse.

Watch Video Solution

3. TP and TQ are tangents drawn from an external point (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ show that (i) $\frac{(ST)^2}{SP.SQ} = x^2 - \frac{1}{a^2} + y^2 - \frac{1}{b^2}$ (ii) $ST.S'T\cos\theta = (CT)^2 - a^2 - b^2$ where, S and S' are the foci, C the center and theta is the angle between the tangents.

Watch Video Solution

Exercise Subjective Type Questions

1. about to only mathematics

Watch Video Solution

2. A straight line PQ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=r^2(b<r<a)$. RS is a focal chord of the ellipse. If RS is parallel to PQ and meets the circle at points R and S. Find the length of RS.

Watch Video Solution

3. about to only mathematics

Watch Video Solution

Exercise Questions Asked In Previous 13 Years Exam

1. The minimum area of the triangle formed by the tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinate axes is ab sq. units (b)

$\frac{a^2+b^2}{2} sq \cdot units$ $\frac{(a+b)^2}{2} sq \cdot units$ (d) $\frac{a^2+ab+b^2}{3}$ sq. units

A. ad sq units

B. $\frac{a^2+b^2}{2}$ sq units

C. $\frac{(a+b)^2}{2}$ sq units

D. $\frac{a^2+ab+b^2}{3}$ sq units

Answer: A

Watch Video Solution

2. about to only mathematics

Watch Video Solution

3. An ellipse has OB as the semi-minor axis, $FandF'$ as its foci, and $\angle FBF'$ a right angle. Then, find the eccentricity of the ellipse.

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{\sqrt{3}}$

Answer: A

Watch Video Solution

4. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then its eccentricity is

A. $\frac{3}{5}$

B. $\frac{1}{2}$

C. $\frac{4}{5}$

D. $\frac{1}{\sqrt{5}}$

Answer: A

Watch Video Solution

5. about to only mathematics

A. $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

B. $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

C. $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

D. $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

Answer: B::C

Watch Video Solution

6. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$ Then the length of the semi-major axis is

A. $\frac{8}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{3}$

Answer: A

Watch Video Solution

7. The line passing through the extremity A of the major exis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets is auxiliary circle at the point M. Then the area of the triangle with vertices at $A, M,$ and O (the origin) is $(a)\frac{31}{10}$ $(b)\frac{29}{10}$ $(c)\frac{21}{10}$ $(d)\frac{27}{10}$

A. $\frac{31}{10}$

B. $\frac{29}{10}$

C. $\frac{21}{10}$

D. $\frac{27}{10}$

Answer: D

Watch Video Solution

8. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of

the given ellipse at points.

$$\left(\pm \frac{\left(3\sqrt{5}\right)}{2} \pm \frac{2}{7} \right)$$ (b)

$$\left(\pm \frac{\left(3\sqrt{5}\right)}{2} \pm \frac{\sqrt{19}}{7} \right) \left(\pm 2\sqrt{3}, \pm \frac{1}{7} \right)$$ (d)

$$\left(\pm 2\sqrt{3} \pm \frac{4\sqrt{3}}{7} \right)$$

A. $\left(\pm \left(3\frac{\sqrt{5}}{2}, \pm \frac{2}{7} \right) \right.$

B. $\left(\pm \left(3\frac{\sqrt{5}}{2}, \pm \frac{\sqrt{19}}{4} \right) \right.$

C. $\left(\pm 2\sqrt{3}, \pm \frac{1}{7} \right)$

D. $\left(\pm 2\sqrt{3}, \pm \left(4\frac{\sqrt{3}}{7} \right) \right.$

Answer: C

Watch Video Solution

9. A triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4\frac{\sin^2 A}{2}\cdot$ If $a, bandc,$ denote the length of the sides of the triangle opposite to the angles $A, B, andC$, respectively, then (a)$b + c = 4a$ (b) $b + c = 2a$ (c)the locus of point A is an ellipse

(d)the locus of point A is a pair of straight lines

A. b+c=4a

B. b+c=2a

C. locus of point A is an ellipse

D. locus of point A is a pair od straight lines

Answer: B::C

Watch Video Solution

10. The conic having parametric representation $x = \sqrt{3}\left(1 - \frac{t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$ is

A. an circle

B. a parabola

C. an ellipse

D. a hyperbola

Answer: C

Watch Video Solution

11. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the ellipse is (1) $x^2 + 16y^2 = 16$ (2) $x^2 + 12y^2 = 16$ (3) $4x^2 + 48y^2 = 48$ (4) $4x^2 + 64y^2 = 48$

A. $x^2 + 12y^2 = 16$

B. $4x^2 + 48y^2 = 48$

C. $4x^2 + 64y^2 = 48$

D. $x^2 + 16y^2 = 16$

Answer: A

Watch Video Solution

12. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at point A and B. Q. The coordinates of A and B are

A. (3,0) amd (0,2)

B. $\left(-\frac{8}{5}, \left(2\frac{\sqrt{161}}{15}\right)\right)$ and $\left(-\frac{9}{8}, \frac{8}{5}\right)$

C. $\left(-\frac{8}{5}, \left(2\frac{\sqrt{161}}{15}\right)\right)$ and $(0, 2)$

D. $(3, 0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Answer: D

Watch Video Solution

13. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at point A and B. Q. The orthocenter of the trianlge PAB is

A. $\left(5, \frac{8}{7}\right)$

B. $\left(\frac{7}{5}, \frac{25}{8}\right)$

C. $\left(\frac{11}{5}, \frac{8}{5}\right)$

D. $\left(\frac{8}{25}, \frac{7}{5}\right)$

Answer: C

Watch Video Solution

14. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the

ellipse at point A and B. Q. The equation of the locus of the points whose distance from the point P and the line AB are equal, is

A.

$$\left(9x^2+y^2-6xy-54x-62y+241=0\right.$$

B.

$$x^2+9y^2+6xy-54x+62y-241=0$$

C.

$$9x^2+9y^2-6xy-54x-62y-241=0$$

D. $x^2+y^2-2xy+27x+31y-120=0$

Answer: A

Watch Video Solution

15. Find the equation of an ellipse hose axes lie along the coordinate axes, which passes through the point (-3,1) and has eccentricity equal to $\sqrt{2/5}$.

A. $5x^2 + 3y^2 - 48 = 0$

B. $3x^2 + 5y^2 - 15 = 0$

C. $5x^2 + 3y^2 - 32 = 0$

D. $3x^2 + 5y^2 - 32 = 0$

Answer: D

Watch Video Solution

16. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

A. $\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: C

Watch Video Solution

17. Statement 1: An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4isy = 2x + 2\sqrt{3}$.

Statement 2: If the line $y = mx + \frac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement 1 is true,

Statement 2 is true,

Statement 2 is a correct explanation for statemennt 1

C. Statement 1 is true,statement 2 is true,

Statement 2 is not a correct explanation for statement 1.

D. Statement 1 is true,

Statement 2 is false.

Answer: B

Watch Video Solution

18. An ellipse is drawn by taking a diameter of the circle $(x-1)^2+y^2=1$ as its semi-minor axis and a diameter of the circle $x^2+(y-2)^2=4$ as its semi-major axis. If the centre of the ellipse is the origin and its

axes are the coordinate axes, then the equation of the ellipse is (1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

A. $4x^2 + y^2 = 4$

B. $x^2 + 4y^2 = 8$

C. $4x^2 + y^2 = 8$

D. $x^2 + 4y^2 = 16$

Answer: D

Watch Video Solution

19. the equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0,3) is

A. $x^2 + y^2 - 6y - 7 = 0$

B. $x^2 + y^2 - 6y + 7 = 0$

C. $x^2 + y^2 - 6y - 5 = 0$

D. $x^2 + y^2 - 6y + 5 = 0$

Answer: A

Watch Video Solution

20. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q.Let the tangents to the ellipse at P and Q meet at R. If $\Delta(h)$ Area of triangle ΔPQR, and $\Delta_1 = \max_{\frac{1}{2} \leq h \leq 1} \Delta(h)$ and $\Delta_2 = \min_{\frac{1}{2} \leq h \leq 1} \Delta(h)$ Then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2$

Watch Video Solution

21. The locus of the foot of prependicular drawn from the center of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

A. $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

B. $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

C. $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$

D. $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

Answer: A

Watch Video Solution

22. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the end of latus rectum. Find the area of quadrilateral so formed

A. $\frac{27}{2}$

B. 27

C. $\frac{27}{4}$

D. 18

Answer: B

Watch Video Solution

23. Let E1 and E2, be two ellipses whose centers are at the origin.The major axes of E1 and E2, lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line x+ y =3 touches the curves S, E1 and E2 at P,Q and R, respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$ .If e1 and e2 are the eccentricities of E1 and E2, respectively, then the correct expression(s) is(are):

A. (a)$e_1^2 + e_2^2 = \frac{43}{40}$

B. (b) $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

C. (c) $|e|_1^2 - e_2^2 \mid = \frac{5}{8}$

D. (d) $e_1 e^2 = \frac{\sqrt{3}}{4}$

Answer: A::B

Watch Video Solution

24. Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0) and (f_2, 0)$ where $f_1 > 0 and f_2 < 0.$ Let $P_1 and P_2$ be two parabolas with a common vertex at (0, 0) and

with foci at $(f_1.0)and$ (2f_2 , 0), respectively. LetT_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangents to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is

Watch Video Solution

25. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of

the triangle MQR to area of the quadrilateral MF1NF2 is

A. $\left(-\frac{9}{10}, 0\right)$

B. $\left(\frac{2}{3}, 0\right)$

C. $\left(\frac{9}{10}, 0\right)$

D. $\left(\frac{2}{3}, \sqrt{6}\right)$

Answer: A

Watch Video Solution

26. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF1NF2 is

A. (a)$3:4$

B. (b)$4:5$

C. (c)$5:8$

D. (d)$2:3$

Answer: C

Watch Video Solution

27. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = - 4, then the equation of the normal to it at (1, 3/2) is :

A. $x + 2y = 4$

B. $2y - x = 2$

C. $4x - 2y = 1$

D. $4x + 2y = 7$

Answer: C

Watch Video Solution