



## MATHS

### BOOKS - ARIHANT MATHS (ENGLISH)

### ESSENTIAL MATHEMATICAL TOOLS

#### Examples

1. Solve  $2x + 1 > 3$ .



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2. solve  $-2 < 2x - 1 < 2$ .

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3. solve the following inequations.

$$\frac{3(x - 2)}{5} \geq \frac{5(2 - x)}{3}$$

$$(ii) \frac{2x - 2}{4} + 9 \geq 3 + \frac{4x}{3}$$

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4. solve for X.  $\frac{4}{x + 1} \leq 3 \leq \frac{6}{x + 1}, (x > 0)$

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5. Explain the following : (i)  $|x| = 5$  (ii)  $|x| = -5$  (iii)

$|x| < 5$  (iv)  $|x| < -5$  (v)  $|x| > -5$  (vi)  $|x| < 5$



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6. solve for ,x where  $f(x) = |x| \geq 0$



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7. solve  $|x - 3| < 5$ .



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8. solve  $|x - 1| \leq 2$ .



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9. Solve the following systems of linear inequations:

$$1 \leq |x - 2| \leq 3$$



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10. solve  $1 \leq |x - 1| \leq 3$ ,



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11. solve  $\left| \frac{2}{x-4} \right| > 1, x \neq 4.$

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12. Solve  $|x - 1| + |x - 2| \geq 4$

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13. Find the interval in which  $f(x)$  is positive or negative :  $f(x) = (x-1)(x-2)(x-3)$

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14. solve  $f(x) = \frac{(x - 1)(2 - x)}{(x - 3)} \geq 0$ .

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15. find the value of x for which

$$f(x) = \frac{(2x - 1)(x - 1)^2(x - 2)^2}{(x - 4)^2} \geq 0.$$

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16. Find the value of x for which

$$f(x) = \frac{(x - 2)^2(1 - x)(x - 3)^2(x - 4)^2}{(x - 1)} \leq 0.$$

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17. Solve:  $\frac{|x| - 1}{|x| - 2} \geq 0, x \neq \pm 2$

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18. Solve:  $\frac{-1}{|x| - 2} \geq 1, \text{ where } x \in R, x \neq \pm 2.$

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19. solve  $\frac{|x + 3| + x}{x + 2} > 1$

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20. If  $f(x) = \frac{(x-1)^3(x+2)^4(x-3)^5(x+6)}{x^2(x-7)^3}$ ,

Solve the following inequality  $f \geq 0$

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21.

Let

$$F(x) = \frac{\left(\sin x - \frac{1}{2}\right)(\ln x - 1)^4(x-2)^3\left(\tan x - \sqrt{3}\right)^5}{(e^x - e^2)(x-3)^6(\sin^2 x - 1)}$$

solve the following inequalities for  $x \in (0, 2\pi)$

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22.

Let

$$f(x) = \frac{(\cos x + |\cos x|) \left(\sin x - \frac{3}{2}\right)^3 (\tan x - 1)^5}{(\cos x - 2)^2 (\tan x - \sqrt{3})^3}$$

Find the interval of  $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  for which (i)  $f(x)$

$> 0$  (ii)  $f(x) < 0$



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23. find  $a$  for which  $3x^2 + ax + 3 > 0, \forall x \in R.$



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24. find  $a$  for which  $ax^2 + x - 1 < 0, \forall x \in R?$



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25. Solve  $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$ .



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26. Solve  $|x + 1| + \sqrt{x} - 1 = 0$ .



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Example

1. Solve

$$|x^2 - 1| + (x - 1)^2 + \sqrt{x^2} - 3x + 2 = 0.$$



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2. Let  $f(x) = x$  and  $g(x) = |x|$  be two real-valued functions, check whether the functions are one -one.



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## Exercise For Session 1

1. All the rational numbers are irrational also.



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2. All the integers are irrational also.



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3. Irrational numbers are real numbers also.



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4. Zero is a natural number.



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5. Sum of two natural numbers is a rational number.



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6. A positive interber is a natural number also.



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7. sum of two rational numbers is

A. rational

B. irrational

C. Both (a) and (b)

D. None of these

**Answer: A**



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**8.** Which one of the following statement is true?

- (i) The sum of two irrational numbers is always an irrational number.
- (ii) The sum of two irrational numbers is always a rational number.
- (iii) The sum of two irrational numbers may be a rational number or irrational number.

(iv) The sum of two irrational numbers is always an integer.

A. rational

B. irrational

C. real

D. None of these

**Answer: C**



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**9. Product of two rational numbers is**

- A. always rational
- B. rational or irrational
- C. always irrational
- D. None of these

**Answer: A**



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**10.** If  $a$  is an irrational number which is divisible by  $b$ , then the number  $b$

- A. must be rational
- B. must be irrational



C. may be rational or irrational

D. None of these

**Answer: C**



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## Exercise For Session 2

1.  $|x - 1| < 2$



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2.  $|x - 3| > 2$



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3. Solve:  $0 < |x - 1| < 3$



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4.  $|x - 1| + |2x - 3| = |3x - 4|$



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5.  $\left| \frac{x - 3}{x^2 - 4} \right| \leq 1.$



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## Exercise For Session 3

1. Solve  $\frac{x - 2}{x + 2} > \frac{2x - 3}{4x - 1}$



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2.  $\frac{2x - 1}{2x^3 + 3x^2 + x} > 0.$



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3.  $\frac{4x}{x^2 + 3} \geq 1.$



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$$4. \frac{(x - 1)(x + 1)(x + 4)(x + 6)}{7x^2 + 8x + 4} > 0$$



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$$5. \frac{x}{x^2 - 5x + 9} \leq 1.$$



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6. Solution of inequality  $|x - 1| < 0$  is

A.  $x=0$

B.  $x=1$

C.  $x \neq 1$

D. No solution

**Answer: D**



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7. Solution of inequality  $x^2 + x + |x| + 1 \leq 0$  is

A. (1,2)

B. (0,1)

C. No solution

D. None of these

**Answer: C**



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8. Solution of inequality  $x^2 + x + |x| + 1 \leq 0$  is

A.  $\left(-\frac{2}{3}, 4\right)$

B.  $(4, \infty)$

C.  $\left(-\frac{2}{3}, 1\right)$

D. None of these

**Answer: D**

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9. Solution of inequality  $\left|x + \frac{1}{x}\right| < 4$  is

A.  $(2 - \sqrt{3}, 2 + \sqrt{3}) \cup (-2 - \sqrt{3}, -2 + \sqrt{3})$

B.  $R - (2 - \sqrt{3}, 2 + \sqrt{3})$

C.  $R - (-\sqrt{2} - \sqrt{3}, 2 + \sqrt{3})$

D. None of these

**Answer: A**

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10. The solution of  $|x^2 + 3x| + x^2 - 2 \geq 0$  is `

A.  $(-\infty, 1)$

B.  $(0,1)$

C.  $\left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$

D. None of these

**Answer: C**



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11. The solution of  $||x| - 1| < |1 - x|, x \in R$  is

A.  $(-1, 1)$



B.  $(0, \infty)$

C.  $(-1, \infty)$

D. None of these

**Answer: D**



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**12.** The solution of  $2^x + 2^{|x|} \geq 2\sqrt{2}$  is

A.  $(-\infty, \log_2(\sqrt{2} + 1))$

B.  $(0, \infty)$

C.  $\left(\frac{1}{2}, \log_2(\sqrt{2} - 1)\right)$

$$D. (-\infty \log_2(\sqrt{2} - 1)] \cup \left[\frac{1}{2}, \infty\right)$$

**Answer: D**



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## Exercise For Session 4

1. Find all values of 'm' which  $(2m - 3)x^2 + 2mx + 4 < 0$  for all real x.



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2. If  $ax^2 - bx + 5 = 0$  does not have two distinct real roots, then find the minimum value of  $5a+b$ .



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3. If  $x, y \in [0, 10]$ , then find the number of solutions  $(x,y)$  of the inequation  $3^{\sec^2 x - 1} \sqrt{9y^2 - 6y + 2} \leq 1$



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**Exercise Single Option Correct Type Questions**

1. For a  $a \leq 0$ , determine all real roots of the equation  $x^2 - 2a|x - a| - 3a^2 = 0$ .

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2. Solve  $|x^2 + 4x + 3| + 2x + 5 = 0$ .

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3. Solve  $|x^2 - 3x - 4| = 9 - |x^2 - 1|$

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4. Solve the equation

$$2^{|x+1|} - 2^x = |2^x - 1| + 1$$



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5. Find the set of all real 'a' such that  $5a^2 - 3a - 2$ ,  $a^2 + a - 2$  and  $2a^2 + a - 1$  are the lengths of the sides of a triangle?



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6. Solve  $(x + 3)^5 - (x - 1)^5 \geq 244$ .



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7. Solve  $||x - 2| - 1| \geq 3$ .

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8. Solve  $1 \leq \frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$ .

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9. Let  $f(x) = \frac{2x}{2x^2 + 5x + 2}$ , find the interval for which  $f(x) > 0$ .

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10. Draw the graph of  $||x||$  and check its injectivity.



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11. Solve the inequality  $|x - 1| + |2 - x| > 3 + x$ .



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12. Solve the equation

$$\sqrt{x^2 + 12y} + \sqrt{y^2 + 12x} = 33, x + y = 23.$$



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13. Solve the equation

$$\sqrt{2x - 1} + \sqrt{3x - 2} = \sqrt{4x - 3} + \sqrt{5x - 4}.$$

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14. If  $x, y$  and  $z$  are real such that

$$x + y + z = 4, x^2 + y^2 + z^2 = 6, x \text{ belongs to}$$

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15.

Solve

$$\left| \frac{x - 1}{3 + 2x - 8x^2} \right| + |1 - x| = \frac{(x - 1)^2}{|3 + 2x - 8x^2|} + 1$$

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16. Let  $f(x) = \frac{x^2 - 2x + 1}{x + 3}$ , find  $x$ : (i)  $f(x) > 0$

(ii)  $f(x) < 0$



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17. Solve the inequation:  $\left| 1 - \left( \frac{|x|}{1 + |x|} \right) \right| \geq \frac{1}{2}$



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Exercise More Than One Correct Option Type Questions

1. If  $\cos x - y^2 - \sqrt{y - x^2 - 1} \geq 0$ , then

A.  $y \geq 1$

B.  $x \in R$

C.  $y=1$

D.  $x=0$

**Answer: C::D**



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2. If  $(\sin \alpha)x^2 - 2x + b \geq 2$ , for all real values of

$x \leq 1$  and  $\alpha \in \left(0, \frac{\pi}{2}\right) \cup (\pi/2, \pi)$ , then possible

real value of  $b$  is /are a. 2 b. 3 c. 4 d. 5

A. 2

B. 3

C. 4

D. 5

**Answer: A::C**



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**3.** If  $|ax^2 + bx + c| \leq 1$  for all  $x$  in  $[0, 1]$  then

A.  $|a| \leq 8$

B.  $|b| \leq 8$

C.  $|c| \leq 1$

D.  $|a| + |b|_{|c|} \leq 17$

**Answer: A::B::D**

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## Exercise Passage Based Questions

1. Let  $f(x) = ax^2 + bx + c$ ,  $a, b, c \in R$ . It is given

$$|f(x)| \leq 1, |x| \leq 1$$

The possible value of  $|a + c|$ , if  $\frac{8}{3}a^2 + 2ab^2$  is

maximum is given by

A. 1

B. 0

C. 2

D. 3

**Answer: A**



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2. Let  $f(x) = ax^2 + bx + C$ ,  $a, b, c \in R$ . It is given

$|f(x)| \leq 1$ ,  $|x| \leq 1$  The possible value of  $|a + c|$ , if

$\frac{8}{3}a^2 + 2b^2$  is maximum, is given by

A. 1

B. 0

C. 2

D. 3

**Answer: A**



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3. Let  $f(x) = ax^2 + bx + c$ ,  $ab, c \in R$ . It is given

$$|f(x)| \leq 1, |x| \leq 1$$

The possible value of  $|a + c|$ , if  $\frac{8}{3}a^2 + 2ab^2$  is

maximum is given by

A. 32

B.  $\frac{32}{3}$

C.  $\frac{2}{3}$

D.  $\frac{16}{3}$

**Answer: B**



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4. Consider the equation  $|2x| - |x - 4| = x + 4$  The least integer satisfying the equation is

A. -4

B. 4

C. 5

D. -5

**Answer: A**



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5. Consider the equation  $|2x| - |x - 4| = x + 4$  Total number of prime numbers less than 20 satisfying the equation is

A. 3

B. 4

C. 5

D. 6



**Answer: D**



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6. Consider a number  $N = 21P53Q4$ .

The number of ordered pairs  $(P, Q)$  so that the number ' $N$ ' is divisible by 9, is

A. 11

B. 12

C. 10

D. 8

**Answer: A**



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7. Consider a number  $n = 21P53Q4$ .

The number of values of  $Q$  so that the number 'N' is divisible by 8, is

A. 4

B. 3

C. 2

D. 6

**Answer: B**



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8. Consider a number  $N = 21P53Q4$ . The number of ordered pairs  $(P, Q)$  so that the number 'N' is divisible by 44, is

A. 2

B. 3

C. 4

D. 5

**Answer: C**



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9. Consider the nine digit number  $n = 73\alpha 4961\beta 0$ .

If  $p$  is the number of all possible distinct values of  $(\alpha - \beta)$ , then  $P$  is equal to

A. 17

B. 18

C. 19

D. 20

**Answer: C**



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10. Consider the nine digit number  $n = 73\alpha 4961\beta$

0. If  $q$  is the number of all possible values of  $\beta$  for which the given number is divisible by 8, then  $q$  is equal to

A. 2

B. 3

C. 4

D. 5

**Answer: A**



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11. Consider the nine digit number  $n = 73\alpha4961\beta$

0. The number of ordered pairs  $(\alpha, \beta)$  for which the given number is divisible by 88, is

A. 1

B. 2

C. 3

D. 4

**Answer: B**



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12. Consider the nine digit number  $n = 73\alpha4961\beta$

0. The number of possible values of  $(\alpha + \beta)$  for which the given number is divisible by 6, is

A. 3

B. 4

C. 6

D. 7

**Answer: D**



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13. Consider the nine digit number  $n = 73\alpha 4961\beta$

0. The number of possible values of  $\beta$  for which

$$i^N = 1 \text{ (where } i = \sqrt{-1}\text{),}$$

A. 2

B. 3

C. 4

D. 5

**Answer: D**



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**14.** The set of integers can be classified into  $k$  classes, according to the remainder obtained when they are divided by  $K$  (where  $K$  is a fixed natural number). The classification enables us to solve even some more difficult problems of number theory e.g.

(i) even, odd classification is based on whether remainder is 0 or 1 when divided by 2.

(ii) when divided by 3, the remainder may be 0,1,2.

Thus, there are three classes.

$n^2 + n + 1$  is never divisible by

A. 0,1

B. 1,2

C. 0,2

D. 0,1,2

**Answer: A**



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**15.** The set of integers can be classified into  $k$  classes, according to the remainder obtained when they are divided by  $K$  (where  $K$  is a fixed natural number). The classification enables solving even some more difficult problems of number theory e.g.

(i) even, odd classification is based on whether remainder is 0 or 1 when divided by 2.

(ii) when divided by 3, the remainder may be 0,1,2.

Thus, there are three classes.

The number obtained, when the square of an integer is divided by 3, is

A. (a)2

B. (b)3

C. (c)1

D. (d)None of these

**Answer: C**



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**16.** The set of integers can be classified into  $k$  classes, according to the remainder obtained when they are divided by  $K$  (where  $K$  is a fixed natural number). The classification enables us to solve even some more difficult problems of number theory e.g.

(i) even, odd classification is based on whether remainder is 0 or 1 when divided by 2.

(ii) when divided by 3, the remainder may be 0,1,2.

Thus, there are three classes.

$n^2 + n + 1$  is never divisible by

A. 16

B. 15

C. 240

D. 720

**Answer: D**



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## Exercise Single Integer Answer Type Questions

1. The number of solutions of the equation

$$|x - 1| - |2x - 5| = 2x$$



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2. The number of integral solution of the equation

$$|x^2 - 7| \leq 9 \text{ are}$$



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3. Find the number of solution of the system of equation  $x+2y=6$  and  $|x-3|=y$



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