MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

HYPERBOLA

Examples

1. To find the equation of the hyperbola from the definition that hyperbola is the locus of a point which moves such that the difference of its

distances from two fixed points is constant with the fixed point as foci

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2. Find the equation of the hyperbola whose directrix is $2x + y = 1$, focus $(1, 2)$ and eccentricity $\sqrt{3}$.

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3. Find the lengths of the transvers and the conjugate axis, eccentricity, the coordinates of foci,

vertices, the lengths of latus racta, and the equations of the directrices of the following hyperbola: $16x^2 - 9y^2 = -144$.

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4. Write the eccentricity of the hyperbola whose latus rectum is half of its transverse axis.

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5. Prove that the point $\left\{\frac{a}{2}\left(t+\frac{1}{t}\right), \frac{b}{2}\left(t-\frac{1}{t}\right)\right\}$ lies on the hyperbola

for all values of $t(t \neq 0)$.

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6. Find the equation to the hyperbola whose foci, are (6,4) and (-4,4) and eccentricity is 2.

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7. Obtain the equation of a hyperbola with coordinate axes as principal axes given that the distances of one of its vertices from the foci are 9 and 1 units.

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8. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola, if its eccentricity is 2.

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9. Let two points P and Q lie on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$

whose centre C be such that CP is perpendicular to

CQ,

a lt b. Then the value of $\frac{1}{CP^2} + \frac{1}{CQ^2}$ is

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10. Find the position of the point (5, -4) relative to the hyperbola $9x^2 - y^2 = 1$.

A. inside

B. outside

C. on the hyperbola

D. none of the above

Answer:

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11. If the line $lx + my + n = 0$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Then

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12. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$.

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13. For what value of λ does the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$?

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14. If it is possible to draw the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having slope 2, then find its range of eccentricity.

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15. Find the equation of the tagent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.

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16. Find the equations to the common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

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17. PQ is a chord joining the points ϕ_1 and ϕ_2 on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If ϕ_1 and $\phi_2 = 2\alpha$, where$alha$ is constant, prove that PQ touches the hyperbola $\frac{x^2}{a^2}\cos^2\alpha - \frac{y^2}{b^2} = 1$

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18. If the line $y = mx + \sqrt{a^2m^2 - b^2}$ touches the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\phi, b\tan\phi)$, show that $\phi = \sin^{-1}\left(\frac{b}{a}m\right)$.

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19. A normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the axes at $MandN$ and lines MP and NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = (a^2 + b^2)$.

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20. The line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. then prove that $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{(a^2 - b^2)^2}{n^2}$

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21. If the normal at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{2a^2} = 1$ meets the transvers axis at G, then prove that $AG\dot{A}'G = a^2\left(e^4 \sec^2\theta - 1\right)$, where $AandA'$ are the vertices of the hyperbola.

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22. Find the locus of the foot of perpendicular from the centre upon any normal to line hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

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23. The locus of the poles of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which subtend a right angle at its centre is

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24. From the points on the circle $x^2 + y^2 = a^2$, tangents are drawn to the hyperbola $x^2 - y^2 = a^2$: prove that the locus of the middle-points $\left(x^2 - y^2\right)^2 = a^2\left(x^2 + y^2\right)$

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25. Prove that the locus of the middle-points of the chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ which pass through a fixed point (α, β) is a hyperbola whose centre is $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$.

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26. If the pair of straight lines $Ax^2 + 2Hxy + By^2 = 0$ be conjugate diameters of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $Aa^2 + Bb^2$.

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27. Find the asymptotes of the curve $xy - 3y - 2x = 0$.

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28. The asymptotes of a hyperbola are parallel to lines $2x + 3y = 0$ and $3x + 2y = 0$. The hyperbola has its centre at (1, 2) and it passes through (5, 3). Find its equation.

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29. If the normal to the rectangular hyperbola $xy = c^2$ at the point $'t'$ meets the curve again at t_1 then $t^3 t_1,$ has the value equal to

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30. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

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31. A ray emerging from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after the first reflection if point P lies in the first quadrant.

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32. The equation of the transverse and conugate axes of a hyperbola are respectively $3x + 4y - 7 = 0$ and $4x - 3y + 8 = 0$ and their respective lengths are 4 and 6. Find the equation of the hyperbola.

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33. If the eccentricity of the hyperbola $x^2 - y^2(\sec)^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2(\sec)^2\alpha + y^2 = 25,$ then a value of α is : (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

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34. Find the area of the triangle formed by any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with its asymptotes.

A. $\sec \lambda$

B. $\cos ec\lambda$

C. $\sec^2 \lambda$

D. $\cos ec^2\lambda$

Answer: A

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35. The equation to the chord joining two points $(x_1, y_1) and (x_2, y_2)$ on the rectangular hyperbola $xy = c^2$ is:

$\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$ $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

(d) $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

A. $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$

B. $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

C. $\frac{x}{y_1 + y_2} + \frac{y}{x_1 + x_2} = 1$

D. $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

Answer: A

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36. Area of the quadrilateral formed with the foci of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ (a) $4(a^2 + b^2)$ (b) $2(a^2 + b^2)$ (c) $(a^2 + b^2)$ (d) $\frac{1}{2}(a^2 + b^2)$

A. $4(a^2 + b^2)$

B. $2(a^2 + b^2)$

C. $(a^2 + b^2$

D. $\frac{1}{2}(a^2+b^2)$

Answer: B

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A. $\left(\frac{a^2+b^2}{a}\right)$

B. $-\left(\frac{a^2+b^2}{a}\right)$

C. $\left(\frac{a^2+b^2}{b}\right)$

D. $-\left(\frac{a^2+b^2}{b}\right)$

Answer: D

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38. Let the major axis of a standard ellipse equals the transverse axis of a standard hyperbola and their director circles have radius equal to 2R and R respectively. If e_1 and e_2, are the eccentricities of the ellipse and hyperbola then the correct relation is

A. (a) $4e_1^2 - e_2^2 = 6$

B. (b) $e_1^2 - 4e_2^2 = 2$

C. (c) $4e_2^2 - e_1^2 = 6$

D. (d) $e_2^2 - 4e_1^2 = 2$

Answer: C

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39. The tangent to the hyperbola $xy = c^2$ at the point P intersects the x-axis at T and y- axis at T'.The normal to the hyperbola at P intersects the x-axis at N and the y-axis at N' . The areas of the triangles PNT and PN'T' are Δ and Δ' respectively, then $\frac{1}{\Delta} + \frac{1}{\Delta}$, is

A. (a)equal to 1

B. (b)depends on t

C. (c)depends on c

D. (d)equal to 2

Answer: C

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40. Let any double ordinate PNP^1 of the hyperbol $\frac{x^2}{9} - \frac{y^2}{4} = 1$ be produced both sides to meet the asymptotes in Q and Q', then $PQ.\ P'Q$ is equal to

A. 9

B. 16

C. 25

D. 41

Answer: B

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41. The coordinates of a point on the hyperbola $\frac{x^2}{24} - \frac{y^2}{18} = 1$ which s nearest to the line $3x + 2y + 1 = 0$ are (6, 3) (b) $(-6, -3)$ $6, -3)$

(d) $(-6, 3)$

A. $(6, 3)$

B. $(-6, -3)$

C. $(6, -3)$

D. $(-6, 3)$

Answer: D

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42. For each positive integer consider the point P with abscissa n on the curve $y^2 - x^2 = 1$. If d_n represents the shortest distance from the point P

to the line $y = x$ then $\lim_{n \to \infty} (nd_n)$ has the value equal to:

A. (a) $\frac{1}{2\sqrt{2}}$

B. (b) $\frac{1}{2}$

C. (c) $\frac{1}{\sqrt{2}}$

D. (d) 0

Answer: A

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43. If two tangents can be drawn the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from $\left(\alpha, \alpha^2\right)$, then

A. $\alpha \in (-\infty, -2)$

B. $\alpha \in (-2, 0)$

C. $\alpha \in (0, 2)$

D. $\alpha \in (2, \infty)$

Answer: A::D

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44. If the ellipse $x^2 + \lambda^2 y^2 = \lambda^2 a^2, \lambda^2 > 1$ is confocal with the hyperbola $x^2 - y^2 = a^2$, then

A. ratio of eccentricities of ellipse and hyperbola is $1 : \sqrt{3}$

B. ratio of major axis of ellipse and transverse axis of hyperbola is $\sqrt{3} : 1$

C. `The ellipse and hyperbola cuts each other orthogonally

D. ratio of length of latusrectumof ellipse and hyperbola is $1 : 3$

Answer: A::B::C

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45. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = C^2$ at four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3),$ and $S(x_4, y_4),$ then $x_1 + x_2 + x_3 + x_4 = 0$ $y_1 + y_2 + y_3 + y_4 = 0$ $x_1x_2x_3x_4 = C^4$ $y_1y_2y_3y_4 = C^4$

A. $\sum x_1 = 0$

B. $\sum y_1 = 0$

C. $\prod x_1 = 0$

D. $\prod y_1 = 0$

Answer: A::B::C::D

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46. . A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. An equation of the line is

A. $9x + 3y - 8 = \infty$

B. $9x - 3y + 8 = 0$

C. $9x + 3y + 8 = 0$

D. $9x - 3y - 8 = 0$

Answer: B::C

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47. The differential equation $\frac{dy}{dx} = \frac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity. $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{5}{3}}$ $\sqrt{\frac{2}{5}}$ (d) $\sqrt{\frac{5}{2}}$

A. $\sqrt{\frac{7}{3}}$

B. $\sqrt{\frac{5}{3}}$

C. $\sqrt{\frac{3}{2}}$

D. $\sqrt{\frac{5}{2}}$

Answer: B::D

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48. A conic C satisfies the differential equation $\left(1+y^2\right)dx - xydy = 0$ and passes through the point $(1, 0)$. An ellipse E which is confocal with C having its eccentricity equal to $\sqrt{\frac{2}{3}}$

Q. Equation of ellipse E is

A. 1

B. 2

C. 3

D. 4

Answer: B

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49. A conic C satisfies the differential equation, $\left(1+y^2\right)dx - xydy = 0$ and passes through the point $(1, 0)$.An ellipse E which is confocal with C having its eccentricity equal to $\sqrt{\frac{2}{3}}$.

A. $\frac{x^2}{3}+\frac{y^2}{1}=1$

B. $\frac{x^2}{1}+\frac{y^2}{3}=1$

C. $\frac{x^2}{4}+\frac{y^2}{9}=1$

D. $\frac{x^2}{9}+\frac{y^2}{4}=1$

Answer: A

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50. A conic C satisfies the differential equation $\left(1+y^2\right)dx-xydy=0$ and passes through the point $(1,0)$. An ellipse E which is confocal with C

having its eccentricity equal to $\sqrt{\frac{2}{3}}$

Q. Equation of ellipse E is

A. $x^2 + y^2 = 4$

B. $x^2 + y^2 = 8$

C. $x^2 + y^2 = 10$

D. $x^2 + y^2 = 13$

Answer: A

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51. For the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to the normal from the centre. Q. The value $\frac{PF \cdot PG}{(CB^2)}$ is equal to

A. (a)4

B. (b)3

C. (c)2

D. (d)1

Answer: D

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52. For the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to the normal from the centre. Q. The value $PF \cdot Pg$ is equal to

A. $(CA)^2$

B. $(CF)^2$

C. $(CB)^2$

D. $CA \cdot CB$

Answer: A

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53. For the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to the normal from the centre. Q. Locus of middle-point of G and g is a hyperbola of eccentricity

A. $\frac{1}{\sqrt{e^2 - 1}}$

B. $\frac{e}{\sqrt{e^2 - 1}}$

C. $2\left(\sqrt{e^2 - 1}\right)$

D. $\frac{e}{2}$

Answer: B

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54. The equation of transverse axis of hyperbola (passing through origin) having asymptotes $3x - 4y = 1$ and $4x - 3y = 6$ is $ax + by - c = 0, a, b \in N$ and $\gcd(a, b, c) = 1$ then the value of $a + b + c$ is ___

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55. If a variable line has its intercepts on the coordinate axes $eande'$, where $\frac{e}{2}ande\frac{\prime}{\square}2$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2+y^2=r^2$, where $r=$ 1 (b) 2 (c) 3 (d) cannot be decided

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56. Statement-I Director circle of hypebola $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ is defined only when $b \geq a$.

Statement-II Director circle of hyperbola $\frac{x^2}{25}-\frac{y^2}{9}=1$ is $x^2+y^2=16$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

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57. Statement 1 : If a circle $S = 0$ intersects a hyperbola $xy = 4$ at four points, three of them being (2, 2), (4, 1) and $\left(6, \frac{2}{3}\right)$, then the coordinates of the fourth point are $\left(\frac{1}{4}, 16\right)$.

Statement 2 : If a circle $S = 0$ intersects a hyperbola $xy = c^2$ at $t_1, t_2, t_3,$ and t_3 then $t_1 - t_2 - t_3 - t_4 = 1$

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

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58. Prove that the perpendicular focal chords of a rectangular hyperbola are equal.

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59. The normal at three points P, Q, R on a rectangular hyperbola intersect at a point T on the curve. Prove that the centre of the hyperbola is the centroid of the triangle PQR.

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60. Find the equation of the hyperbola, whose asymptotes are the straight lines $(x + 2y + 3) = 0, (3x + 4y + 5) = 0$ and which passes through the point (1-1).

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61. In both an ellipse and hyperbola , prove that the focal distance of any point and the perpendicular from the centre upon the tangent at it meet on a circle whose centre is the focus and whose radius is the semi-transverse axis.

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62. A circle with centre $(3\alpha, 3\beta)$ and of variable radius cuts the rectangular hyperbola $x^2 - y^2 = 9a^2$ at the points P, Q, S, R. Prove that

the locus of the centroid of triangle PQR is $(x-2\alpha)^2-(y-2\beta)^2=a^2$.

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Example

1. Show that the equation $7x^2-9y^2+54x-28y-116=0$ represent a hyperbola. Find the coordinate of the centre, lenghts of transverse and conjugate axes, eccentricity, latusrectum, coordinates of foci, vertices and directrices of the hyperbola.

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2. If SY and S'Y' be drawn perpendiculars from foci to any tangent to a hyperbola. Prove that y and Y' lie on the auxiliary circle and that product of these perpendicular is constant.

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3. If a pair of conjugate diameters meets the hyperbola and its conjugate in P and D respectively, then prove that $CP^2 - CD^2 = a^2 - b^2$.

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4. For the hyperbola $x^2 - y^2 = a^2$, prove that the triangle CPD is isosceles and has constant area, where CP and CD are a pair of its conjugate diameter.

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5. If the lines $lx + my + n = 0$ passes through the extremities of a pair of conjugate diameters of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, show that $a^2l^2 - b^2m^2 = 0$.

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6. If any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with centre C, meets its director circle in P and Q, show that CP and CQ are conjugate semi-diameters of the hyperbola.

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7. Show that the tangent at any point of a hyperbola cuts off a triangle of constant area from the asymptotes and that the portion of it

intercepted between the asymptotes is bisected at the point of contact.

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Jee Type Solved Examples Subjective Type Questions

1. Consider hyperbola $xy = 22$ to find the equation of tangent at point $(2, 11)$.

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2. The tangent at the point P of a rectangular hyperbola meets the asymptotes at L and M and C is the centre of the hyperbola. Prove that $PL = PM = PC.$

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3. If the normals at four points $P(x_i y_i), i = 1, 2, 3, 4$ on the rectangular hyperbola $xy = c^2$, meet at the point Q(h, k), then prove that $x_1 + x_2 + x_3 + x_4 = h$

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Exercise For Session 1

1. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is 1 (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$

A. 1

B. $\frac{1}{2}$

C. -1

D. $\sqrt{2}$

Answer: D

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2. If e_1 and e_2 represent the eccentricity of the curves $6x^2 - 9y^2 = 144$ and $9x^2 - 16y^2 = 144$ respectively . Then $\frac{1}{e_1^2} + \frac{1}{e_2^2}$ is equal to

A. $e_1^2 - e_2^2 = 1$

B. $e_1^2 - e_2^2 < 3$

C. $e_1^2 - e_2^2 = 3$

D. $e_1^2 - e_2^2 > 3$

Answer: B

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3. The transverse axis of a hyperbola is of length $2a$ and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio $2:1$. The equation of the hyperbola is

A. $4x^2 - 5y^2 = 4a^2$

B. $4x^2 - 5y^2 = 5a^2$

C. $5x^2 - 4y^2 = 4a^2$

D. $5x^2 - 4y^2 = 5a^2$

Answer: D

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4. The eccentricity of the hyperbola whose latus-rectum is 8 and length of the conjugate axis is equal to half the distance between the foci, is

A. $\frac{2}{\sqrt{3}}$

B. $\frac{3}{\sqrt{3}}$

C. $\frac{4}{\sqrt{3}}$

D. $\frac{5}{\sqrt{3}}$

Answer: A

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5. The straight line $x + y = \sqrt{2}P$ will touch the hyperbola $4x^2 - 9y^2 = 36$ if (a) $p^2 = 2$ (b) $p^2 = 5$ (c) $5p^2 = 2$ (d) $p^2 = \frac{2}{5}$

A. $p^2 = 2$

B. $p^2 = 5$

C. $p^2 = \frac{2}{5}$

D. $p^2 = \frac{2}{5}$

Answer: D

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6. The equation of the tangent parallel to $y - x + 5 = 0$ drawan to $\frac{x^2}{3} - \frac{y^2}{2} = 1$ is

A. $x - y - 1 = 0$

B. $x - y + 2 = 0$

C. $x + y - 1 = 0$

D. x+y+2=0`

Answer: A

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7. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, then the point $\left(\frac{1}{e}, \frac{1}{e'}\right)$ lies on the circle:

A. x^(2)+y^(2)=1`

B. $x^2 + y^2 = 2$

C. $x^2 + y^2 = 3$

D. $x^2 + y^2 = 4$

Answer: A

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8. If e and e' are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola `5x^(2) - 4y^(2) = 45 respectively , then ee' is equal to

A. -1

B. 1

C. -4

D. 9

Answer: B

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9. The equation $\frac{x^2}{10-\lambda}+\frac{y^2}{6-\lambda}=1$ represents

A. a hyperbola if $\lambda < 6$

B. an ellipse if $\lambda > 6$

C. a hyperbola if $6 < \lambda < 10$

D. an ellipse if $0 < \lambda < 6$

Answer: C::D

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10. Find the centre, eccentricity, foci and directrices of the hyperbola : $x^2 - 3y^2 - 2x = 8$.

A. $\frac{2}{\sqrt{3}}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{2}$

Answer: (c)

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11. For hyperbola $x^2 \sec^2 \alpha - y \cos ec^2 \alpha = 1,$ which of the following remains constant with change in $'\alpha'$ abscissa of vertices (b) abscissa of foci eccentricity (d) directrix

A. abscissae of vertices

B. abscissae of foci

C. eccentricity

D. directrix

Answer: B

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12. If the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then find the value

A. `1

B. 5

C. 7

D. 9

Answer: (c)

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13. Find the equation of the hyperbola whose foaci are $(0, 5)$ and $(-2, 5)$ and eccentricity 3.

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14. Prove that the straight lines $\frac{x}{a} - \frac{y}{b} = m$ and $\frac{x}{a} + \frac{y}{b} = \frac{1}{m}$, where a and b are given positive real numbers and 'm' is a parameter, always meet on a hyperbola.

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15. Find the centre, eccentricity and length of axes of the hyperbola $3x^2 - 5y^2 - 6x + 20y - 32 = 0$.

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16. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is (a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $\frac{4}{5}$

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17. If the line $y = 3x + \lambda$ touches the hyperbola $9x^2 - 5y^2 = 45$, then λ=

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18. Find the equation of tangents to the curve $4x^2 - 9y^2 = 1$ which are parallel to $4y = 5x + 7$.

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Exercise For Session 2

1. Find the equation of tangent to the hyperbola $16x^2 - 25y^2 = 400$ perpendicular to the line $x - 3y = 4$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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2. If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then the value of p is

A. $\frac{1}{9}$

B. $\frac{1}{3}$

C. 9

D. 18

Answer: C

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3. If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ is parallel to $3x - 4y + 4 = 0$, then the value of θ, is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{5\pi}{12}$

Answer: A

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4. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola $x^2 - 2y^2 = 4$, then the point of contact is $(-2, \sqrt{6})$ (b) $(-5, 2\sqrt{6})$ $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right)$ (d) $(4, -\sqrt{6})$

A. $(-2, \sqrt{6})$

B. $(-5, 2\sqrt{6})$

C. $\left(\frac{1}{2}, \frac{1}{\sqrt{6}}\right.$

D. $(4, -\sqrt{6})$

Answer: D

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5. Find the equation of the chord of the hyperbola $25x^2 - 16y^2 = 400$ which is bisected at the point (5, 3).

A. $115x - 47y = 434$

B. $125x - 48y = 481$

C. $127x - 49y = 488$

D. $155x - 67y = 574$

Answer: B

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6. Find the value of m for which $y = mx + 6$ is tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{49} = 1$

A. $\sqrt{\frac{17}{20}}$

B. $-\sqrt{\frac{17}{21}}$

C. $\sqrt{\frac{20}{17}}$

D. $-\sqrt{\frac{21}{17}}$

Answer: A

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7. P is a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and N is the foot of the perpendicular from P on the transverse axis. The tantent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then OT.ON is equal to

A. a^2

B. b^2

C. e^2

D. `b^(2)la

Answer: A

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8. about to only mathematics

A. $9x^2 - 8y^2 - 18x + 9 = 0$

B. $9x^2 - 8y^2 - 18x - 9 = 0$

C. $9x^2 - 8y^2 + 18x + 9 = 0$

D. $9x^2 - 8y^2 + 18x - 9 = 0$

Answer: B

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9. about to only mathematics

A. $\frac{a^2+b^2}{a}$

B. `-(a^(2)+b^(2))/(a)

C. $\frac{a^2+b^2}{b}$

D. $-\frac{a^2+b^2}{b}$

Answer: D

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10. The tangent at a point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point $(0, -b)$ and the normal at P passes through the point $\left(2a\sqrt{2}, 0\right)$. Then the eccentricity of the hyperbola is

A. $\frac{5}{4}$

B. $\frac{3}{2}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: C

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11. A tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $PandQ$. Show that the locus of the midpoint of PQ is $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2} - \frac{y^2}{b^2}$.

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12. A line through the origin meets the circle $x^2 + y^2 = a^2$ at P and the hyperbola $x^2 - y^2 = a^2$ at Q. Prove that the locus of the point of

intersection of tangent at P to the circle with the tangent at Q to the hyperbola is a straight line.

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13. Normal are drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at point $\theta_1 and th \eta_2$ meeting the conjugate axis at $G_1 and G_2$, respectively. If $\theta_1 + \theta_2 = \frac{\pi}{2}$, prove that $C G_1 \dot{C} G_2 = \frac{a^2 e^4}{e^2 - 1}$, where C is the center of the hyperbola and e is the eccentricity.

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14. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of their middlepoints is the curve, $y^2(x - a) = x^3$.

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Exercise For Session 3

1. The diameter of $16x^2 - 9y^2 = 144$ which is conjugate to $x = 2y$ is

A. $y = \frac{16}{9}x$

B. $y = \frac{32}{9}x$

C. $x = \frac{16}{9}y$

D. $x = \frac{32}{9}y$

Answer: B

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2. Tangents drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1,$ then tangents are at angle

A. (a) $\frac{\pi}{6}$

B. (b) $\frac{\pi}{4}$

C. (c) $\frac{\pi}{3}$

D. (d) $\frac{\pi}{2}$

Answer: D

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3. If $H = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0, C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$ and $A = \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ then H, A and C are in

A. (a) AP

B. (b) GP

C. (c) HP

D. (d) AGP

Answer: A

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4. Find the angle between the asymptotes of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

A. $\tan^{-1}\left(\frac{2}{3}\right)$

B. $\tan^{-1}\left(\frac{3}{2}\right)$

C. $2\tan^{-1}\left(\frac{2}{3}\right)$

D. $2\tan^{-1}\left(\frac{3}{2}\right)$

Answer: D

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5. If e and e_1, are the eccentricities of the hyperbolas $xy = c^2$ and $x^2 - y^2 = c^2$, then $e^2 + e_1^2$ is equal to

A. a) 2

B. b) 4

C. c) 6

D. d) 8

Answer: D

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6. Find the product of the length of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes.

A. $\frac{1}{2}$

B. 2

C. $\frac{2}{3}$

D. $\frac{3}{2}$

Answer: C

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7. The number of points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 3$ from which mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$ is/are (a)0 (b) 2 (c) 3 (d) 4

A. 0

B. 2

C. 3

D. 4

Answer: A

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8. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda(\lambda \in R^+)$, then the locus of point P is (a) $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ (c) $xy = \lambda c^2$ (d) none of these

A. $x^2 = \lambda c^2)$

B. $y^2 = \lambda c^2)$

C. $xy = \lambda c^2$

D. None of these

Answer: A

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9. If $S = 0$ is the equation of the hyperbola $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$, then the value of K for which $S + K = 0$ represents its asymptotes is (a)20 (b) -16 (c) -22 (d) 18

A. 20

B. 18

C. -16

D. -22

Answer: D

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10. A ray emanating from the point $\left(\sqrt{41}, 0\right)$ is incident on the hyperbola $16x^2 - 25y^2 = 400$ at the point P with abscissa10.Find the equation of a

reflected ray after first reflection and point P lies in 2nd quadrant is

A. $4\sqrt{3}-\left(10-\sqrt{41}\right)y+4\sqrt{123}=0$

B. $4\sqrt{3}+\left(10-\sqrt{41}\right)y-4\sqrt{123}=0$

C. $4\sqrt{3}+\left(10-\sqrt{41}\right)y+4\sqrt{123}=0$

D. $4\sqrt{3}-\left(10-\sqrt{41}\right)y-4\sqrt{123}=0$

Answer: B

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11. If a ray of light incident along the line $3x + \left(5 - 4\sqrt{2}\right)y = 15$ gets reflected from the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1,$ then its reflected ray goes along the line

A. $x\sqrt{2} - y + 5 = 0$

B. $y\sqrt{2} - x + 5 = 0$

C. $y\sqrt{2} - x - 5 = 0$

D. None of these

Answer: D

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12. The equation of the transvers and conjugate axes of a hyperbola are, respectively, $x + 2y - 3 = 0$ and $2x - y + 4 = 0$, and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

a) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$

b) $\frac{2}{5}(x - y - 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

c) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

d) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

A. $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 5$

B. $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 5$

C. $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

D. $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$

Answer: B

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13. Find the equation of that diameter which bisects the chord $7x + y - 2 = 0$ of the hyperbola $\frac{x^2}{3} - \frac{y^2}{7} = 1.$

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14. Find the equation of the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and which passes through the origin.

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15. The asymptotes of the hyperbola centre of the point $(1, 2)$ are parallel to the lines $2x + 3y = 0$ and $3x + 2y = 0$. If the hyperbola passes through the points $(5, 3)$, show that its equation is $(2x + 3y - 8)(3x + 2y + 7) = 154$

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16. If the pair of straight lines $Ax^2 + 2Hxy + By^2 = 0$ be conjugate diameters of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $Aa^2 = Bb^2$.

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17. A circle cuts the rectangular hyperbola $xy = 1$ in the points $(x_r, y_r), r = 1, 2, 3, 4$.

Prove that $x_1x_2x_3x_4 = y_1y_2y_3y_4 = 1$

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Exercise Single Option Correct Type Questions

1. P is any point on the hyperbola $x^2 - y^2 = a^2$. If F_1 and F_2 are the foci of the hyperbola and $PF_1 \cdot PF_2 = \lambda(OP)^2$. Where O is the origin, then λ is equal to

A. 1

B. $\sqrt{2}$

C. 2

D. 3

Answer: A

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2. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda\left(\lambda \in R^+\right)$, then the locus of point P is (a) $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ (c) $xy = \lambda c^2$ (d) none of these

A. $x^2 - y^2 = \lambda c^2$

B. $y^2 = \lambda c^2)$

C. $xy = \lambda c^2$

D. $x^2 = \lambda c^2$

Answer: D

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3. If $xy = \lambda^2 - 9$ be a rectangular hyperbola whose branches lie only in the second and fourth quadrant, then

A. $|\lambda| \geq 3$

B. $|\lambda| < 3$

C. $\lambda \in R - \{-3, 3\}$

D. None of these

Answer: B

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4. If there are two points A and B on rectangular hyperbola $xy = c^2$ such that abscissa of $A =$ ordinate of B, then locusof point of intersection of tangents at A and B is (a) $y^2 - x^2 = 2c^2$ (b) $y^2 - x^2 = \frac{c^2}{2}$ (c) $y = x$ (d) non of these

A. $y^2 = x^2 + 2c^2$

B. $y^2 = x^2 + \frac{c^2}{2}$

C. $y = x$

D. $y = 3x$

Answer: C

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5. A series of hyperbola are drawn having a common transverse axis of length 2a. Prove that the locus of point P on each hyperbola, such that its distance from the transverse axis is equal to its distance from an asymptote, is the curve $\left(x^2 - y^2\right)^2 = \lambda x^2\left(x^2 - a^2\right)$, then λ equals

A. $\left(x^2 - y^2\right)^2 = 4x^2\left(x^2 - a^2\right)$

B. $\left(x^2-y^2\right)^2=x^2\left(x^2-a^2\right)$

C. $\left(x^2-y^2\right)^2=4y^2\left(x^2-a^2\right)$

D. $\left(x^2-y^2\right)^2=y^2\left(x^2-a^2\right)$

Answer: A

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6. If a rectangular hyperbola $(x-1)(y-2)=4$ cuts a circle $x^2+y^2+2gx+2fy+c=0$ at points $(3,4),(5,3),(2,6)$ and $(-1,0)$, then the value of $(g+f)$ is equal to

A. a) -3

B. b) -9

C. c) 8

D. d) 9

Answer: A

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7. If p, q, r, s ae rational numbers and the roots of $f(x) = 0$ are eccentricities of a parabola and a rectangular hyperbola, where

$f(x0 = px^3 + qx^2 + rx + s, thenp + q + r + s =$

p b. $-p$ c. $2p$ d. 0

A. -1

B. 0

C. 1

D. data inadequate

Answer: B

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8. From a point on the line $y = x + c$, c(parameter), tangents are drawn to the hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1$ such that chords of contact pass through a fixed point (x_1, y_1). Then , $\frac{x_1}{y_1}$ is equal to

A. 2

B. 3

C. 4

D. None of these

Answer: A

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9. Two conics $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $x^2 = -\frac{a}{b}y$ intersect, if

A. $0 < b \leq \frac{1}{2}$

B. $0 < a \leq \frac{1}{2}$

C. $a^2 < b^2$

D. $a^2 > b^2$

Answer: B

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10. The number of points outside the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ from where two perpendicular tangents can be drawn to the hyperbola are: (a) 0 (b) 1 (c) 2 (d) none of these

A. 0

B. 1

C. 2

D. None of these

Answer: A

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11. Let $A = (-3, 4)$ and $B = (2, -1)$ be two fixed points. A point C moves such that $\tan\left(\frac{1}{2}\angle ABC\right) : \tan\left(\frac{1}{2}\angle BAC\right) = 3:1$

Thus, locus of C is a hyperbola, distance between whose foci is

A. 5

B. $5\sqrt{2}$

C. $\frac{5}{2}$

D. $\frac{5}{\sqrt{2}}$

Answer: B

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12. A point P is taken on the right half of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having its foci as S_1 and S_2. If the internal angle bisector of the angle $\angle S_1 P S_2$ cuts the x-axis at poin $Q(\alpha, 0)$ then range of α is

A. a. $[-a, a]$

B. b. $[0, a]$

C. c. $(0, a]$

D. d. $[-a, 0]$

Answer: C

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13. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 120^0 and the product of perpendiculars drawn from the foci upon its any tangent is 9, then the locus of the point of intersection of perpendicular tangents of the hyperbola can be $x^2 + y^2 = 6$ (b) $x^2 + y^2 = 9$ $x^2 + y^2 = 3$ (d) $x^2 + y^2 = 18$

A. $x^2 + y^2 = 3$

B. $x^2) + y^2 = 6$

C. $x^2 + y^2 = 9$

D. $x^2 + y^2 = 18$

Answer: D

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14. If $\alpha + \beta = 3\pi$, then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through which of the following points? Focus (b) Center One of the endpoints of the transverse exis. One of the endpoints of the conjugate exis.

A. focus

B. centre

C. one of the end point of the transverse axis

D. one of the end of the conjugate axis

Answer: B

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15. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b)$ and $x^2 - y^2 = c^2$ cut at right angles, then:

A. $a^2 + b^2 = 2c^2$

B. $b^2 - a^2 = 2c^2$

C. $a^2 - b^2 = 2c^2$

D. $a^2b^2 = 2c^2$

Answer: C

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16. Chords of the hyperbola $x^2 - y^2 = a^2$ touch the parabola $y^2 = 4ax$. Prove that the locus of their middle-points is the curve $y^2(x - a) = x^3$.

A. $y^2(x + a) = x^3$

B. $y^2(x - a) = x^3$

C. $y^2(x+2a) = 3x^3$

D. $y^2(x-2a) = 2x^3$

Answer: B

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17. about to only mathematics

A. $9x^2 + 12y^2 = 108$

B. $9\left(x - \frac{1}{3}\right)^2 + 12(y-1)^2 = 1$

C. $9\left(x - \frac{1}{3}\right)^2 + 4(y-1)^2 = 36$

D. None of these

Answer: B

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18. The equation of the line passing through the centre of a rectangular hyperbola is $x - y - 1 = 0$. If one of its asymptotoes is $3x - 4y - 6 = 0$, the equation of the other asymptote is

A. $4x - 3y + 8 = 0$

B. $4x + 3y + 17 = 0$

C. $3x - 2y + 15 = 0$

D.

Answer: B

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19. The condition that a straight line with slope m will be normal to parabola $y^2 = 4ax$ as well as a tangent to rectangular hyperbola $x^2 - y^2 = a^2$ is

A. a) $m^6 - 4m^2 + 2m - 6y = 0$

B. b) $m^4 + 3m^3 + 2m + 1 = 0$

C. c) $m^6 - 2m = 0$

D. d) $m^6 + 4m^4 + 3m^2 + 1 = 0$

Answer: D

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20. Find the locus of the midpoints of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to y = 2x.

A. $3x - 4y = 4$

B. $3y - 4x + 4 = 0$

C. $4x - 4y = 3$

D. $3x - 4y = 2$

Answer: A

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21. The co-ordinates of the centre of the hyperbola, $x^2 + 3xy + 3y^2 + 2x + 3y + 2 = 0$ is $(-1, 0)$ (b) (1, 0) $(-1, 1)$ (d) $(1, -1)$

A. $(-1, 0)$

B. $(1, 0)$

C. $(-1, 1)$

D. $(1, -1)$

Answer: A

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22. Let F_1, F_2 are the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and F_3, F_4 are the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectivley then the statement which holds true is:

A. `their equations of their asymptots are different

B. $e_h > e_c$

C. `area of the quadrilateral formed by their foci is 50sq. Units

D. their auciliary circles will have the same equation

Answer: C

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23. Locus of the point of intersection of the tangents at the points with eccentric angles ϕ and $\frac{\pi}{2} - \phi$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

A. $x = a$

B. $y = b$

C. $x = ab$

D. $y = ab$

Answer: B

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24. Latusrectum of the conic satisfying the differential equation $xdy + ydx = 0$ and passing through the point (2, 8) is

A. $4\sqrt{2}$

B. 8

C. $8\sqrt{2}$

D. 16

Answer: C

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25. The point of intersection of the curve whose parametrix equations are $x = t^2 + 1, y = 2t$ and $x = 2s, y = \frac{2}{s}$, is given by

A. $(1, -3)$

B. $(2, 2)$

C. $(-2, 4)$

D. $(1, 2)$

Answer: B

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26. If the tangent and normal to a rectangular hyperbola cut off intercepts x_1 and x_2 on one axis and y_1 and y_2 on the other, then

A. $x_1y_1 + x_2y_2 = 0$

B. $x_1y_2 + x_2y_1 = 0$

C. $x_1x_2 + y_1y_2 = 0$

D. None of these

Answer: C

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27. The focus of rectangular hyperbola $(x - a) \cdot (y - b) = c^2$ is

A. (a) $(h - p, k - p)$

B. (b) $(h-p, k+p)$

C. (c) $(h+p, k-p)$

D. (d) None of these

Answer: A

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28. The equation of a hyperbola conjugate to the hyperbola $x^2+3xy+2y^2+2x+3y=0$ is

A. $x^2+3xy+2y^2+2x+3y+1=0$

B. $x^2+3xy+2y^2+2x+3y+2=0$

C. $x^2 + 3xy + 2y^2 + 2x + 3y + 3 = 0$

D. $x^2 + 3xy + 2y^2 + 2x + 3y + 4 = 0$

Answer: B

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29. about to only mathematics

A. -2

B. 0

C. 2

D. 4

Answer: B

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30. Let C be a curve which is the locus of the point of intersection of lines $x = 2 + m$ and $my = 4 - m$. A circle $s \equiv (x-2)^2 + (y+1)^2 = 25$ intersects the curve C at four points: $P, Q, R, and S$. If O is center of the curve C, then $OP^2 + OP^2 + OR^2 + OS^2$ is (a) 50 (b) 100 (c) 25 (d) $\frac{25}{2}$

A. 25

B. 50

C. 100

D. 200

Answer: C

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Exercise More Than One Correct Option Type Questions

1. The equation of common tangent to the parabola $y^2 = 8x$ and hyperbola $3x^2 - y^2 = 3$ is

A. $2x - y + 1 = 0$

B. $2x - y - 1 = 0$

C. $2x + y + 1 = 0$

D. $2x + y - 1 = 0$

Answer: A::C

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2. If the length of minor axis of the ellipse $\frac{x^2}{k^2a^2} + \frac{y^2}{b^2} = 1$ is equal to the length of transverse axis of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,and the

equation of ellipse is confocal with hyperbola then the value k is equal to

A. $-\sqrt{2}$

B. $\sqrt{2}$

C. $-\sqrt{3}$

D. $(\sqrt{3})$

Answer: C::D

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3. If $(a\sec\theta, b\tan\theta)$ and $(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $\tan.\frac{\theta}{2}\tan.\frac{\phi}{2} = \frac{1-e}{1+e}$.

A. $\frac{e-1}{e+1}$

B. $\frac{1-e}{1+e}$

C. $\frac{1+e}{1-e}$

D. $\frac{e+1}{e-1}$

Answer: B::C

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4. If foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ coincide with the foci of $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and eccentricity of the hyperbola is 3. then

A. $a^2 + b^2 = 9$

B. there is no directrix circle to the hyperbola

C. centre of the directrix circle is (0, 0)

D. Length of the latusrecum of the hyperbola =16

Answer: A::B::D

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5. Find all the aspects of hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0.$

A. length of the transverse axis=$2\sqrt{3}$

B. length of the conjugate axis =8

C. centre at$(1, -2)$

D. eccentricity $= \sqrt{19}$

Answer: A::B::C

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6. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then $a > 0, b > 0$ $a > 0, b < 0$ $a\langle 0, b\rangle 0$ (d) $a < 0, b < 0$ none of these

A. $a > 0, b \geq 0$

B. $a > 0, b < 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

Answer: B::C

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7. about to only mathematics

A. $(x_4, -y_4)$

B. (x_4, y_4)

C. $(-x_4, y_4)$

D. $(-x_4, -y_4)$

Answer: D

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8. The line $y = x + 5$ touches

A. the parabola $y^2 = 20x$

B. the ellipse $9x^2 + 16y^2 = 144$

C. the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$

D. the circle $x^2 + y^2 = 25$

Answer: A::B::C

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9. The coordinates of a point common to a directrix and an asymptote of the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$ are

A. $\left(\frac{25}{\sqrt{41}}, \frac{20}{\sqrt{41}}\right)$

B. $\left(\frac{-25}{\sqrt{41}}, \frac{-20}{\sqrt{41}}\right)$

C. $\left(\frac{25}{3}, \frac{20}{3}\right)$

D. $\left(\frac{-25}{3}, \frac{-20}{3}\right)$

Answer: A::B

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10. If (5, 12) and (24, 7) are the foci of a hyperbola passing through the origin, then (where e is eccentricity and LR is Latus Rectum)

A. $e = \frac{\sqrt{386}}{12}$

B. $e = \frac{\sqrt{386}}{13}$

C. latusrectum $= \frac{121}{3}$

D. latusrectum $= \frac{121}{6}$

Answer: A::D

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11. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn.

A. If n=1, then e=$\sqrt{2}$

B. if $n > 1$, then $0 < e < \sqrt{2}$

C. if n=0, then $e > \sqrt{2}$

D. None of these

Answer: A::B::C

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12. Which of the following equations in parametric form can represent a hyperbola, where t is a parameter?

A. $A: x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$

B. $B: \frac{tx}{a} - \frac{y}{b} + t = 0$ and $\frac{x}{a} + \frac{ty}{b} - 1 = 0$

C. $C: x = e^t + e^{-t}$ and $y = e^t - e^{-t}$

D.

$$D: x^2 - 6 = 2\cos t \text{ and } y^2 + 2 = 4\cos^2\left(\frac{t}{2}\right)$$

Answer: A::C::D

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13. Length of common tangents to the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is

A. $y = x + \sqrt{a^2 - b^2}$

B. $y = x - \sqrt{a^2 - b^2}$

C. $y = -x + \sqrt{a^2 - b^2}$

D. $y = -x - \sqrt{a^2 - b^2}$

Answer: A::B::C::D

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14. Given ellipse $\frac{x^2}{16} + \frac{y^2}{17} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$, if the ordinate of one of the points of intersection is produced to cut asymptote at P, then which of the following is true?

A. They have the same foci

B. Square of the ordinate of point of intersection is $\frac{63}{25}$

C. Sum of the squares of coordinate of P is 16

D. P lies on the auxiliary circle formed by ellipse

Answer: A::B::C::D

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15. Solutions of the differential equation $\left(1-x^2\right)\frac{dy}{dx}+xy=ax$ where a $\in$ R is

A. a conic which is an ellipse

B. centre of the conic is $(0, a)$

C. length of one of the principal axes is 1

D. length of one of the principal axes is equal to

2

Answer: A::B::D

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Exercise Passage Based Questions

1. The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin . The point of the tangency being (a, b) then find the value of $\sin^{-1}\left(\frac{a}{b}\right)$

A. $\frac{5\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$

Answer: C

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2. The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin. The point of tangency being (a, b).

Length of the latusrectum of the conic is

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) 4

A. 1

B. $\sqrt{2}$

C. 2

D. 4

Answer: C

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3. The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin. The point of tangency being (a, b).

Q. If e be the eccentricity of the conic, then the value of $\left(1 + e^2 + e^4\right)$ is

A. 3

B. 7

C. $\frac{7}{4}$

D. 21

Answer: B

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4. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals . The locus of P is a curve C.

the equation of the curve C is

A. $x^2 = 2y$

B. $x^2 = 4y$

C. $x^2 = 6y$

D. $x^2 = 8y$

Answer: B

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5. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola $xy = 4$ is equal to the sum of the ordinates of feet of normals. The locus of P is a curve C.

Q.If the tangent to the curve C cuts the coordinate axes at A and B, then , the locus of the middle point of AB is

A. $x^2 + 2y = 0$

B. $x^2 = y$

C. $2x^2 + y = 0$

D. $x^2 = 2y$

Answer: C

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6. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola $xy = 4$ is equal to the sum of the ordinates of feet of normals. The locus of P is a curve C.

Q. The area of the equilateral triangle inscribed in the curve C having one vertex as the vertex of curve C is

A. (a) $8\sqrt{3}$ sq. units

B. (b) $12\sqrt{3}$ sq. units

C. (c) $27\sqrt{3}$ sq. units

D. (d) $16\sqrt{3}$ sq. units

Answer: D

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7. Let P(x, y) be a variable point such that

$$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$$

which represents a hyperbola.

The locus of the intersection of two perpendicular tangents to the hyperbola is

A. $\frac{5}{4}$

B. $\frac{4}{3}$

C. $\frac{5}{3}$

D. $\frac{3}{2}$

Answer: C

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8. Let P(x, y) be a variable point such that $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}=4\right.$ which represents a hyperbola.

Q. Locus of point of intersection of two perpendicular tangents to the hyperbola is

A. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{1}{4}$

B. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{3}{4}$

C. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{5}{4}$

D. $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{7}{4}$

Answer: D

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9. Let P(x, y) be a variable point such that $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=4$ which represents a hyperbola.

Q. If origin is shifted to point $\left(3, \frac{7}{2}\right)$ and axes are rotated in anticlockwise through an angle θ, so that the equation of hyperbola reduces to its standard form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then θ equals

A. $\tan^{-1}\left(\frac{4}{3}\right)$

B. $\tan^{-1}\left(\frac{3}{4}\right)$

C. $\tan^{-1}\left(\frac{5}{4}\right)$

D. $\tan^{-1}\left(\frac{4}{5}\right)$

Answer: B

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10. Let $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose eccentricity is e. Let θ be the angle between its asymptotes. Tangents are drawn to the hyperbola at some arbitrary points R. These tangent meet the coordinate axes at the points A and B respectively. The rectangle OABC (O being the origin) is

completedm, then

Q.Locus of point C is

A. $\frac{b^2}{x^2} - \frac{a^2}{y^2} = 1$

B. $\frac{b^2}{x^2} + \frac{a^2}{y^2} = 1$

C. $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

D. $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

Answer: D

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11. Let $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose eccentricity is e. Let θ be the angle between its asymptotes. Tangents are drawn to the hyperbola at some arbitrary points R. These tangent meet the coordinate axes at the points A and B respectively. The rectangle OABC (O being the origin) is completedm, then

Q. If $\cos^2\left(\frac{\theta_1 + \theta_2}{2}\right) = \lambda \cos^2\left(\frac{\theta_1 - \theta_2}{2}\right)$, then λ is equal to

A. $\frac{a^2 + b^2}{a^2}$

B. $\frac{a^2 + b^2}{b^2}$

C. $\frac{a^2+b^2}{ab}$

D. $\frac{a^2+b^2}{2ab}$

Answer: A

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12. If e is the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and θ is the angle between the asymptotes, then $\cos.\frac{\theta}{2}$ is equal to

A. $\frac{1}{2e}$

B. $\frac{1}{e}$

C. $\frac{1}{e^2}$

D. $\frac{1}{2e^2}$

Answer: B

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13. The vertices of $\triangle ABC$ lie on a rectangular hyperbola such that the orhtocentre of the triangle is $(2, 3)$ and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola

intersect at the point (1, 1). Q. The equation of the rectangular hyperbola is

A. $xy - 5 = y - x$

B. $xy - 1 = x + y$

C. $xy = x + y + 1$

D. $xy - 11 = \ - x - y$

Answer: C

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14. The vertices of $\triangle ABC$ lie on a rectangular hyperbola such that the orhtocentre of the triangle is $(2, 3)$ and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1). Q. The number of real tangents that can be drawn from the point (1, 1) to the rectangular hyperbola is

A. a. 0

B. b. 2

C. c. 3

D. d. 4

Answer: B

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Exercise Single Integer Answer Type Questions

1. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ are given to be confocal and length of mirror axis of the ellipse is same as the conjugate axis of the hyperbola. If e_1 and e_2 represents the eccentricities of ellipse and hyperbola respectively, then the value of $e_1^{-2}+e_1^{-2}$ is

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2. If abscissa of orthocentre of a triangle inscribed in a rectangular hyperbola $xy = 4$ is $\frac{1}{2}$, then the ordinate of orthocentre of triangle is

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3. Normals drawn to the hyperbola $xy = 2$ at the point $P(t_1)$ meets the hyperbola again at $Q(t_2)$, then minimum distance between the point P and Q is

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4. The normal at P to a hyperbola of eccentricity $\frac{3}{2\sqrt{2}}$ intersects the transverse and conjugate axes at M and N respectively. The locus of mid-point of MN is a hyperbola, then its eccentricity.

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5. If radii of director circle of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are in the ratio $1:3$ and $4e_1^2-e_2^2=\lambda$, where e_1 and e_2

are the eccetricities of ellipse and hyperbola respectively, then the value of λ is

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6. The shortest distance between the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $4x^2 + 4y^2 = a^2(b > a)$ is f(a, b), then the value of $f(4, 6) + f(2, 3)$ is

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7. ABC is a triangle such that $\angle ABC = 2\angle BAC$. If AB is fixed and locus of C is a hyperbola, then the

eccentricity of the hyperbola is

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8. Point P lie on hyperbola $2xy = 1$. A triangle is contructed by P, S and S' (where S and S' are foci). The locus of ex-centre opposite S (S and P lie in first quandrant) is $(x + py)^2 = \left(\sqrt{2} - 1\right)^2 (x - y)^2 + q$, then the value of $p + q$ is

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9. Chords of the circle $x^2 + y^2 = 4$, touch the hyperbola $\frac{x^2}{4} - \frac{y^2}{16} = 1$.The locus of their middle-points is the curve $\left(x^2 + y^2\right)^2 = \lambda x^2 - 16y^2$, then the value of λ is

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10. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angle θ and ϕ to the x-axis. If $\tan\theta \cdot \tan\phi = 2$, then the value $2\alpha^2 - \beta^2$ is _____.

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Hyperbola Exercise 8 Matching Type Questions

1. Any given statement in column I can have correct matching with one or more statement(s) given in column II. Match the following.

	Column I		Column II
(A)	If λ be the length of the latusrectum of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, then 3λ is divisible by	(p)	4
(B)	If the chord $x\cos\alpha + y\sin\alpha = p$ of the hyperbola $\frac{x^2}{16} - \frac{y^2}{18} = 1$ subtends a right angle at the centre, a circle touches the given chord and concentric with hyperbola, then the diameter of circle is divisible by	(q)	6
(C)	For the hyperbola $xy = 8$ any tangent of it at P meets coordinate axes at Q and R, then the area of triangle CQR is divisible by (where 'C' is centre of the hyperbola)	(r)	8
(D)	For the hyperbola $x^2 - 3y^2 = 9$, acute angle between its asymptotes is $\frac{\pi\lambda}{24}$, then λ is divisible by	(s)	16

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2. Any given statement in column I can have correct matching with one or more statement(s) given in column II. Q. Match the following.

	Column I		Column II
A.	If e_1 and e_2 are the eccentricities of ellipse and hyperbola respectively, then the values of λ are	(p)	$2\sqrt{2}$
B.	If both e_1 and e_2 are the eccentricities of the hyperbolas, then the values of λ are	(q)	$2\sqrt{3}$
C.	If e_1 and e_2 are the eccentricities of the hyperbola and conjugate hyperbola, then the values of λ are	(r)	$2\sqrt{5}$
D.	If e_1 is the eccentricity of the hyperbola for which there exist infinite points from which perpendicular tangents can be drawn and e_2 is the eccentricity of the hyperbola in which no such points exist, then the values of λ are	(s)	$2\sqrt{6}$

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Exercise Statement I And Ii Type Questions

1. Statement-I $\frac{5}{3}$ and $\frac{5}{4}$ are the eccentricities of two conjugate hyperbolas.

Statement-II If e_1 and e_2 are the eccentricities of two conjugate hyperbolas, then $e_1 e_2 > 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

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2. Statement-I A hyperbola and its conjugate hyperbola have the same asymptotes.

Statement-II The difference between the second degree curve and pair of asymptotes is constant.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

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3. Statement-I The equation of the directrix circle to the hyperbola $5x^2 - 4y^2 = 20$ is $x^2 + y^2 = 1$.

Statement-II Directrix circle is the locus of the point of intersection of perpendicular tangents.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

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4. Statement-I Two tangents are drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\frac{x^2}{25} - \frac{y^2}{16} = 1$, then the angle between tangnets is $\frac{\pi}{2}$.

Statement-II $x^2 + y^2 = 9$ is the directrix circle of $\frac{x^2}{25} - \frac{y^2}{16} = 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

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5. Statement-I If eccentricity of a hyperbola is 2, then eccentricity of its conjugate hyperbola is $\frac{2}{\sqrt{3}}$.

Statement-II if $e \text{ and } e_1$ are the eccentricities of two conjugate hyperbolas, then $ee_1 > 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

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6. Statement-I The line $4x - 5y = 0$ will not meet the hyperbola $16x^2 - 25y^2 = 400$.

Statement-II The line $4x - 5y = 0$ is an asymptote ot the hyperbola.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

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7. Statement-I The point (5, -3) inside the hyperbola $3x^2 - 5y^2 + 1 = 0$.

Statement-II The point (x_1, y_1) inside the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1 < 0$.

A. a) Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. b) Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. c) Statement-I is true, Statement-II is false.

D. d) Statement-I is false, Statement-II is true

Answer: C

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8. Statement-I A hyperbola whose asymptotes include $\frac{\pi}{3}$ is said to be equilateral hyperbola.

Statement-II The eccentricity of an equilateral hyperbola is $\sqrt{2}$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

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Exercise Subjective Type Questions

1. Given the base of a triangle and the ratio of the tangent of half the base angles .Show that the vertex moves on a hyperbola whose foci are the extremities of a diameter

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2. A, B, C are three points on the rectangular hyperbola $xy = c^2$, The area of the triangle formed by the points A, B and C is

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3. If a hyperbola be rectangular, and its equation be $xy = c^2$, prove that the locus of the middle points of chords of constant length 2d is $(x^2 + y^2)(xy - c^2) = d^2xy$.

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4. If four points be taken on a rectangular hyperbola such that the chord joining any two is perpendicular to the chord joining the other two, and if $\alpha, \beta, \gamma, \delta$ be the inclinations to either asymptotes of the straight lines joining these

points to the centre, then $\tan\alpha\tan\beta\tan\gamma\tan\delta$ is equal to

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5. A circle cuts two perpendicular lines so that each intercept is of given length. The locus of the centre of the circle is conic whose eccentricity is

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6. Let the tangent at a point P on the ellipse meet the major axis at B and the ordinate from it meet

the major axis at A. If Q is a point on the AP such that $AQ = AB$, prove that the locus of Q is a hyperbola. Find the asymptotes of this hyperbola.

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7. A series of hyperbola is drawn having a common transverse axis of length 2a. Then the locus of a point P on each hyperbola, such that its distance from the transverse axis is equal to its distance from an asymptotes, is

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Hyperbola Exercise 10 Subjective Type Questions

1. If there are two points A and B on rectangular hyperbola $xy = c^2$ such that abscissa of A =ordinate of B, then the locus of points of intersection of tangents at A and B is

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2. (a) Prove that any line parallel to either of the asymptotes of a hyperbola shall meet it in one point at infinity.

(b) Prove that the asymptotes of a hyperbola are

the diagonals of the rectangle formed by the lines drawn parallel to the axes at the vertices of the hyperbola [i.e., at $(\pm a, 0)$ and $(0, \pm b)$].

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3. From the point (x_1, y_1) and (x_2, y_2), tangents are drawn to the rectangular hyperbola $xy = c^2$. If the conic passing through the two given points and the four points of contact is a circle, then show that $x_1x_2 = y_1y_2$ and $x_1y_2 + x_2y_1 = 4c^2$.

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Exercise Questions Asked In Previous 13 Years Exam

1. Find the locus of a point $P(\alpha, \beta)$ moving under the condition that the line $y = ax + \beta$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

A. an ellipse

B. a circle

C. a parabola

D. a hyperbola

Answer: D

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2. Let a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axis of the given ellipse. Also, the product of the eccentricities of the given ellipse and hyperbola is 1. Then,

A. the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

B. the equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

C. focus of hyperbola is $(5,0)$

D. vertex of hyperbola is $\left(5\sqrt{3},0\right)$

Answer: A::C

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3. A hyperbola having the transverse axis of length $2 \sin \theta$ is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

A. $x^2 \cos ec^2\theta - y^2 \sec^2 \theta = 1$

B. $x^2 \sec^2 \theta - y^2 \cos ec^2\theta = 1$

C. $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$

D. $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$

Answer: A

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4. Two branches of a hyperbola

A. a) have a common tangent

B. b) have a common normal

C. c) do not have a common tangent

D. d) do not have a common normal

Answer: B::C

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5. For the hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1; \left(0 < \alpha < \frac{\pi}{4}\right)$. Which of the following remains constant when alpha varies?

A. abscissae of vertices

B. abscissae of foci

C. eccentricity

D. directrix

Answer: B

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A. $1-\sqrt{\frac{2}{3}}$

B. $\sqrt{\frac{3}{2}}-1$

C. $1+\sqrt{\frac{2}{3}}$

D. $\sqrt{\frac{3}{2}}+1$

Answer: B

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7. An ellipse intersects the hyperbola $2x^2-2y=1$ orthogonally. The eccentricity of the ellipse is

reciprocal to that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then (a) the foci of ellipse are $(\pm 1, 0)$ (b) equation of ellipse is $x^2 + 2y^2 = 2$ (c) the foci of ellipse are $(t2, 0)$ (d) equation of ellipse is $\left(x^2 2y\right)$

A. equation of ellipse $x^2 + 2y^2 = 2$

B. the foci of the ellipse are $(\pm 1, 0)$

C. equation of ellipse is $x^2 + 2y^2 = 4$

D. the foci of ellipse are $\left(\pm \sqrt{2}, 0\right)$

Answer: A::B

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8. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at points A and B.

The equation of a common tangent with positive slope to the circle as well as to the hperbola is

A. $2x - \sqrt{5}y - 20 = 0$

B. $2x - \sqrt{5}y + 4 = 0$

C. $3x - 4y + 8 = 0$

D. $4x - 3y + 4 = 0$

Answer: B

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9. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. Then the equation of the circle with AB as its diameter is

A. $x^2 + y^2 - 12x + 24 = 0$

B. $x^2 + y^2 + 12x + 24 = 0$

C. $x^2 + y^2 + 24x - 12 = 0$

D. $x^2 + y^2 - 24x - 12 = 0$

Answer: A

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11. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at point P intersects the x-axis at (9, 0), then find the eccentricity of the hyperbola.

A. $\sqrt{\frac{5}{2}}$

B. $\sqrt{\frac{3}{2}}$

C. $\sqrt{2}$

D. $\sqrt{3}$

Answer: B

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A. the equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

B. a focus of the hyperbola is $(2, 0)$

C. the eccentricity of the hyperbola is $\sqrt{\frac{5}{2}}$

D. the equation of the hyperbola is $x^2 - 3y^2 = 3$

Answer: B::D

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13. Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallet to the sraight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are (A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ (C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

A. $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

B. $\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

C. $\left(3\sqrt{3}, -2\sqrt{2}\right)$

D. $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

Answer: A::B

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14. Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$ Suppose that H and S touch each other at a point $(P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$ The common tangent to H and S at P intersects the x-axis at point M. If (l,m) is the centroid of the triangle ΔPMN then the correct expression is (A) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$ (B)

$$\frac{dm}{dx_1} = \frac{x_!}{3\left(\sqrt{x_1^2 - 1}\right)} \Bigg) f \text{ or } x_1 > 1 \quad \text{(C)}$$

$$\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2} f \text{ or } x_1 > 1 \quad \text{(D)}$$

$$\frac{dm}{dy_1} = \frac{1}{3} f \text{ or } y_1 > 0$$

A. $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2} \Bigg)$ for $x_1 > 1$

B. $\frac{dm}{dx_1} = \frac{x_1}{3\sqrt{x_1^2 - 1}}$ for $x_1 > 1$

C. $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2} \Bigg)$ for $x_1 > 1$

D. $\frac{dm}{dx_1} = \frac{1}{3}$ for $y_1 > 0$

Answer: A::B::D

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15. The eccentricity of the hyperbola whose latus-rectum is 8 and length of the conjugate axis is equal to half the distance between the foci, is

A. $\frac{2}{\sqrt{3}}$

B. $\sqrt{3}$

C. $\frac{4}{3}$

D. $\frac{4}{\sqrt{3}}$

Answer: A

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16. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point

A. $(-\sqrt{2}, -\sqrt{3})$

B. $(3\sqrt{2}, 2\sqrt{3})$

C. $(2\sqrt{2}, 3\sqrt{3})$

D. $(\sqrt{3}, \sqrt{2})$

Answer: C

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17. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ then which of the following CANNOT be sides of a right angled triangle?

A. $2a, 8, 1$

B. $a, 4, 1$

C. $a, 4, 2$

D. $2a, 4, 1$

Answer: A::B::C

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Hyperbola Exercise 11 Questions Asked In Previous 13 Years Exams

1. Matching the information given in the three columns of the following table column 1, 2 and 3 contain conics, equations of tangents in the conics and points of contact, respectively.

	Column 1		Column 2		Column 3
(I)	$x^2 + y^2 = a^2$	(i)	$my = m^2x + a$	(P)	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II)	$x^2 + a^2y^2 = a^2$	(ii)	$y = mx + a\sqrt{m^2+1}$	(Q)	$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III)	$y^2 = 4ax$	(iii)	$y = mx + \sqrt{a^2m^2-1}$	(R)	$\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV)	$x^2 - a^2y^2 = a^2$	(iv)	$y = mx + \sqrt{a^2m^2+1}$	(S)	$\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q. The tangent to a suitable conic (column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then

which of the following options is the correct combination?

A. (IV)(iii)(S)

B. (II)(iv)(R)

C. (IV)(iv)(S)

D. (II)(iii)(R)

Answer: B

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2. Matching the information given in the three columns of the following table column 1, 2 and 3 contain conics, equations of tangents in the conics and points of contact, respectively.

	Column 1		Column 2		Column 3
(I)	$x^2 + y^2 = a^2$	(i)	$my = m^2x + a$	(P)	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II)	$x^2 + a^2y^2 = a^2$	(ii)	$y = mx + a\sqrt{m^2+1}$	(Q)	$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III)	$y^2 = 4ax$	(iii)	$y = mx + \sqrt{a^2m^2-1}$	(R)	$\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV)	$x^2 - a^2y^2 = a^2$	(iv)	$y = mx + \sqrt{a^2m^2+1}$	(S)	$\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q. For $a = \sqrt{2}$), if a tangent is drawn to a suitable conic (Column 1) at the points of contact (-1, 1), then which of the following options is the only correct combination for obtaining its equation?

A. (III)(i)(P)

B. (I)(i)(P)

C. (II)(ii)(Q)

D. (I)(ii)(Q)

Answer: D

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3. Matching the information given in the three columns of the following table column 1, 2 and 3 contain conics, equations of tangents in the conics and points of contact, respectively.

	Column 1		Column 2		Column 3
(I)	$x^2 + y^2 = a^2$	(i)	$my = m^2x + a$	(P)	$\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II)	$x^2 + a^2y^2 = a^2$	(ii)	$y = mx + a\sqrt{m^2+1}$	(Q)	$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III)	$y^2 = 4ax$	(iii)	$y = mx + \sqrt{a^2m^2-1}$	(R)	$\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV)	$x^2 - a^2y^2 = a^2$	(iv)	$y = mx + \sqrt{a^2m^2+1}$	(S)	$\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

Q. For $a = \sqrt{2}$), if a tangent is drawn to a suitable conic (Column 1) at the points of contact (-1, 1), then which of the following options is the only correct combination for obtaining its equation?

A. (III)(i)(P)

B. (III)(ii)(Q)

C. (II)(ii)(R)

D. (I)(ii)(Q)

Answer: A

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