MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

INVERSE TRIGONOMETRIC FUNCTIONS

Examples

1. Find domain of $\sin^{-1}(2x^2 - 1)$

A. $\Rightarrow x \in [-1, 1]$

B. $\Rightarrow x \in [0, 1]$

C. $\Rightarrow x \in [-1, 0]$

D. $\Rightarrow x \in [-2, 0]$

Answer: A

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2. Evaluate the following: $\sin^{-1}\left(\frac{\sin\pi}{4}\right)$ (ii) $\cos^{-1}\left(\cos 2\frac{\pi}{3}\right)$ $\tan^{-1}\left(\frac{\tan\pi}{3}\right)$

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3. Evaluate the following $(i)\sin^{-1}(\sin 7)$ $(ii)\sin^{-1}(\sin(-5))$

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4. Prove that: $\sec^2\left(\tan^{-1}2\right)+\operatorname{cosec}^2\left(\cot^{-1}3\right)=15.$

A. 15

B. 10

C. -15

D. -10

Answer: A

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5. Evaluate the following

$(i)\sin^{-1}\left(\sin.\left(\frac{-3\pi}{4}\right)\right) \qquad (ii)\cot^{-1}(\cot(-4))$

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6. Evaluate the following

$(i)\sin\left(\cos ec^{-1}\frac{5}{3}\right) \qquad (ii)\cot\left(\tan^{-1}.\ \frac{3}{4}\right)$

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7. Find the value of $\tan\left\{\cot^{-1}\left(\frac{-2}{3}\right)\right\}$

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

Answer: D

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8. Solve $\sin^{-1}x - \cos^{-1}x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

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9. Prove that: $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{8}{19}\right) = \frac{\pi}{4}$

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10. Prove that : $\tan^{-1}2+\tan^{-1}3=\frac{3\pi}{4}$

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11. Prove that : $\frac{\tan^{-1}1}{2}+\frac{\tan^{-1}1}{5}+\frac{\tan^{-1}1}{8}=\frac{\pi}{4}$

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12. Prove that

$\tan^{-1}.\frac{3}{4}+\tan^{-1}.\frac{3}{5}-\tan^{-1}.\frac{8}{19}=\frac{\pi}{4}$

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13. Prove that :

$\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$

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14. If $x^2 + y^2 + z^2 = r^2$, *then* $\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$ is equal to π (b) $\frac{\pi}{2}$ (c) 0 (d) none of these

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15. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

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16. Solve for x , $\tan^{-1}(x+1) + \tan^{-1} x + \tan^{-1}(x-1) = \tan^{-1} 3x$

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17. The value of $\sum_{m=1}^{\infty} \tan^{-1}\left(\frac{2m}{m^4 + m^2 + 2}\right)$ is

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18. If two angles of a triangle are $\tan^{-1}(2) and tan^{-1}(3)$, then find the third angle.

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19. Solve the following equations

$(i)\tan^{-1}.\frac{x-1}{x-2}=\tan^{-1}.\frac{x+1}{x+2}=\frac{\pi}{4}$

(ii) $\tan^{-1}2\times\tan^{-1}3x=\frac{\pi}{4}$

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20. If $\frac{\cos^{-1}x}{a}+\frac{\cos^{-1}y}{b}=\alpha$, then $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=s\in^2\alpha$

(b) $\cos^2\alpha$ (c) $tan^2\alpha$ (d) $\cot^2\alpha$

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21. If $\cos^{-1}\lambda + \cos^{-1}\mu + \cos^{-1}\gamma = 3\pi$, then find the value of $\lambda\mu + \mu\gamma + \gamma\lambda$

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22. If $\Sigma_{i=1}^{2n} \cos^{-1} x_i = 0$,then find the value of $\Sigma_{i=1}^{2n} x_i$

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23. If $\Sigma_{i=1}^{2n} \sin^{-1} x_i = n\pi$, then find the value of $\Sigma_{i=1}^{2n} x_i$.

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24. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$,then find the value of $\Sigma \frac{(x^{101} + y^{101})(}{(x^{303} + y^{303})(}$

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25. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$

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26. The sum of the infinte series $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)+\dots\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$

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27. Evaluate the following :

$\sin^2\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$

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28. Evaluate: $\left\{\frac{2\tan^{-1}1}{5}-\frac{\pi}{4}\right\}$ (ii) $\tan\left\{\frac{1}{2}\frac{\cos^{-1}\left(\sqrt{5}\right)}{3}\right\}$

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29. Evaluate: $\cos\left(2\cos^{-1}x+\sin^{-1}x\right)atx=\frac{1}{5}$.

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30. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, show that $x^4+y^4+z^4+4x^2y^2z^2=2\left(x^2y^2+y^2z^2+z^2x^2\right)$

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31. Draw the graph of $f(x)=4x^3-3x$ and hence draw the graph of $g(x)=\cos^{-1}\left(4x^3-3x\right)$.

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32. Prove that

$$4\tan^{-1}.\ \frac{1}{5}-\tan^{-1}.\ \frac{1}{70}+\tan^{-1}.\ \frac{1}{99}=\frac{\pi}{4}$$

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34. Solve: $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0$

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35. Find the value of

$$\tan\left\{\frac{1}{2}\sin^{-1}\left(\frac{2x}{1+x^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right\}$$

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36. Let $f(x) = \sin x + \cos x + \tan x + \sin^{-1} x + \cos^{-1} x + \tan^{-1} x\cdot$

Then find the maximum and minimum values of $f(x)\cdot$

A. $\frac{\pi}{2} + \cos 1$

B. $\frac{\pi}{2} + \sin 1$

C. $\frac{\pi}{4} + \tan 1 + \cos 1$

D. $\frac{\pi}{4} + \tan 1 + \sin 1$

Answer: A

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37. The value of $5 \cdot \cot\left(\Sigma_{k=1}^{5} \cot^{-1}\left(k^2 + k + 1\right)\right)$ is equal to

A. $\frac{5}{2}$

B. 7

C. -7

D. $\frac{7}{2}$

Answer: B

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38. If the equation $5\arctan(x^2+x+k)+3arc\cot(x^2+x+k)=2\pi,$ has two distinct solutions,then the range of $k,$ is

A. $\left(0,\frac{5}{4}\right]$

B. $\left(-\infty,\frac{5}{4}\right)$

C. $\left(\frac{5}{4},\infty\right)$

D. $\left(-\infty,\frac{5}{4}\right]$

Answer: B

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39. If $f(x)=x^{11}+x^9-x^7+x^3+1$ and $f\left(\sin^{-1}(\sin 8)\right)=\alpha,\alpha$ is constant, then $f\left(\tan^{-1}(tan8)\right)$ is equal to α (b) $\alpha-2$ (c) $\alpha+2$ (d) $2-\alpha$

A. α

B. $\alpha - 2$

C. $\alpha + 2$

D. $2 - \alpha$

Answer: D

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40. The number of values of x for which $\sin^{-1}\left(x^2 - \frac{x^4}{3} + \frac{x^6}{9}\right) + \cos^{-1}\left(x^4 - \left(\frac{x^8}{3} + \frac{x^{12}}{9}..\right)\right) = \frac{\pi}{2}$, where \`0lt=|x|

A. 1

B. 2

C. 3

D. 4

Answer: C

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41. Suppose $3\sin^{-1}(\log_2 x) + \cos^{-1}(\log_2 y) = \pi/2$ and $\sin^{-1}(\log_2 x) + 2\cos^{-1}(\log_2 y) = 11\pi/6$. then the value of $\frac{1}{x^{-2}} + \frac{1}{y^{-2}}$ equals .

A. 6

B. 7

C. 5

D. $\frac{7}{2}$

Answer: A

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42. Find the domain and range of $f(x)=\sin^{-1}(\log[x])+\log(\sin^{-1}[x])$,

where $[\cdot]$ denotes the greatest integer function.

A. 1

B. 2

C. 0

D. $\left\{\log.\dfrac{\Pi}{2}\right\}$

Answer: D

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43. $\Sigma_{n=1}^{5}\sin^{-1}(\sin(2n-1))$ is

A. 1

B. 2

C. 3

D. 4

Answer: A

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44. If α and $\beta(\alpha > \beta)$ are roots of the equation $x^2 - \sqrt{2}x + \sqrt{3 - 2\sqrt{2}} = 0$, then the value of $\left(\cos^{-1}\alpha + \tan^{-1}\alpha + \tan\right.$

is equal to

A. $\frac{3\pi}{8}$

B. $\frac{5\pi}{8}$

C. $\frac{7\pi}{8}$

D. $\frac{\pi}{3}$

Answer: A

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45. If the mapping $f(x) = mx + c, m > 0$ maps $[-1, 1]$ onto $[0, 2]$, then $\tan\left(\tan^{-1}. \frac{1}{7} + \cot^{-1} 8 + \cot^{-1} 18\right)$ is equal to

A. $f\left(\frac{2}{3}\right)$

B. $f\left(\frac{1}{3}\right)$

C. $f\left(\frac{-1}{3}\right)$

D. $f\left(\frac{-2}{3}\right)$

Answer: D

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46. If

$(\sin^{-1} a)^2 + (\cos^{-1} b)^2 + (\sec^{-1} c)^2 + (\cos ec^{-1} d)^2 = \frac{5\pi^2}{2}$, then the va

A. $-\pi^2$

B. $-\frac{\pi^2}{2}$

C. 0

D. $\frac{\pi^2}{2}$

Answer: C

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47. If $f(x) = \Sigma_{r=1}^{n} \tan^{-1}\left(\frac{1}{x^2 + (2r-1)x + (r^2 - r + 1)}\right)$, then $\left| \lim_{n \to \infty} f'(0\right.$

is

A. 1

B. 2

C. 3

D. 4

Answer: A

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48. The range of the function

$f(x) = \sec^{-1}(x) + \tan^{-1}(x)$,is

A. $(0, \pi)$

B. $\left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$

C. $\left(0, \frac{3\pi}{4}\right]$

D. None of these

Answer: A

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49. The solution set of inequality $(\cot^{-1}x)(\tan^{-1}x) + \left(2 - \frac{\pi}{2}\right)\cot^{-1}x - 3\tan^{-1}x - 3\left(2 - \frac{\pi}{2}\right) > 0,$ is

A. $x \in (\tan 2, \tan 3)$

B. $x \in (\cot 3, \cot 2)$

C. $x \in (-\infty, \tan 2) \cup (\tan 3, \infty)$

D. $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

Answer: B

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50. Let $f(x) = \sin(\sin^{-1} 2x) + \cos ec(\cos ec^{-1} 2x) + \tan(\tan^{-1} 2x)$, then which one of the following statements is/are incorrect ?

A. f(x) is odd function

B. f(x) is injective

C. Range of f (x) contains only two integers.

D. The value of $f'\left(\frac{1}{2}\right)$ is equal to 6.

Answer: D

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51. If $f(x) = \cos^{-1}(\cos(x+1))$ and $g(x) = \sin^{-1}(\sin(x+2))$, then

A. $f(1) + g(1) = (\pi - 1)$

B. $f(1) > g(1)$

C. $f(2) > g(2)$

D. $f(2) < g(2)$

Answer: A::B::C

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52. Which of the following is/are correct?

A.

$$\cos\left(\cos\left(\cos^{-1} 1\right)\right) < \sin\left(\sin^{-1}(\sin(\pi - 1))\right) < \sin\left(\cos^{-1}(\cos(2\pi -$$

B.

$$\cos\left(\cos\left(\cos^{-1} 1\right)\right) < \sin\left(\cos^{-1}(\cos(2\pi - 2))\right) < \sin\left(\sin^{-1}(\sin(\pi -$$

C.

$$\sum_{t=1}^{5000} \cos^{-1}(\cos(2t\pi - 1)) = \sum_{t=1}^{2500} \cot^{-1}(\cot(t\pi + 2)) \quad \text{, where } \quad t \in I$$

D.

$$\cot^{-1} \cot \cos ec^{-1} \cos ec \sec^{-1} \sec \tan \tan^{-1} \cos \cos^{-1} \sin^{-1} \sin 4 = 4$$

Answer: A::B::C::D

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53. Let x_1 and $x_2(x_1 > x_2)$ be roots of the equation $\sin^{-1}\left(\cos\left(\tan^{-1}\left(\cos ec\left(\cot^{-1} x\right)\right)\right)\right) = \frac{\pi}{6}$, then

A. $\sin^{-1}. \frac{1}{x^1} + \cos^{-1}. \frac{1}{x_2} = \pi$

B. $\sin^{-1}\left(\frac{1}{x_1}\right) + \cos^{-1}\left(\frac{1}{x_2}\right) = 0$

C. $\sin^{-1}. \frac{1}{x_1} + \sin^{-1}\left(\frac{1}{x_2}\right) = 0$

D. $\cos^{-1}\left(\frac{1}{x_1}\right) + \cos^{-1}\left(\frac{1}{x_2}\right) = \pi$

Answer: A::C::D

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54. Suppose f, g and h be three real valued function defined on R Let $f(x) = 2x + |x|$ $g(x) = \frac{1}{3}(2x - |x|)$ $h(x) = f(g(x))$ The range of the function $k(x) = 1 + \frac{1}{\pi}\left(\cos^{-1} h(x) + \cot^{-1}(h(x))\right)$ is equal to

A. $\left[\frac{1}{4}, \frac{7}{4}\right]$

B. $\left[\frac{5}{4}, \frac{11}{4}\right]$

C. $\left[\frac{1}{4}, \frac{5}{4}\right]$

D. $\left[\frac{7}{4}, \frac{11}{4}\right]$

Answer: B

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55. Suppose f, g, and h be three real valued function defined on R.

Let $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$

The domain of definition of the function $l(x) = \sin^{-1}(f(x) - g(x))$ is equal to

A. $\left(\frac{3}{8}, \infty\right]$

B. $(-\infty, 1]$

C. $[-1, 1]$

D. $\left(-\infty, \frac{3}{8}\right]$

Answer: D

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56. In ΔABC, if $\angle B = \sec^{-1}\left(\frac{5}{4}\right) + \cos ec^{-1}\sqrt{5}$,

$\angle C = \cos ec^{-1}\left(\frac{25}{7}\right) + \cot^{-1}\left(\frac{9}{13}\right)$ and c = 3

$\tan A, \tan B, \tan C$ are in

A. AP

B. GP

C. HP

D. neither AP, GP nor HP

Answer: A

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57. In ΔABC, if $\angle B = \sec^{-1}\left(\frac{5}{4}\right) + \cos ec^{-1}\sqrt{5}$,
$\angle C = \cos ec^{-1}\left(\frac{25}{7}\right) + \cot^{-1}\left(\frac{9}{13}\right)$ and c = 3

The distance between orthocentre and centroid of triangle with sides $a^2, b^{\frac{4}{3}}$ and c is equal to

A. $\frac{5}{2}$

B. $\frac{5}{3}$

C. $\frac{10}{3}$

D. $\frac{7}{2}$

Answer: B

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58. Let x_1, x_2, x_3 be the solution of $\tan^{-1}\left(\frac{2x+1}{x+1}\right)+\tan^{-1}\left(\frac{2x-1}{x-1}\right)=2\tan^{-1}(x+1)$ where $x_1 < x_2$ is equal to

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59. If the range of function $f(x)=\left(\pi\sqrt{2}+\cos^{-1}\alpha\right)x^2+2\left(\cos^{-1}\beta\right)x+\pi\sqrt{2}-\cos^{-1}\alpha$ is $[0,\infty)$ then find the value of $|\alpha-\beta|+2\alpha\beta+1$.

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60. Consider $f(x)=\sin^{-1}[2x]+\cos^{-1}([x]-1)$ (where $[.]$ denotes greatest integer function .) If domain of $f(x)$ is $[a,b)$ and the range of $f(x)$ is $\{c,d\}$ then $a+b+\frac{2d}{c}$ is equal to (where $c < d$)

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61. Let $f(x) = \min\left(\tan^{-1}x, \cot^{-1}x\right)$ and $h(x) = f(x+2) - \pi/3$. Let x_1, x

be the integers in the range of h (x) , then the value of $\left(\cos^{-1}(\cos x_1) + \sin^{-1}(\sin x_2)\right)$ is equal to

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62. If the area enclosed by the curves $f(x) = \cos^{-1}(\cos x)$ and $g(x) = \sin^{-1}(\cos x)$ in $x \in [9\pi/4, 15\pi/4]$ is

(where a and b are coprime), then the value of b is ___

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63. Consider the curve $y = \tan^{-1}x$ and a point $A\left(1, \frac{\pi}{4}\right)$ on it. If the variable point $P_i(x_i, y_i)$ moves on the curve for $i = 1, 2, 3, \ldots N(n \in N)$ such that $y_i = \sum_{m=1}^{i} \tan^{-1}\left(\frac{1}{2m^2}\right)$ and

$B(x, y)$ be the limiting position of variable point P_n as $n \to \infty$, then the value of reciprocal of the slope of AB will be ____

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64. If

$$\tan^{-1} x + \tan^{-1}. \frac{\sqrt{1-y^2}}{y} = \frac{\pi}{3} \text{ and } \sin^{-1} y - \cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) =$$

is

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65. If

$$A = \frac{1}{1}\cot^{-1}\left(\frac{1}{1}\right) + \frac{1}{2}\cot^{-1}\left(\frac{1}{2}\right) + \frac{1}{3}\cot^{-1}\left(\frac{1}{3}\right) \text{ and } B = 1\cot^{-1}(1)$$

where $a, b, c, d \in N$ are in their lowest form , find $(b - a - c - d)$

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66. Statement I If α, β are roots of $6x^2 + 11x + 3 = 0$, then $\cos^{-1}\alpha$ exists but not $\cot^{-1}\beta(\alpha > \beta)$.

Statement II Domain of $\cos^{-1}x$ is $[-1, 1]$.

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67. Statement I If $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4} - \tan^{-1}z$ and $x + y + z = 1$, then arithmetic mean of odd powers of x, y, z is equal to 1/3 .

Statement II For any x, y, z we have

$$xyz - xy - yz - zx + x + y + z = 1 + (x-1)(y-1)(z-1)$$

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68. Match the principal values of $\cos^{-1}\left(8x^4 - 8x^2 + 1\right)$ given in column I with the corresponding intervals of x given in column II . For which it holds .

	Column I		Column II
A	$4\cos^{-1}x$	$p.$	$0 \le x \le \frac{1}{\sqrt{2}}$
B	$4\cos^{-1}x - 2\pi$	$q.$	$\frac{1}{\sqrt{2}} \le x \le 1$
C	$2\pi - 4\cos^{-1}x$	$r.$	$-1 \le x \le -\frac{1}{\sqrt{2}}$
D	$4\pi - 4\cos^{-1}x$	$s.$	$-\frac{1}{\sqrt{2}} \le x \le 0$

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69. If $A = 2\tan^{-1}(2\sqrt{2} - 1)$ and $B = 3\sin^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{3}{5}\right)$, then which is greater ?

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70. Solve $\left(\tan^{-1}x\right)^2 + \left(\cot^{-1}x\right)^2 = \frac{5\pi^2}{8}$

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71. If $\left[\sin^{-1}\left(\cos^{-1}\left(\sin^{-1}\left(\tan^{-1}x\right)\right)\right)\right]=1$, where $[\cdot]$ denotes the greatest integer function, then $x\in$

A. $[\tan\sin\cos 1, \tan\sin\cos\sin 1]$

B. $(\tan\sin\cos 1, \tan\sin\cos\sin 1)$

C. $[-1, 1]$

D. $[\sin\cos\tan 1, \sin\cos\sin\tan 1]$

Answer: A

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72. If $\tan^{-1}y=4\tan^{-1}x\left(|x|<\frac{\tan(\pi)}{8}\right)$. Find y as an algebraic function of x, and, hence, prove that $\tan\pi/8$ is a root of the equation $x^4-6x^2+1=0$

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73. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta + x^2 . \cos 2\beta - x \cos \beta - \sin \beta = 0$, then $\tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$ is equal to

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74. Find the number of positive integral solution of the equation $\tan^{-1} x + \frac{\cos^{-1}(y)}{\sqrt{1 - y^2}} = \frac{\sin^{-1}(3)}{\sqrt{10}}$

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75. If $\cot^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{6}, n \in N$, then the maximum value of n is :

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76. If $\cot^{-1}\left(\frac{n^2 - 10n + 21.6}{\pi}\right) > \frac{\pi}{6}$, where $xy < 0$ then the possible values of z is (are) 3 (b) 2 (c) 4 (d) 8

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77. The set of values of k for which $x^2 - kx + \sin^{-1}(\sin 4) > 0$ for all real x is

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78. The least and the greatest values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are $\frac{-\pi}{2}, \frac{\pi}{2}$ (b) $\frac{-\pi^3}{8}, \frac{\pi^3}{8}$ $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ (d) none of these

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79. If x_r is given by , $x_{r+1} = \sqrt{\frac{1}{2}(1 + x_r)}$. Then, show: $\cos^{-1} x_0 = \frac{\sqrt{1 -}}{x_1 x_2}$ ......up to infinity .

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80. Express:

$$\cot^{-1}\left(\frac{y}{(1-x^2-y^2)}\right)=2\tan^{-1}\sqrt{\frac{3-4x^2}{4x^2}}-\frac{\tan^{-1}\sqrt{3-4x^2}}{x^2}$$ as a rational integral equation in x and y.

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82. If $x=\cos ec\left[\tan^{-1}\left\{\cos\left(\cot^{-1}\left(\sec\left(\sin^{-1}a\right)\right)\right)\right\}\right]$ and $y=\sec\left[\cot^{-1}\left\{\sin\left(\tan^{-1}\left(\cos ec\left(\cos^{-1}a\right)\right)\right)\right\}\right]$ then find the relation between x and y

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83. Show taht

$$2\tan^{-1}\left(\tan\left(\frac{\alpha}{2}\right)\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right)=\tan^{-1}\left(\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}\right)$$

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84. Solve the following equation for x :

$$3\frac{\sin^{-1}(2x)}{1+x^2}-4\frac{\cos^{-1}(1-x^2)}{1+x^2}+2\frac{\tan^{-1}(2x)}{1-x^2}=\frac{\pi}{3}$$

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85. Solve the equation : $2\tan^{-1}(2x-1)=\cos^{-1}x.$

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Jee Type Solved Examples Subjective Type Examples

1. Solve the equation $\sin^{-1}yx+\sin^{-1}6\sqrt{3}x=\frac{-\pi}{2}.$

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Exercise For Session 1

1. Find the value of the following

$$\sin\left[\frac{\pi}{3}-\sin^{-1}.\left(\frac{1}{2}\right)\right]$$

A. $-1/2$

B. 1

C. $1/2$

D. $1/4$

Answer: C

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2. Find the value of : $\cos ec\left[\sec^{-1}\left(\sqrt{2}\right)+\cot^{-1}(1)\right]$

A. 1

B. -2

C. 0

D. -1

Answer: D

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3. Find the domain of the following

$y = \sec^{-1}(x^2 + 3x + 1)$

A. $(-\infty, -3] \cup [-2, -1] \cup [0, \infty)$

B. $(-\infty, -3] \cup [-2, -1]$

C. $(-\infty, -3] \cup [0, \infty)$

D. None of these

Answer: A

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4. Find the domain of the following $y=\cos^{-1}\left(\frac{x^2}{1+x^2}\right)$

A. `[-1,1]

B. R

C. $[0, 1]$

D. $[-1, 5]$

Answer: B

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5. Find the domain of the following $y=\tan^{-1}\left(\sqrt{x^2-1}\right)$

A. $(-\infty, -2]\cup[1, \infty)$

B. `(-infty,-1]

C. $(-\infty, -1]\cup[2, \infty)$

D. $(-\infty, -1] \cup [1, \infty)$

Answer: D

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Exercise For Session 2

1. What is the value of $\cos^{-1}\left(\frac{\cos(2\pi)}{3}\right) + \sin^{-1}\left(\frac{\sin(2\pi)}{3}\right)$?

A. $\frac{\pi}{2}$

B. $-\pi$

C. π

D. 0

Answer: C

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2. The value of $\sin^{-1}\left(\frac{\sin(3\pi)}{5}\right)$

A. $\frac{2\pi}{5}$

B. (2π)

C. $\frac{\pi}{5}$

D. None of these

Answer: A

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3. Find the value of $\sin^{-1}\left(\cos.\frac{33\pi}{5}\right)$.

A. $-\frac{\pi}{5}$

B. $-\frac{\pi}{10}$

C. $\frac{\pi}{5}$

D. $\frac{\pi}{10}$

Answer: B

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4. Find $\sin^{-1}(\sin\theta).\ \cos^{-1}(\cos\theta), \tan^{-1}(\tan\theta), \cot^{-1}(\cot\theta)$ for $\theta \in \left(\frac{5\pi}{2}, 3\pi\right)$

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Exercise For Session 3

1. Find the value of : $\cos\left\{\sin\left(\sin^{-1}\frac{\pi}{6}\right)\right\}$

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2. Find the value of : $\sin\left\{\cos\left(\cos^{-1}\frac{3\pi}{4}\right)\right\}$

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3. Evaluate the following :

$$\sin^2\left(\cos^{-1}.\ \frac{1}{2}\right)+\cos^2\left(\sin^{-1}.\ \frac{1}{3}\right).$$

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4. The value of $\tan^2(\sec^{-1}2)+\cot^2(\text{cosec}^{-1}3)$ is

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5. Find the solutions of the equation $\cos(\cos^{-1}x)=\cos ec(\cos ec^{-1}x)$.

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Exercise For Session 4

1. Evaluate the following

$$\tan^{-1}\left\{\tan\left(-\frac{7\pi}{8}\right)\right\}.$$

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2. Evaluate the following

$$\tan^{-1}\left\{\cot\left(-\frac{1}{4}\right)\right\}$$

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3. Evaluate the following

$$\sec\left(\cos^{-1}\left(\frac{2}{3}\right)\right).$$

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4. Evaluate the following

$$\cos ec\left(\sin^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$$

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5. Evaluate the following

$$\cos\left[\cos^{-1}\left(\frac{-1}{3}\right)-\sin^{-1}\left(\frac{1}{3}\right)\right].$$

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6. If $\sin^{-1}x=\pi/5$, for some $x\in(-1,1)$, then find the value of $\cos^{-1}x$

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7. If $\sec^{-1}x=\cos ec^{-1}y$, then find the value of $\cos^{-1}.\frac{1}{x}+\cos^{-1}.\frac{1}{y}$

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8. Prove that $\tan^{-1}x+\tan^{-1}.\frac{1}{x}=\begin{cases}\pi/2 & \text{if} x>0\\ -\pi/2 & \text{if } x<0\end{cases}$

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9. Solve the following

$5\tan^{-1}x + 3\cot^{-1}x = 2\pi$

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10. If $4\sin^{-1}x + \cos^{-1}x = \pi$, then what is the value of x ?

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Exercise For Session 5

1. Show that $\sin^{-1}.\ \frac{3}{5} + \sin^{-1}.\ \frac{15}{17} = \pi - \sin^{-1}.\ \frac{84}{85}$

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2. Evaluate $\sin^{-1}. \frac{4}{5} + \sin^{-1}. \frac{5}{13} + \sin^{-1}. \frac{16}{65}$

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3. If $\tan^{-1} 4 + \tan^{-1} 5 = \cot^{-1} \lambda$, then find $'\lambda'$

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4. If $x \in \left(0, \frac{\pi}{2}\right)$, then show that $\cos^{-1}\left(\frac{7}{2}(1 + \cos 2x) + \sqrt{(\sin^2 x - 48\cos^2 x)} \sin x\right) = x - \cos^{-1}(7 \cos x$

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5. Solve the following

$\sin^{-1}. \frac{1}{5} + \sin^{-1}. \frac{2}{3} = \sin^{-1} x$

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6. Solve the following

$$\sin^{-1} x + \sin^{-1} 2x = \frac{2\pi}{3}$$

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Exercise For Session 6

1. Evaluate the following :

$$\tan\left(\cos ec^{-1} \frac{\sqrt{41}}{4}\right)$$

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2. Evaluate the following :

$$\sec\left(\cot^{-1}. \frac{16}{63}\right)$$

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3. Find the value of $\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right)$

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4. Find the value of $\sin\left(\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right)$

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5. Show that $\cot\left[\sin^{-1}\sqrt{\frac{13}{17}}\right] = \sin\left[\tan^{-1}.\ \frac{2}{3}\right]$

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Exercise For Session 7

1. Sketch for the curve $y = \sin^{-1}(3x - 4x^3)$

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2. Draw the graph of $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$.

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3. Draw the graph of the following

$y = \cos^{-1}(2x^2 - 1)$

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4. Draw the graph of $y = \sin^{-1}\left(2x\sqrt{1 - x^2}\right)$

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5. Draw the graph of $y = \tan^{-1}\left(\frac{2x}{1 - x^2}\right)$

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6. Draw the graph of $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

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7. Draw the graph of $y = \cos^{-1}\left(\frac{1+x^2}{1+x^2}\right)$

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Exercise Single Option Correct Type Questions

1. Find $\cot^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$ in terms of cos

A. $\cos^{-1}(x^2)$

B. $\frac{\pi}{2} - \frac{1}{2}\cos^{-1}(x^2)$

C. $\frac{\pi}{3} - \frac{1}{2}\cos^{-1}(x^2)$

D. None of these

Answer: D

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2. The value of $\cos\left(\frac{1}{2}\cos^{-1}.\frac{1}{8}\right)$ is equal to

A. $3/4$

B. $-3/4$

C. $1/16$

D. $1/4$

Answer: A

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3. solve $\sin^{-1}(\sin 5) > x^2 - 4x$

A. $x = 2 - \sqrt{9 - 2\pi}$

B. $x = 2 + \sqrt{9 - 2\pi}$

C. $x \in \left(2 - \sqrt{9 - 2\pi}, 2 + \sqrt{9 - 2\pi}\right)$

D. $x > 2 + \sqrt{9 - 2\pi}$

Answer: C

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4. The value of

$$\sin^{-1}\left\{\left(\sin.\ \frac{\pi}{3}\right)\frac{x}{\sqrt{(x^2 + k^2 - kx)}}\right\} - \cos^{-1}\left\{\left(\cos.\ \frac{\pi}{6}\right)\frac{x}{\sqrt{(x^2 + k^2 - kx)}}\right.$$

is

A. $\tan^{-1}\left(\frac{2x^2 + sk - k^2}{x^2 - 2xk + k^2}\right)$

B. $\tan^{-1}\left(\frac{x^2 + 2xk - 2k^2}{x^2 - 2xk - k^2}\right)$

C. $\tan^{-1}\left(\frac{x^2 + 2xk - 2k^2}{2x^2 - 2xk + 2k^2}\right)$

D. None of the above

Answer: D

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5. Find the smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right), 0 \le x \le 1$

A. $\left(0, \frac{\pi}{4}\right)$

B. $\left(0, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

D. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Answer: A

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6. Sum of infinite terms of the series $\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+...$ is

A. $\pi/4$

B. $\tan^{-1} 2$

C. $\tan^{-1} 3$

D. $\tan^{-1} 4$

Answer: B

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7. Solution of equation $\cot^{-1} x + \sin^{-1}. \frac{1}{\sqrt{5}} = \frac{\pi}{4}$ is

A. $x = 3$

B. $x = 1/\sqrt{5}$

C. $x = 0$

D. None of these

Answer: A

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8. Solution set of the inequality

$\left(\cot^{-1}x\right)^2 - \left(5\cot^{-1}x\right) + 6 > 0$ is

A. $(\cot 3, \cot 2)$

B. $(-\infty, \cot 2) \cup (\cot 2, \infty)$

C. $(\cot 2, \infty)$

D. None of the above

Answer: B

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9. Find the sum of the series :

$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \ldots. + \tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) + \ldots\ldots\infty$

A. $\pi / 4$

B. $\pi / 2$

C. π

D. None of these

Answer: A

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10. If $x+\frac{1}{x}=2$, the principal value of $\sin^{-1}x$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer: B

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11. If $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then the value of $\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\sin 2x}{5 + 3\cos 2x}\right)$ is

A. $x/2$

B. $2x$

C. $3x$

D. x

Answer: D

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12. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$

A. $\frac{2\pi}{3}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. π

Answer: B

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13. $\sin\left[\tan^{-1}.\frac{1-x^2}{2x}+\cos^{-1}.\frac{1-x^2}{1+x^2}\right]$ is

A. 1

B. 0

C. -1

D. None of these

Answer: A

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14. If $\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)-\cos^{-1}\left(\frac{1-b^2}{1+b^2}\right)=2\tan^{-1}x$, then x is

A. $\frac{a-b}{1+ab}$

B. $\frac{b-a}{1+ab}$

C. $\frac{a+b}{1-ab}$

D. None of these

Answer: A

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15. If $\left|\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right| < \frac{\pi}{3}$, then

A. $x \in \left[-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

B. $x \in \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

C. $x \in \left[0, \frac{1}{\sqrt{3}}\right]$

D. None of these

Answer: B

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16. The value of

$$\cos^{-1}\left[\cot\left(\sin^{-1}\left(\sqrt{\frac{2-\sqrt{3}}{4}}\right)+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\sqrt{2}\right)\right.$$

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

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17. If $\frac{\tan^{-}(x)}{\pi} < \frac{\pi}{3}x \in N$ then the maximum vlaue of x is

A. 2

B. 5

C. 7

D. None of these

Answer: B

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18. If $\tan^{-1}.\ \frac{\sqrt{(1+x^2)}-\sqrt{(1-x^2)}}{\sqrt{(1+x^2)}+\sqrt{(1-x^2)}}=\alpha$, then x^2 is

A. $\cos 2\alpha$

B. $\sin 2\alpha$

C. $\tan 2\alpha$

D. $\cot 2\alpha$

Answer: B

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19. If $\cos ec^{-1}(\cos ecx)$ and $\cos ec\left(\cos ec^{-1}x\right)$ are equal functions, then the maximum range of value of x is

A. $\left[-\frac{\pi}{2}, -1\right] \cup \left[1, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

C. $(-\infty, -1] \cup [1, \infty)$

D. $[-1, 0) \cup (0, 1]$

Answer: A

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20. The value of $\lim_{|x| \to \infty} \cos\left(\tan^{-1}\left(\sin\left(\tan^{-1} x\right)\right)\right)$ is equal to

A. -1

B. $\sqrt{2}$

C. $-\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{2}}$

Answer: D

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21. Complete solution set of $\left(\cot^{-1}x\right)+2\left(\tan^{-1}x\right)=0,$ where [] denotes the greatest integer function, is equal to $(0, \cot 1)$ (b) $(0, \tan 1)$ $(\tan 1, \infty)$ (d) $(\cot 1, \tan 1)$

A. $(0, \cot 1)$

B. $(0, \tan 1)$

C. $(\tan 1, \infty)$

D. $(\cot 1, \tan 1)$

Answer: D

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22. If $\sin^{-1}:[-1,1]\rightarrow\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ and $\cos^{-1}:[-1,1]\rightarrow[0,\pi]$ be two bijective functions, respectively inverse of bijective functions $\sin:\left[\frac{\pi}{2},\frac{3\pi}{2}\right]\rightarrow[-1,1]$ and $\cos:[0,\pi]\rightarrow[-1,1]$ then $\sin^{-1}x+\cos$

is

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. not a constant

Answer: D

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23. If $a\sin^{-1}x-b\cos^{-1}x=c$, then $a\sin^{-1}x+b\cos^{-1}x$ is equal to 0 (b) $\frac{\pi ab+c(b-a)}{a+b}$ $\frac{\pi}{2}$ (d) $\frac{\pi ab+c(a-b)}{a+b}$

A. 0

B. $\dfrac{\pi ab + c(b-a)}{a+b}$

C. $\dfrac{\pi}{2}$

D. $\dfrac{\pi ab + c(a-b)}{a+b}$

Answer: D

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24. The number of integer x satisfying $\sin^{-1}|x-2| + \cos^{-1}(1-|3-x|) = \dfrac{\pi}{2}$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

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25. The value of α such that $\frac{\sin^{-1} 2}{\sqrt{5}}, \frac{\sin^{-1} 3}{\sqrt{10}}, \sin^{-1} \alpha$ are the angles of a triangle is $\frac{-1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

A. $\frac{-1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{2}}$

Answer: D

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26.

$Let \left| \tan^{-1} x \tan^{-1} 2x \tan^{-1} 3x \tan^{-1} 3x \tan^{-1} x \tan^{-1} 2x \tan^{-1} 2x \tan^{-1} \right.$

of values of x satisfying the equation is 1 (b) 2 (c) 3 (d) 4

A. 1

B. 2

C. 3

D. 4

Answer: A

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27. If α is the only real root of the equation $x^3 + bx^2 + cx + 1 = 0 (b < c)$, then find the value of $\tan^{-1}\alpha + \tan^{-1}(\alpha^{-1})$

A. $-\pi$

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: A

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28. Let $u = \cot^{-1}\sqrt{\cos 2\theta} - \tan^{-1}\sqrt{\cos 2\theta}$, then the value of $\sin u$ is

A. $\cos 2\theta$

B. $\sin 2\theta$

C. $\tan^2\theta$

D. $\cot^2\theta$

Answer: C

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29. Let

$$f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}xx \geq 0, \; = \; -2\tan^{-1}xx > 0$$

function fx) is continuous everywhere but not differentiable at x equals to

A. 1

B. -1

C. 0

D. $\frac{1}{\sqrt{2}}$

Answer: C

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30. Let $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) \forall x \in R.$ The function $f(x)$ is continuous everywhere but not differentiable at x is/ are

A. $0, 1$

B. $-1, 1$

C. $-1, 0$

D. $0, 2$

Answer: B

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31. Let $f: \overrightarrow{R0, \frac{\pi}{2}}$ be defined by $f(x) = \tan^{-1}\left(x^2 + x + a\right)\cdot$ Then the set of values of a for which f is onto is $(0, \infty)$ (b) $[2, 1]$ (c) $\left[\frac{1}{4}, \infty\right]$ (d) none of these

A. $(81, \infty)$

B. $[81, \infty)$

C. $(-\infty, 81)$

D. $(-\infty, 81]$

Answer: A

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32. Let $f(x) = \sin^{-1} 2x + \cos^{-1} 2x + \sec^{-1} 2x$. Then the sum of the maximum and minimum values of f(x) is

A. π

B. 2π

C. 3π

D. $\frac{\pi}{2}$

Answer: B

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33. If $\tan^{-1}.\frac{b}{c+a}+\tan^{-1}.\frac{c}{a+b}=\frac{\pi}{4}$ where a, b, c , are the sides of $\Delta ABC, \text{then} \Delta ABC$ is

A. Acute - angled triangle

B. Obtuse - angled triangle

C. Right- angled triangle

D. Equilateral triangle

Answer: C

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34. Solutions of $\sin^{-1}(\sin x) = \sin x$ are if $x \in (0, 2\pi)$

A. 4 real roots

B. 2 positive real roots

C. 2 negative real roots

D. 5 real roots

Answer: D

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35. The equation $\frac{e^{\sin^{-1} x}}{\pi} = \frac{y}{\log y}$ has

A. Unique solution

B. Infinite many solution

C. $x = 1$

D. $y = e$

Answer: B

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36. Let $f(x) = 1 + 2\sin\left(\frac{e^x}{e^x + 1}\right)$ $x \geq 0$ then $f^{-1}(x)$ is equal to (assuming f is bijectve)

A. $\log\left(\frac{\sin^{-1}\left(\frac{x-1}{2}\right)}{1 - \sin^{-1}\left(\frac{x-1}{2}\right)}\right)$

B. $\log\left(\frac{\sin\left(\frac{x-1}{2}\right)}{1 - \sin\left(\frac{x-1}{2}\right)}\right)$

C. $e^{\frac{\sin^{-1}.\left(\frac{x-1}{2}\right)}{1 - \sin^{-1}\left(\frac{x-1}{2}\right)}}$

D. $e^{\frac{\sin\left(\frac{x-1}{2}\right)}{1 - \sin\left(\frac{x-1}{2}\right)}}$

Answer: A

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37. $\cos^{-1}\left(\cos\left(2\cot^{-1}\left(\sqrt{2}-1\right)\right)\right)$ is equal to

A. $\sqrt{2}-1$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. None of these

Answer: C

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38. The maximum value of $f(x)=\tan^{-1}\left(\frac{\left(\sqrt{12}-2\right)x^2}{x^4+2x^2+3}\right)$ is (A) 18° (B) 36° (C) 22.5° (D) 15°

A. 18°

B. 36°

C. 22.5°

D. 15°

Answer: D

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39. If $\dfrac{\tan^{-1}\left(\sqrt{1+x^2-1}\right)}{x}=4^0$ then $x=\tan 2^0$ (b) $x=\tan 4^0$ $x=\dfrac{\tan 1}{4^0}$ (d) $x=\tan 8^0$

A. $x=\tan 2^{\circ}$

B. $x=\tan 4^{\circ}$

C. $x=\tan(1/4)^{\circ}$

D. $x=\tan 8^{\circ}$

Answer: D

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40. If $\tan^{-1}(\sin^2\theta - 2\sin\theta + 3) + \cot^{-1}(5^{\sec} \hat{} (2y) + 1) = \frac{\pi}{2}$, then value of $\cos^2\theta - \sin\theta$ is equal to 0 (b) -1 (c) 1 (d) none of these

A. 0

B. -1

C. 1

D. None of the above

Answer: C

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41. The number of solution of the equation $\left|\tan^{-1}|x|\right| = \sqrt{(x^2+1)^2 - 4x^2}$ is

A. 1

B. 2

C. 3

D. 4

Answer: D

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42. For any real number $x \geq 1$, the expression

$\sec^2(\tan^{-1}x) - \tan^2(\sec^{-1}x)$ is equal to

A. 1

B. 2

C. $2x^2$

D. $2\sqrt{2}$

Answer: B

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43. Let $f: R \rightarrow \left[0, \frac{\pi}{2}\right)$ be defined by $f(x) = \tan^{-1}(3x^2 + 6x + a)$. If $f(x)$ is an onto function . then the value of a si

A. 1

B. 2

C. 3

D. 4

Answer: C

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44. The value of expression $\tan^{-1}\left(\frac{\sqrt{2}}{2}\right) + \sin^{-1}\left(\frac{\sqrt{5}}{5}\right) - \cos^{-1}\left(\frac{\sqrt{10}}{10}\right)$

A. $\cot^{-1}\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\right)$

B. $\cot^{-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

C. $-\pi+\cot^{-1}\left(\frac{1+\sqrt{2}}{1-\sqrt{2}}\right)$

D. $\pi-\cot^{-1}\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)$

Answer: C

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45. The value of $\sec\left(2\cot^{-1}2+\cos^{-1}.\frac{3}{5}\right)$ is equal to

A. $\frac{25}{24}$

B. $-\frac{24}{7}$

C. $\frac{25}{7}$

D. $-\frac{25}{7}$

Answer: D

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46. Which one of the following statement is meaningless ?

A. $\cos^{-1}\left(\text{In}\left(\frac{2e+4}{3}\right)\right)$

B. $\cos ec^{-1}\left(\frac{\pi}{3}\right)$

C. $\cot^{-1}\left(\frac{\pi}{2}\right)$

D. $\sec^{-1}(\pi)$

Answer: A

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47. The value of $\sec\left(\sin^{-1}\left(\sin\left(\frac{-50\pi}{9}\right)\right)+\cos^{-1}\left(\frac{\cos(31\pi)}{9}\right)\right)$

A. $\sec.\ \frac{10\pi}{9}$

B. $\sec.\ \frac{\pi}{9}$

C. 1

D. -1

Answer: B

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48. The number k is such that $\tan\{arc\tan(2) + arc\tan(20k)\} = k$. The sum of all possible values of k is

A. $-\frac{19}{40}$

B. $-\frac{21}{40}$

C. 0

D. $\frac{1}{5}$

Answer: A

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49. The value of $\Sigma_{r=2}^{\infty}\tan^{-1}\left(\frac{1}{r^2 - 5r + 7}\right)$, is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

Answer: C

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50. If

$$x = \tan^{-1} 1 - \cos^{-1}\left(-\frac{1}{2}\right) + \frac{\sin^{-1} 1}{2}, y = \cos\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right),$$

then

A. $x = \pi y$

B. $y = \pi x$

C. $\tan x = -(4/3)y$

D. $\tan x = (4/3)y$

Answer: C

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51. Prove that: `tan^(-1)(1/2tan2A)+tan^(-1)(cota)+tan^(-1)(cot^3A)={ when 0

A. $4\tan^{-1}(1)$

B. $2\tan^{-1}(2)$

C. 0

D. None of these

Answer: A

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52. $\sum_{n=1}^{\infty}\left(\tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right)\right)$ is equal to (A) $\tan^{-1}(2)+\tan^{-1}(3)$ (B) $4\tan^{-1}(1)$ (C) $\frac{\pi}{2}$ (D) $\sec^{-1}(-\sqrt{2})$

A. $\tan^{-1}.\frac{1}{2}+\tan^{-1}.\frac{2}{3}$

B. $4\tan^{-1}1$

C. $\frac{\pi}{2}$

D. $\sec^{-1}(-\sqrt{2})$

Answer: A

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53. Number of solutions (s) of the equations

$\cos^{-1}(1-x)-2\cos^{-1}x=\frac{\pi}{2}$ is

A. 3

B. 2

C. 1

D. 0

Answer: C

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54. There exists a positive real number of x satisfying $\cos\left(\tan^{-1}x\right)=x\cdot$ Then the value of $\cos^{-1}\left(\frac{x^2}{2}\right) is\ \frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{2\pi}{5}$ (d) $\frac{4\pi}{5}$

A. $\frac{\pi}{10}$

B. $\frac{\pi}{5}$

C. $\frac{2\pi}{5}$

D. $\frac{4\pi}{5}$

Answer: C

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55. The range of values of p for which the equation $\sin\cos^{-1}\left(\cos\left(\tan^{-1}x\right)\right)=p$ has a solution is

A. $\left(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$

B. $[0, 1)$

C. $\left[\frac{1}{\sqrt{2}}, 1\right)$

D. $(-1, 1)$

Answer: B

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56. Number of solutions of the equation $\log_{10}\left(\sqrt{5\cos^{-1}x - 1}\right) + \frac{1}{2}\log_{10}\left(2\cos^{-1}x + 3\right) + \log_{10}\sqrt{5} = 1$ is

A. 0

B. 1

C. more than one but finite

D. infinite

Answer: B

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57. Solve $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(3x-2)$

A. $\left\{\frac{1}{2}, 1\right\}$

B. $\left[\frac{1}{2}, 1\right]$

C. $\left[\frac{1}{3}, 1\right]$

D. $\left\{\frac{1}{3}, 1\right\}$

Answer: A

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58. The set of values of x, satisfying the equation $\tan^2\left(\sin^{-1}x\right) > 1$ is -

A. $[-1, 1]$

B. $\left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

C. $(-1, 1) - \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$

D. $[-1, 1] - \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

Answer: C

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59. The solution set of equation $\sin^{-1}\sqrt{1-x^2} + \cos^{-1}x = \cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) - \sin^{-1}x$, is

A. $[-1, 1] - \{0\}$

B. $(0, 1] \cup \{-1\}$

C. $[-1, 0) \cup \{1\}$

D. $[-1, 1]$

Answer: C

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60. If $\cos^{-1}.\frac{x}{a} - \sin^{-1}.\frac{y}{b} = \theta(a, b, \neq 0)$, then the maximum value of $b^2x^2 + a^2y^2 + 2abxy\sin\theta$ equals

A. ab

B. $(a+b)^2$

C. $2(a+b)^2$

D. a^2b^2

Answer: D

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61. The value of $\Sigma_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{r^2+5r+7}\right)$ is equal to

A. $\tan^{-1}3$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2} - \cos^{-1}.\frac{1}{\sqrt{10}}$

D. $\cot^{-1} 2$

Answer: C

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62. The range of the function ,

$f(x) = \tan^{-1}\left(\frac{1+x}{1-x}\right) - \tan^{-1} x$ is

A. $\{\pi/4\}$

B. $\{-(\pi/4), 3\pi/4\}$

C. $\{\pi/4, -(3\pi/4)\}$

D. $\{3\pi/4\}$

Answer: C

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63. Let $g : \overrightarrow{R0, \frac{\pi}{3}}$ be defined by $g(x) = \cos^{-1}\left(\frac{x^2 - k}{1 + x^2}\right)$. Then find the possible values of k for which g is a subjective function.

A. $\left\{\frac{1}{2}\right\}$

B. $\left(-1, -\frac{1}{2}\right]$

C. $\left\{-\frac{1}{2}\right\}$

D. $\left[-\frac{1}{2}, 1\right)$

Answer: C

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64. Number of values of x satisfying simultaneously $\sin^{-1} x = 2\tan^{-1} x$ and $\tan^{-1}\sqrt{x(x-1)} + \cos ec^{-1}\sqrt{1 + x - x^2} = \frac{7}{2}$, is

A. 0

B. 1

C. 2

D. 3

Answer: C

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65. Number of values of x satisfying the equation $\cos(3arc\cos(x-1))=0$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: D

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66. Which one of the following function contains only one integer in its range ?

[Note sgn (k) denotes the signum function of k.]

A. a. $f(x) = \frac{1}{2}\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

B. b. $g(x) = sgn\left(x + \frac{1}{x}\right)$

C. c. $h(x) = \sin^2 x + 2\sin x + 2$

D. d. $k(x) = \cos^{-1}(x^2 - 2x + 2)$

Answer: D

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67. If range of the function $f(x) = \tan^{-1}(3x^2 + bx + 3), x \in R$ is $\left[0, \frac{\pi}{2}\right)$, then square of sum of all possible values of b will be

A. 0

B. 18

C. 72

D. None of these

Answer: A

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Exercise More Than One Correct Option Type Questions

1. Let $\theta = \tan^{-1}\left(\tan.\ \frac{5\pi}{4}\right)$ and $\phi = \tan^{-1}\left(-\tan.\ \frac{2\pi}{3}\right)$ then

A. $\theta > \phi$

B. $4\theta - 3\phi = 0$

C. $\theta + \phi = \frac{7\pi}{12}$

D. None of these

Answer: B::C

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2. Let $f(x) = e^{\cos} \,\hat{}\, ((-1))\left\{\sin\left(x + \frac{\pi}{3}\right)\right\}$. Then, $f\left(\frac{8\pi}{9}\right) = e^{5\pi/18}$ (b) $e^{13\pi/18}$ (c) $e^{-2\pi/18}$ (d) none of these

A. $f\left(\frac{8\pi}{9}\right) = e^{5\pi/18}$

B. $f\left(\frac{8\pi}{9}\right) = e^{13\pi/18}$

C. $f\left(-\frac{7\pi}{4}\right) = e^{\pi/12}$

D. $f\left(-\frac{7\pi}{4}\right) = e^{11\pi/12}$

Answer: B::C

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3. The value of $\tan\left[\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right]$ is $\frac{6}{17}$ (b) $\frac{7}{16}$ (c) $\frac{16}{7}$ (d) none of these

A. $a + b = 23$

B. $a - b = 11$

C. $3b = a + 1$

D. $2a = 3b$

Answer: A::B::C

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4. Let $f(x) = \sin^{-1} x + \cos^{-1} x$. Then $\frac{\pi}{2}$ is equal to

A. $f\left(-\frac{1}{2}\right)$

B. $f(k^2 - 2k + 3), k \in R$

C. $f\left(\frac{1}{1+k^2}\right), k \in R$

D. $f(-2)$

Answer: A::C

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5. The solution of $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is

A. $n\pi - 1$

B. $n\pi$

C. $n\pi + 1$

D. $2n\pi + 1$

Answer: A::B::C

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6. If $(\sin^{-1}x + \sin^{-1}w)(\sin^{-1}y + \sin^{-1}z) = \pi^2$, then $D = \begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix} (N_1, N_2, N_3, N_4 \in N)$

A. has a maximum value of 2

B. has a minimum value of 0

C. 16 different D are possible

D. has a minimum value of -2

Answer: A::C::D

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7. Indicate the relation which can hold in their respective domain for infinite values of x

A. $\tan\left|\tan^{-1}x\right| = |x|$

B. $\cot\left|\cot^{-1}x\right| = |x|$

C. $\tan^{-1}|\tan x| = |x|$

D. $\sin\left|\sin^{-1}x\right| = |x|$

Answer: A::B::C::D

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8. To the equation $2^{2\pi/\cos^{-1}x} - \left(a+\frac{1}{2}\right)2^{\frac{\pi}{\cos^{-1}x}} - a^2 = 0$ has only one real root, then

A. $1 \leq a \leq 3$

B. $a \geq 1$

C. $a \leq -3$

D. $a \geq 3$

Answer: B::C

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9. $\sin^{-1}(\sin 3) + \sin^{-1}(\sin 4) + \sin^{-1}(\sin 5)$ when simplified reduces to

A. an irrational number

B. a rational number

C. an even prime

D. a negative integer

Answer: B::D

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10. $2\tan\left(\tan^{-1}(x)+\tan^{-1}(x^3)\right), wherex \in R-\{-1,1\}$, is equal to $\dfrac{2x}{1-x^2}t\left(2\tan^{-1}x\right)\tan\left(\cot^{-1}(-x)-\cot^{-1}(x)\right)\tan\left(2\cot^{-1}x\right)$

A. $\dfrac{2x}{1-x^2}$

B. $\tan\left(2\tan^{-1}x\right)$

C. $\tan\left(\cot^{-1}(-x)-\cot^{-1}(x)\right)$

D. $\tan\left(2\cot^{-1}x\right)$

Answer: A::B::C

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11. Let $f(x)=\sin^{-1}|\sin x|+\cos^{-1}(\cos x)$. Which of the following statement(s) is / are TRUE ?

A. $f(f(3))=\pi$

B. f (x) is periodic with fundamental period 2π

C. f (x) is neither even nor odd

D. Range of f(x) is $[0, 2\pi]$

Answer: A::B

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12. If $f(x) = \sin^{-1} x . \cos^{-1} x . \tan^{-1} x . \cot^{-1} x . \sec^{-1} x . \cos ec^{-1} x$, then which of the following statement (s) hold(s) good?

A. (a) The graph of $y = f(x)$ does not lie above x-axis

B. (b) The non-negative difference between the maximum and the minimum value of the function $y = f(x)$ is $\frac{3\pi^6}{64}$

C. (c) The function $y = f(x)$ is not injective.

D. (d) Number of non-negative integers in the domain of $f(x)$ is 2.

Answer: A::B

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13. Let

$\alpha = 3\cos^{-1}\left(\frac{5}{\sqrt{28}}\right) + 3\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ and $\beta = 4\sin^{-1}\left(\frac{7\sqrt{2}}{10}\right) - 4\tan$

, then which of the following does not hold (s) good ?

A. $\alpha < \pi \quad \text{but} \quad \beta > \pi$

B. $\alpha > \pi \quad \text{but} \quad \beta < \pi$

C. Both $\alpha \quad \text{and} \quad \beta$ are equal

D. $\cos(\alpha + \beta) = 0$

Answer: A::B::D

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14. Let function f(x) be defined as

$f(x) = \left|\sin^{-1} x\right| + \cos^{-1}\left(\frac{1}{x}\right)$. Then which of the following is /are TRUE.

A. f(x) is injective in its domain.

B. f(x) is many - one in its domain.

C. Range of f is singleton set

D. sgn (f(x)) =1 , where sgn x denotes signum function of x.

Answer: A::D

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15. Which of the following pairs of function are identical?

A. $f(x) = \sin(\tan^{-1} x), g(x) = \frac{x}{\sqrt{1+x^2}}$

B. $f(x) = sgn(\cot^{-1} x), g(x) = \sec^2 x - \tan^2 x$,where sgn x denotes signum function of x.

C. $f(x) = e^{\text{In}\left(\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)\right)}, g(x) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

D. $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right), g(x) = 2\tan^{-1} x$

Answer: A::C

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16. The value of $\Sigma_{n=1}^{\infty} \cot^{-1}(n^2+n+1)$ is also equal to

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17. Let $f: I - \{-1, 0, 1\} \to [-\pi, \pi]$ be defined as $f(x) = 2\tan^{-1}x - \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then which of the following statements (s) is (are) correct ?

A. f(x) is bijective

B. f(x) is injective but not surjective

C. f(x) is neither injective nor surjective

D. f(x) is an odd function

Answer: C::D

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18. If $\log x = \frac{-1}{3}$, $\log y = \frac{2}{5}$ and $P = \log\left(\sin\left(arc\cos\sqrt{1-x^2}\right)\right)$ $Q = \log\left(\cos\left(arc\tan\left(\frac{\sqrt{1-x^2y^2}}{xy}\right)\right)\right)$, then

A. a. $P = \frac{-1}{9}$

B. b. $P + Q = \frac{-4}{15}$

C. c. $P - Q = \frac{-2}{5}$

D. d. $\frac{P}{Q} = -5$

Answer: B::C::D

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Exercise Statement I And Ii Type Questions

1. Let S denotes the set consisting of four functions and $S = \{[x], \sin^{-1} x, |x|, \{x\}\}$ where , $\{x\}$ denotes fractional part and [x] denotes greatest integer function , Let A, B , C are subsets of S.

Suppose

A : consists of odd functions (s)

B : consists of discontinuous function (s)

and C: consists of non-decreasing function(s) or increasing function (s).

If $f(x) \in A \cap C, g(x) \in B \cap C, h(x) \in B$ but not C and $l(x) \in$ neither A nor B nor C .

Then, answer the following.

The function f (x) is

A. periodic

B. even

C. odd

D. neither odd nor even

Answer: B

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2. Let S denotes the set consisting of four functions and $S = \{[x], \sin^{-1} x, |x|, \{x\}\}$ where , $\{x\}$ denotes fractional part and [x] denotes greatest integer function , Let A, B , C are subsets of S.

Suppose

A : consists of odd functions (s)

B : consists of discontinuous function (s)

and C: consists of non-decreasing function(s) or increasing function (s).

If $f(x) \in A \cap C, g(x) \in B \cap C, h(x) \in B$ but not C and $l(x) \in$ neither A nor B nor C .

Then, answer the following.

The function f (x) is

A. $\{-1, 0, 1\}$

B. $\{-1, 0\}$

C. $\{0, 1\}$

D. $\{-2, -1, 0, 1\}$

Answer: D

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3. Let S denotes the set consisting of four functions and $S=\{[x], \sin^{-1}x, |x|, \{x\}\}$ where , $\{x\}$ denotes fractional part and [x] denotes greatest integer function , Let A, B , C are subsets of S.

Suppose

A : consists of odd functions (s)

B : consists of discontinuous function (s)

and C: consists of non-decreasing function(s) or increasing function (s).

If $f(x) \in A \cap C, g(x) \in B \cap C, h(x) \in B \text{ but not C and } l(x) \in$ neither A nor B nor C .

Then, answer the following.

The range of $f(h(x))$ is

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left[0, \frac{\pi}{2}\right)$

C. $\left(0, \frac{\pi}{2}\right]$

D. $\left[0, \frac{\pi}{2}\right]$

Answer: B

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4. Let f be a real - valued function defined on R (the set of real numbers)

such that $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$

The value of f(10) is equal to

A. $6\pi - 20$

B. $7\pi - 20$

C. $20 - 7\pi$

D. $20 - 6\pi$

Answer: B

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5. Let f be a real - valued function defined on R (the set of real numbers)

such that $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$

The area bounded by curve $y = f(x)$ and x- axis from $\frac{\pi}{2} \leq x \leq \pi$ is equal to

A. $\frac{\pi^2}{4}$

B. $\frac{\pi^2}{2}$

C. π^2

D. $\frac{\pi^2}{8}$

Answer: B

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6. Let f be a real - valued function defined on R (the set of real numbers)

such that $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$

Number of values of x in interval (0, 3) so that f(x) is an integer, is equal to

A. 1

B. 2

C. 3

D. 0

Answer: C

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7. Consider a real-valued function $f(x) = \sqrt{\sin^{-1} x + 2} + \sqrt{1 - \sin^{-1} x}$ then The domain of definition of $f(x)$ is

A. $[-1, 1]$

B. $[\sin 1, 1]$

C. $[-1, \sin 1]$

D. $[-1, 0]$

Answer: C

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8. Consider a real - valued function

$$f(x) = \sqrt{\sin^{-1} x + 2} + \sqrt{1 - \sin^{-1} x}$$

The range of f (x) is

A. $[0, \sqrt{3}]$

B. $[1, \sqrt{2}]$

C. $[1, \sqrt{6}]$

D. $[\sqrt{3}, \sqrt{6}]$

Answer: D

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9. Given that ,

$$\tan^{-1}\left(\frac{2x}{1 - x^2}\right) = \begin{cases} 2\tan^{-1} x, |x| \leq 1 \\ -\pi + 2\tan^{-1} x, x > 1 \\ \pi + 2\tan^{-1} x, x < -1 \end{cases}$$

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x, |x| \le 1 \\ \pi - 2\tan^{-1}x, x > 1 \text{ and} \\ -(\pi + 2\tan^{-1}), x < -1 \end{cases}$$

$$\sin^{-1}x + \cos^{-1}x = \pi/2 \quad \text{for} \quad -1 \le x \le 1$$

$\sin^{-1}\left(\frac{4x}{x^2+4}\right) + 2\tan^{-1}\left(-\frac{x}{2}\right)$ is independent of x, then

A. $x \in [-3, 4]$

B. $x \in [-2, 2]$

C. $x \in [-1, 1]$

D. $x \in [1, \infty)$

Answer: B

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10. If $\cos^{-1}. \frac{6x}{1+9x^2} = -\frac{\pi}{2} + 2\tan^{-1}3x$, then find the values of x

A. $\left(\frac{1}{3}, \infty\right)$

B. $(-1, \infty)$

C. $(-\infty, m-1)$

D. None of these

Answer: A

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11. If $(x-1)(x^2+1) > 0,$ then find the value of $\sin\left(\frac{1}{2}\tan^{-1}. \frac{2x}{1-x^2} - \tan^{-1}x\right)$

A. 1

B. $\frac{1}{\sqrt{2}}$

C. -1

D. None of these

Answer: C

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12. For $x, y, z, t \in R$, $\sin^{-1} x + \cos^{-1} y + \sec^{-1} z \geq t^2 - \sqrt{2\pi t} + 3\pi$

The value of $x + y + z$ is equal to

A. 1

B. 0

C. 2

D. -1

Answer: D

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13. For $x, y, z, t \in R$, $\sin^{-1} x + \cos^{-1} y + \sec^{-1} z \geq t^2 - \sqrt{2\pi t} + 3\pi$

The principal value of $\cos^{-1}(\cos 5t^2)$ is

A. $\frac{3\pi}{2}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer: B

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14. For $x, y, z, t \in R, \sin^{-1} x + \cos^{-1} y + \sec^{-1} z \geq t^2 - \sqrt{2\pi t} + 3\pi$

The value of $\cos^{-1}(\min\{x, y, z\})$ is

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{\pi}{3}$

Answer: C

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Exercise Passage Based Questions

1. Let $f(x) = \tan^{-1}\left(\frac{(x-2)}{x^2+2x+2}\right)$,then $26f'(1)$ is

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2. Let $f(x) = (arc\tan x)^3 + (arc\cot x)^3$.If the range of f (x) is $[a, b)$, then find the value of $\frac{b}{7a}$.

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3. If $\sum_{n=0}^{\infty} 2\cot^{-1}\left(\frac{n^2+n+4}{2}\right) = k\pi$, then find the value of k.

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4. Find the number of solution of the equation

$\tan\left(\Sigma_{r=1}^{5}\cot^{-1}(2r^2)\right) = \frac{5x+6}{6x+5}$.

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5.

$$\lim_{z \to 0}\left[\left\{\max\left(\sin^{-1}x+\cos^{-1}x\right)^2,\ \min\left(x^2+4x+7\right)\right\}.\ \frac{\sin^{-1}z}{z}\right]$$

is equal to (where [.] denotes greatest integer function)

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6. If $\sin(30^\circ + \text{arc} \ \tan x) = \frac{13}{14}$ and $0 < x < 1$, the value of x is $\frac{a\sqrt{3}}{b}$, where a and b are positive integers with no common factors . Find the value of $\left(\frac{a+b}{2}\right)$.

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7. Let $f: R \to \left(0, \frac{2\pi}{3}\right]$ defined as $f(x) = \cot^{-1}(x^2 - 4x + \alpha)$ Then the smallest integral value of α such that, $f(x)$ is into function is

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8. Let L denotes the number of subjective functions $f : A \to B$, where set A contains 4 elementset B contains 3 elements. M denotes number of elements in the range of the function $f(x) = \sec^{-1}(sgmx) + \cos ec^{-1}(sgnx)$ where $sgnx$ denotes signum function of x. And N denotes coefficient of t^5 in $(1 + t^2)^5(1 + t^3)^8$. The value of $(LM + N)$ is λ, then the value of $\frac{\lambda}{19}$ is

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9. Number of solution (s) of the equations $\cos^{-1}(\cos x) = x^2$ is

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10. If

$\cos^{-1}(x) + \cos^{-1}(y) + \cos^{-1}(z) = \pi\left(\sec^2(u) + \sec^4(v) + \sec^6(w)\right), wher$

are least non-negative angles such that `u

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11. Let $f(x) = \cos\left(\tan^{-1}\left(\sin\left(\cot^{-1}x\right)\right)\right)$. The simplest form of f(x) can be written as $\left(\dfrac{x^2 + A}{x^2 + B}\right)^{1/2}$. Then the value of $(A + B)$ is ....

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12. Prove that : $\tan^{-1}\left\{\dfrac{\cos 2\alpha \sec 2\beta + \cos 2\beta \sec 2\alpha}{2}\right\} = \tan^{-1}\{tan^2(\alpha + \beta)tan^2(\alpha - \beta$

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13. The least natural number 'n' for which $(n - 2)x^2 + 8x + n + 4 > \sin^{-1}(\sin 12) + \cos^{-1}(\cos 12)\ \forall x \in R$ is

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14. The least value of n for which

$(n - 2)x^2 + 8x + n + 4 > \sin^{-1}(\sin 12) + \cos^{-1})(\cos 12),\ \forall x \in R$, where

, is ....

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15. The number of real solution of the equation $\sqrt{1+\cos 2x} = \sqrt{2}\sin^{-1}(\sin x), \ -\pi \le x \le \pi$, is

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Exercise Matching Type Questions

1. Statement I $y = \tan^{-1}(\tan x)$ and $y = \cos^{-1}(\cos x)$ does not have nay solution , if

Statement II $y = \tan^{-1}(\tan x) = x - \pi, x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ and $y = \cos^{-1}(\cos x) = \Big\{ 2$

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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2. Statement I $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$

Statement II $\sin^{-1}x > \tan^{-1}y \text{ for } x > y, \forall x, y \in (0, 1)$

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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3. Statement I $\cos ec^{-1}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right) > \sec^{-1}\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$

Statement II $\cos ec^{-1} x > \sec^{-1} x, \text{ if } \ 1 \leq x < \sqrt{2}$

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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4. Let $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ Statement I $f'(2)=-\frac{2}{5}$ and Statement II $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\pi-2\tan^{-1}x,\ \forall x>1$

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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5. Statement I $\sin^{-1}2x+\sin^{-1}3x=\frac{\pi}{3}$

$\Rightarrow x=\sqrt{\frac{3}{76}}$ only.

and

Statement II Sum of twp negative angles cannot be positive.

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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6. Statement I Number of roots of the equation $\cot^{-1}x\cos^{-1}2x+\pi=0$ is zero.

Statement II Range of $\cot^{-1}x$ and $\cos^{-1}x$ is $(0,\pi)$ and $[0,\pi]$, respectively.

A. Statement I is True, Statement II is True, Statement II is a correct explanation for statement I

B. Statement I is True, Statement II is True, Statement II is NOT a correct explanation for Statement I

C. Statement I is True, Statement II is False

D. Statement I is False, Statement II is True.

Answer: A

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Exercise 6

1. Let

$t_1 = \left(\sin^{-1} x\right)^{\sin^{-1} x}, t_2 = \left(\sin^{-1} x\right)^{\cos^{-1} x}, t_3 = \left(\cos^{-1} x\right)^{\sin^{-1} x}, t_4 = \left(\cos\right.$

,

Match the follwing items of Column I with Column II

Column I	Column II
A. $x \in (0, \cos 1)$	(p) $t_1 > t_2 > t_4 > t_3$
B. $x \in \left(\cos 1, \frac{1}{\sqrt{2}}\right)$	(q) $t_4 > t_3 > t_1 > t_2$
C. $x \in \left(\frac{1}{\sqrt{2}}, \sin 1\right)$	(r) $t_2 > t_1 > t_4 > t_3$
D. $x \in (\sin 1, 1)$	(s) $t_3 > t_4 > t_1 > t_2$

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Exercise Subjective Type Questions

1. If $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$, then x is equal to $\frac{7}{13}$ (b) $\frac{4}{3}$ (c) 13 (d) $\frac{13}{7}$

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2. Solve the equation $\frac{\tan^{-1}(x+1)}{x=1} + \frac{\tan^{-1}(x-1)}{x} = \tan^{-1}(-7)$

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3. Let a, b and c be positive real numbers. Then prove that `tan^(-1) sqrt((a(a + b + c))/(bc)) + tan^(-1) sqrt((b (a + b + c))/(ca)) + tan^(-1) sqrt((c(a + b+ c))/(ab)) =' pi'

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4. Find the sum $\cos ec^{-1}\sqrt{10} + \cos ec^{-1}\sqrt{50} + \cos ec^{-1}\sqrt{170} + \dots. + \cos ec^{-1}\sqrt{(n^2+1)(}$

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5. If $\sin^{-1} x_i \in [0, 1]\,\forall i = 1, 2, 3, .28$ then find the maximum value of $\sqrt{\sin^{-1} x_1}\sqrt{\cos^{-1} x_2} + \sqrt{\sin^{-1} x_2}\sqrt{\cos^{-1} x_3} + \sqrt{\sin^{-1} x_3}\sqrt{\cos^{-1} x_4} + \ + \sqrt{\sin^{-1} x_{28}}\sqrt{\cos^{-1} x_1}$

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6. Find the value of $\sum_{r=1}^{10}\sum_{s=1}^{10}\tan^{-1}\left(\frac{r}{s}\right)$.

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7. Find the value

$$\lim_{n\to\infty}\sum_{k=2}^{n}\cos^{-1}\left(\frac{1+\sqrt{(k-1)k(k+1)(k+2)}}{k(k+1)}\right)$$

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8. If $m\frac{\tan(\alpha-\theta)}{\cos^2\theta}=n\frac{\tan\theta}{\cos^2(\alpha-\theta)}$ the prove that $2\theta=\alpha-\left[\tan^{-1}\left(\frac{n-m}{n+m}\right)\tan\alpha\right]$

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9. If the quadratic equation ,

$4^{\sec^2\alpha}x^2+2x+\left(\beta^2-\beta+\frac{1}{2}\right)=0$ have real roots, then find all the possible value of $\cos\alpha+\cos^{-1}\beta$.

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Exercise 7

1. Prove that : $\tan^{-1}\left(e^{i\theta}\right)=\frac{n\pi}{2}+\frac{\pi}{4}+\frac{i}{2}\ln\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$,where n is an integer.

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Exercise Questions Asked In Previous 13 Years Exam

1. If $\alpha=3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta=3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct options (s) is (are)

A. $\cos\beta>0$

B. $\sin\beta<0$

C. $\cos(\alpha+\beta) > 0$

D. $\cos\alpha < 0$

Answer: B::C::D

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2. about to only mathematics

A. $\frac{x}{\sqrt{1+x^2}}$

B. x

C. $x\sqrt{1+x^2}$

D. $\sqrt{1+x^2}$

Answer: C

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3. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is

A. $\frac{3x - x^3}{1 - 3x^2}$

B. $\frac{3x + x^3}{1 - 3x^2}$

C. $\frac{3x - x^3}{1 + 3x^2}$

D. $\frac{3x + x^3}{1 + 3x^2}$

Answer: A

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4. The value of $\cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$ is

A. $\frac{23}{25}$

B. $\frac{25}{23}$

C. $\frac{23}{24}$

D. $\frac{24}{23}$

Answer: B

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5. If x, y, z are in A.P. and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in A.P. then show that x=y=z and y≠0

A. $x = y = z$

B. $2x = 3y = 6z$

C. $6x = 3y = 2z$

D. $6x = 4y = 3z$

Answer: A

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6. The value of $\sec\left(2\cot^{-1}2+\cos^{-1}.\frac{3}{5}\right)$ is equal to

A. $\frac{5}{17}$

B. $\frac{6}{17}$

C. $\frac{3}{17}$

D. $\frac{4}{17}$

Answer: B

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7. If $\sin^{-1}\left(\frac{x}{5}\right)+\cos ec^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then the value of x is

A. 1

B. 3

C. 4

D. 5

Answer: B

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8. If $\cos^{-1}x-\frac{\cos^{-1}y}{2}=\alpha, then 4x^2-4xy\cos\alpha+y^2$ is equal to 4 (b) $2\sin^2\alpha$ (c) $-4\sin^2\alpha$ (d) $4\sin^2\alpha$

A. $-4\sin^2\alpha$

B. $4\sin^2\alpha$

C. 4

D. $2\sin 2\alpha$

Answer: B

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