MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

LIMITS

Examples

1. $\lim\limits_{x \to 2} \dfrac{x^6 - 24x - 16}{x^3 + 2x - 12}$

▶ **Watch Video Solution**

2. Evaluate $\lim\limits_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$.

▶ **Watch Video Solution**

3. Evaluate $\displaystyle\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$

> ▶ Watch Video Solution

4. Evaluate $\displaystyle\lim_{x \to 1} \frac{x^2 + x \log_e x - \log_e x - 1}{(x^2) - 1}$

> ▶ Watch Video Solution

5. Evaluate $\displaystyle\lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$

> ▶ Watch Video Solution

6. The value of $\displaystyle\lim_{x \to 2a} \frac{\sqrt{x - 2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ is

> ▶ Watch Video Solution

7. Evaluate $\lim\limits_{x \to 2} \dfrac{x^3 - 2^3}{x - 2}$

8. Evaluate $\lim\limits_{x \to 1} \dfrac{x + x^2 + \ldots\ldots + x^n - n}{x - 1}$

9. The value of $\lim\limits_{x \to 1} \dfrac{y^3}{x^3 - y^2 - 1}$ as $(x, y) \to (1, 0)$ along the line $y = x - 1$ is

A. 1

B. -1

C. 0

D. Doesn't exist

Answer:

10. Evaluate $\lim\limits_{x \to \infty} \dfrac{x^2 + 5}{x^2 + 4x + 3}$

11. Evaluate $\lim\limits_{n \to \infty} \dfrac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$

12. $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$

13. Evaluate : $\lim\limits_{x \to \infty} \sqrt{x^2 + x + 1} - \sqrt{x^2 + 1}$

14. Evaluate $\lim\limits_{x \to \infty} \dfrac{ax^2 + b}{x + 1}$ when $a \geq 0$

▶ Watch Video Solution

15. If $\lim\limits_{x \to \infty} \left(\dfrac{x^2 + 1}{x + 1} - ax - b \right) = 0$, find the values of a and b.

▶ Watch Video Solution

16. If $\lim\limits_{x \to \infty} \left\{ \dfrac{x^2 + 1}{x + 1} - ax - b \right\} = 2$, then

▶ Watch Video Solution

17. If $\lim\limits_{x \to \infty} \left(\dfrac{x^2 + 1}{x + 1} - ax - b \right) = 0$, find the values of a and b.

▶ Watch Video Solution

18. Let $S_n = 1 + 2 + 3 + + n$ and

$$P_n = \frac{S_2}{S_2 - 1} \frac{\dot{S}_3}{S_3 - 1} \frac{\dot{S}_4}{S_4 - 1} \cdots \frac{S_n}{S_n - 1}$$ Where $n \in N, (n \geq 2).$ Then

$(\lim)_{n \to \infty} P_n =$ _ _ _ _

> **▶ Watch Video Solution**

---

19. If $(\lim)_{n \to \infty} \dfrac{1}{\left(\sin^{-1} x\right)^n + 1} = 1, then f \in d the value of x.$

A. $(- \sin 1, \sin 1)$

B. $(- 1, 1)$

C. $(0, 1)$

D. $(- 1, 0)$

Answer: A::C

> **▶ Watch Video Solution**

20. Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

▶ Watch Video Solution

21. Solve $\lim\limits_{x \to 0} \dfrac{1 - \cos(1 - \cos x)}{\sin^4 x}$

▶ Watch Video Solution

22. Evaluate $\lim\limits_{x \to \infty} \left(2^{-x} \sin(2^x)\right)$

▶ Watch Video Solution

23. Evaluate $\lim\limits_{x \to \infty} e^x \sin(d / e^x)$.

▶ Watch Video Solution

24. Evaluate $\displaystyle \lim_{x \to \infty} \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$

> ▶ Watch Video Solution

25. the value of $\displaystyle \lim_{x \to y} \dfrac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ equals

> ▶ Watch Video Solution

26. $\displaystyle (\lim)_{x \to \infty} \left\{ (x + 5) \tan^{-1}(x + 5) - (x + 1)\tan^{-1}(x + 1) \right\}$ is equal to π (b) 2π (c) $\dfrac{\pi}{2}$ (d) none of these

A. π

B. 2π

C. $\dfrac{\pi}{2}$

D. None of these

Answer: B

27. Evaluate $\lim\limits_{x \to 0} \dfrac{\sin(\pi \cos^2 x)}{x^2}$

28. Let $a = \min\left\{x^2 + 2x + 3, x \in R\right\}$ and $b = \lim\limits_{\theta \to 0} \dfrac{1 - \cos\theta}{\theta^2}$ then the value of $\sum\limits_{r=0}^{n} a^r \cdot b^{n-r}$ is :

29. $\lim\limits_{n \to \infty} \cos\dfrac{x}{2} \cdot \cos\dfrac{x}{4} \cdot \cos\dfrac{x}{8}......\cos\dfrac{x}{2^n}$

30. Evaluate $\lim\limits_{x \to a} \dfrac{\log\{1 + (x - a)\}}{(x - a)}$

31. Evaluate $\lim\limits_{h \to 0} \dfrac{\log_{10}(1 + h)}{h}$

32. Evaluate $\lim\limits_{x \to 0} \dfrac{\log(5 + x) - \log(5 - x)}{x}$.

33. $(\lim)_{x \to 0} \dfrac{1n(1 + 2h) - 21n(1 + h)}{h^2} = \_ \_$

34. The value of $\lim\limits_{x \to \infty} \left(x - x^2 \log_e \left(1 + \dfrac{1}{x} \right) \right)$ is _____.

35. Evaluate $\lim\limits_{x \to 0} \dfrac{a^x - b^x}{x}$

36. Evaluate $\lim\limits_{x \to 0} \dfrac{(ab)^x - a^x - b^x + 1}{x^2}$

37. $\lim\limits_{x \to 0} \dfrac{e^{\tan x} - e^x}{\tan x - x} =$

38. Evaluate $\lim\limits_{x \to 0} \dfrac{ae^x - b}{x} = 2$. Find a and b

39. Find the constant a, b and c such that

$$\lim_{x \to 0} \frac{axe^x - b\log(1+x) + cxe^{-x}}{x^2 \sin x} = 2$$

40. Evaluate $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$

41. $\lim_{x \to 1} (\log_3 3x)^{\log_x 3} =$

42. Evaluate $\lim_{x \to a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$

43. Evaluate the following limits :

$$\underset{x \to \infty}{\text{Lim}} \left(\frac{x + 6}{x + 1} \right)^{x+4}$$

▶ Watch Video Solution

44. Evaluate $\underset{x \to 0}{\lim} \left(1 + \tan^2 \sqrt{x} \right)^{\frac{1}{2x}}$

▶ Watch Video Solution

45. Evaluate $\underset{x \to 0}{\lim} \left\{ \tan\left(\frac{\pi}{4} + x \right) \right\}^{\frac{1}{x}}$

▶ Watch Video Solution

46. Evaluate $\underset{x \to 0}{\lim} \left(\frac{1 + 5x^2}{1 + 3x^2} \right)^{1/x^2}$

▶ Watch Video Solution

47. Evaluate $\lim\limits_{x \to \infty} \left(\dfrac{x-3}{x+2} \right)^x$

48. $\lim\limits_{x \to 0} \sin^2 \left(\dfrac{\pi}{2 - px} \right)^{\sec^2 \left(\left(\frac{\pi}{2 - px} \right) \right)}$

A. $e^{-a/b}$

B. a^{-a^2/b^2}

C. $a^{2a/b}$

D. $e^{4a/b}$

Answer: B

49. Evaluate: $(\lim)_{x \frac{7}{2}} \left(2x^2 - 9x + 8 \right)^{\cot(2x-7)}$

A. $e^{5/2}$

B. $e^{-5/2}$

C. $e^{7/2}$

D. $e^{3/2}$

Answer: A::C

> ▶ Watch Video Solution

---

50. The value of $\lim\limits_{x \to 1} \left(\tan\left(\dfrac{\pi}{4} + \log x \right) \right)^{\frac{1}{\log x}}$ is equal to

A. e

B. e^{-1}

C. e^2

D. e^{-2}

Answer: A::C

> ▶ Watch Video Solution

51. $\lim\limits_{x \to 0} \left(\sin \dfrac{x}{m} + \cos \dfrac{3x}{m} \right)^{(2m) / x}$ is

A. e

B. 1

C. e^{-1}

D. e^2

Answer: A::C::D

▶ **Watch Video Solution**

52. The value of $\lim\limits_{n \to \infty} \left(\dfrac{a - 1 + \sqrt[n]{b}}{a} \right)^n$ $(a > 0, b > 0)$ is equal to

A. $\sqrt[a]{b}$

B. $\sqrt[b]{a}$

C. $\sqrt{b}$

D. $\sqrt{a}$

Answer: A::C

> ▶ **Watch Video Solution**

53. Evaluate $\lim\limits_{x \to 0} |x|^{\sin x}$

> ▶ **Watch Video Solution**

54. Evaluate $\lim\limits_{n \to \infty} (\pi n)^{2/n}$

> ▶ **Watch Video Solution**

55. evaluate $\lim\limits_{n \to \infty} \left(\dfrac{e^n}{\pi}\right)^{\frac{1}{n}}$

> ▶ **Watch Video Solution**

56. Evaluate $\lim\limits_{x \to 0} (\cos ecx)^x$.

57. Slove $\lim\limits_{x \to 0} \dfrac{(1+x)^{1/x} - e}{x}$

58. Evaluate the left-and right-hand limits of the function $f(x) = \begin{cases} \dfrac{|x-4|}{x-4}, & x \neq 4, 0, x = 4, \, atx = 4 \end{cases}$

59. Let $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2. \end{cases}$

Find $\lim\limits_{x \to 1} f(x)$.

60. Show that $\lim\limits_{x \to 0} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist.

61. $\lim\limits_{x \to 1} \dfrac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$,

A. (a) exists and equals to $\sqrt{2}$

B. (b) exists and equals to $-\sqrt{2}$

C. (c) does not exist

D. (d) None of these

Answer: Hence $\lim\limits_{x \to 1} f(x)$ **doesn't exist.**

62. Solve (i) $\lim\limits_{x \to 1} \sin x$ (ii) $\lim\limits_{x \to 0^+} \left[\dfrac{\sin x}{x} \right]$

(iii) $\lim\limits_{x \to 0^-} \left[\dfrac{\sin x}{x} \right]$

(where [.] denotes greatest integer function)

63. Solve (i) $\lim\limits_{x \to \infty} \tan^{-1} x$

(ii) $\lim\limits_{x \to -\infty} \tan^{-1} x$

(where [.] denotes greatest integer function)

▶ Watch Video Solution

64. Solve (i) $\lim\limits_{x \to 0^+} \left[\dfrac{\tan x}{x} \right]$

(ii) $\lim\limits_{x \to 0^-} \left[\dfrac{\tan x}{x} \right]$

(where [.] denotes greatest integer function)

▶ Watch Video Solution

65. Solve (i) $\lim\limits_{x \to 1^-} \sin\left(\sin^{-1} x\right)$ (ii) $\lim\limits_{x \to \pi/2} \sin^{-1}(\sin x)$

▶ Watch Video Solution

66. Solve (i) $\lim\limits_{x \to 0} \cot x$

(ii) $\lim\limits_{x \to +\infty} \cot^{-1} x$

(where $[.]$ denotes greatest integer function)

▶ **Watch Video Solution**

---

67. Solve $\lim\limits_{x \to 0} \left[\sin \dfrac{|x|}{x} \right]$, where $e[.]$ denotes greatest integer function.

▶ **Watch Video Solution**

---

68. Solve $\lim\limits_{x \to 0} \left[\sin \dfrac{|x|}{x} \right]$, where $e[.]$ denotes greatest integer function.

▶ **Watch Video Solution**

---

69. $\lim\limits_{x \to 0} \left[\dfrac{-2x}{\tan x} \right]$, where [.] denotes greatest integer function is

A. -1

B. 4

C. 5

D. None of these

Answer: A::C::D

▶ Watch Video Solution

---

70. Evaluate $\lim\limits_{x \to 0} x^3 \cos\dfrac{2}{x}$.

▶ Watch Video Solution

---

71. Evaluate $\lim\limits_{x \to \infty} \dfrac{x^2\left(2 + \sin^2 x\right)}{x + 100}$

▶ Watch Video Solution

---

72. $\lim\limits_{n \to \infty} \left(\dfrac{1}{n^2 + 1} + \dfrac{2}{n^2 + 2} + \dfrac{3}{n^2 + 3} + \ldots \dfrac{n}{n^2 + n}\right)$

73. The value of the $\lim\limits_{x \to 0} \dfrac{x}{a}\left[\dfrac{b}{x}\right](a \neq 0)(where[\cdot]$ denotes the greatest integer function) is

A. a

B. b

C. $\dfrac{b}{a}$

D. $1 - \dfrac{b}{a}$

Answer: A::C

74. If $[x]$ denotes the greatest integer less than or equal to x, then evaluate $\lim\limits_{n \to \infty} \dfrac{1}{n^2}([1.\,x] + [2.\,x] + [3.\,x] + \ldots + [n.\,x])$.

75. $\lim\limits_{(x \to 0)} \left(\dfrac{1}{x^5} \displaystyle\int_0^x e^{-t^2} dt - \dfrac{1}{x^4} + \dfrac{1}{3x^2} \right)$ is equal to

76. Evaluate $\lim\limits_{x \to 0} \dfrac{x - \left(\int_0^x \cos t\, dt \right)}{x^3 - 6x}$

77. Evaluate $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \cos^2 t\, dt}{x \sin x}$

78. $\lim\limits_{n \to \infty} \dfrac{1}{n} \displaystyle\sum_{r=1}^{2n} \dfrac{r}{\sqrt{n^2 + r^2}}$ equals

79. The value of

$$\lim_{n \to \infty} \left[\frac{1}{n^2 + 1^2} + \frac{2}{n^2 + 2^2} + \ldots\ldots\ldots + \frac{n}{n^2 + n^2} \right], \text{is}$$

▶ Watch Video Solution

80. Evaluate: $(\lim)_{n \to \infty} \left[\dfrac{n!}{n^n} \right]^{1/n}$

A. $\dfrac{1}{e}$

B. e

C. e^2

D. $\dfrac{1}{e^2}$

Answer: A::C

▶ Watch Video Solution

81. The value of $\lim_{x \to 0} \dfrac{x \sin(\sin \times) - \sin^2 x}{x^6}$ equals

A. (a) $\dfrac{1}{6}$

B. (b) $\dfrac{1}{12}$

C. (c) $\dfrac{1}{18}$

D. (d) $\dfrac{1}{24}$

Answer: A::B::C

▶ **Watch Video Solution**

82. If $\displaystyle\lim_{x \to 0} \dfrac{\log_e \cot\left(\frac{\pi}{4} - K_1 x\right)}{\tan K_2 x} = 1$, then

A. $K_1 = K_2$

B. $2K_1 = K_2$

C. $K_1 = 2K_2$

D. $K_1 = 4K_2$

Answer: A::B

▶ Watch Video Solution

83. Let

$$f(\theta) = \frac{1}{\tan^2 \theta}\left\{(1 + \tan\theta)^3 + (2 + \tan\theta)^3 + \ldots\ldots + (10 + \tan\theta)^3\right\} - 10\,t$$

Then, $\lim\limits_{\theta \to \frac{\pi}{2}} f(\theta)$ is equal to

A. 170

B. 166

C. 165

D. None of these

Answer: A

84. For positive integers k=1,2,3,....n,, let S_k denotes the area of $\triangle AOB_k$

such that $\angle AOB_k = \dfrac{k\pi}{2n}$, OA=1 and $OB_k = k$ The value of the

$\lim\limits_{n \to \infty} \dfrac{1}{n^2} \sum\limits_{k-1}^{n} S_k$ is

A. $\dfrac{2}{\pi^2}$

B. $\dfrac{4}{\pi^2}$

C. $\dfrac{8}{\pi^2}$

D. $\dfrac{1}{(2\pi)^2}$

Answer: B

> ▶ Watch Video Solution

85. Evaluate: $\displaystyle(\lim)_{x\to 2}\dfrac{\sqrt{(x+7)}-3\sqrt{(2x-3)}}{(x+6)^{\frac{1}{3}}-2(3x-5)^{\frac{1}{3}}}$

A. $\dfrac{33}{23}$

B. $\dfrac{34}{23}$

C. $\dfrac{54}{25}$

D. None of these

Answer: B::C::D

▶ Watch Video Solution

86.   Let

$g(x)|f(x+c)f(x+2c)f(x+3c)f(c)f(2c)f(3c)f'(c)f'(2c)f'(3c)|$,

where c is constant, then find $\left(\lim \right)_{x \to 0} \dfrac{g(x)}{x}$

A. 0

B. 1

C. 2

D. 3

Answer: A::C::D

87. Let $\lim\limits_{x \to 1} \dfrac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. The value of $f(101)$ equals

A. 5050

B. 5151

C. 4950

D. 101

Answer: A::B

---

88. $\displaystyle \lim_{x \to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}$, $wheren = 100$, is equal to :

$\dfrac{5050}{\pi e}$ (b) $\dfrac{100}{\pi e}$ (c) $-\dfrac{5050}{\pi e}$ (d) $-\dfrac{4950}{\pi e}$

A. $\dfrac{5050}{\pi e}$

B. $\dfrac{100}{\pi e}$

C. $-\dfrac{5050}{\pi e}$

D. $-\dfrac{4950}{\pi e}$

Answer: A

89. The value of $\lim\limits_{n \to \infty} \dfrac{1.\, n + 2.\,(n-1) + 3.\,(n-2) + \ldots + n.1}{1^2 + 2^2 + \ldots + n^2}$ is

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. $\dfrac{1}{2}$

D. $\dfrac{1}{6}$

Answer: A

▶ Watch Video Solution

90. $\lim\limits_{x \to \infty} \dfrac{\cot^{-1}\left(x^{-a} \log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}$, $(a > 1)$, is equal to

A. 1

B. 0

C. $\dfrac{\pi}{2}$

D. Doesn't exist

Answer: A::B

> ▶ Watch Video Solution

91. Find slope of tangent if equation of the curve is $y = 3x^4 - 4x$ at $x = 4$

> ▶ Watch Video Solution

92. $\lim\limits_{x \to \infty} \dfrac{\cot^{-1}\left(\sqrt{x+1} + \sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}} =$

A. 1

B. 0

C. $\dfrac{\pi}{2}$

D. None of these

Answer: A

---

93. Evaluate $\lim\limits_{n \to \infty} \cos\left(\pi\sqrt{n^2 + n}\right)$ when n is an integer.

A. 1

B. -1

C. 0

D. Doesn't exist

Answer:

---

94. The value of $\lim\limits_{x \to 0} \dfrac{(\tan(\{x\} - 1))\sin\{x\}}{\{x\}(\{x\} - 1)}$ is where $\{x\}$ denotes the fractional part function

A. 1

B. $\tan 1$

C. $\sin 1$

D. Doesn't exist

Answer: C::D

▶ Watch Video Solution

---

95. $\lim\limits_{x \to 0^-} \left(- In(\{x\} + |[x]|) \right)^{\{x\}}$ is equal to

A. 0

B. 1

C. In2

D. $In\dfrac{1}{2}$

Answer: A::B

▶ Watch Video Solution

96. $\lim\limits_{x \to \infty} \dfrac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$ is equal to

A. 0

B.

C. -1

D. Non- existent

Answer: A

▶ Watch Video Solution

97. The value of $\lim\limits_{x \to 0} (\cos ax)^{\cos ec^2 bx}$ is

A. a) $e^{\left(-\frac{8b^2}{a^2}\right)}$

B. b) $e^{\left(-\frac{8a^2}{b^2}\right)}$

C. c) $e^{-\frac{a^2}{2b^2}}$

D. d) $e^{-\frac{b^2}{2a^2}}$

Answer: C

> ▶ **Watch Video Solution**

98. $\displaystyle\lim_{n \to \infty} \sum_{r=1}^{n} \frac{r}{n^2 + n + 4}$ equals

A. 0

B. $1/3$

C. $1/2$

D. 1

Answer: C

> ▶ **Watch Video Solution**

99. The value of $\lim\limits_{n \to \infty} \left(\sqrt{n^2 + n + 1} - \left[\sqrt{n^2 + n + 1} \right] \right)$ where [.]

denotes the greatest integer function is

A. 0

B. $1/2$

C. $2/3$

D. $1/4$

Answer: A::B

▶ **Watch Video Solution**

100. $\lim\limits_{x \to 1} \dfrac{\sin^2 \left(x^3 + x^2 + x - 3 \right)}{1 - \cos(x^2 - 4x + 3)}$ has the value equal to

A. 18

B. $9/2$

C. 9

D. None of these

Answer: A

---

101. The graph of the function $y = f(x)$ has a unique tangent at the point $(e^a, 0)$ through which the graph passes then $\underline{\lim} \ (x \to e^a) \dfrac{\log_e\{1 + 7f(x)\} - \sin f(x)}{3f(x)}$

A. 1

B. 2

C. 7

D. None of these

Answer: B

102. If $f(x) = \dfrac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct?

A. $\lim\limits_{x \to 1} f(x)$ exists $\Rightarrow a = -2$

B. $\lim\limits_{x \to -2} f(x)$ exists $\Rightarrow a = 13$

C. $\lim\limits_{x \to 1} f(x) = 4/3$

D. $\lim P(x \to -2) f(x) = -1/3$

Answer: A::C

▶ **Watch Video Solution**

103. Let $f(x) = \begin{cases} \dfrac{\tan^2[x]}{x^2 - [x]^2} & \text{for} \quad x > 0 \\ 1 & \text{for} \quad x = 0 \\ \dfrac{1}{\sqrt{|x| \cot |x|}} & \text{for} \quad x < 0 \end{cases}$ where $[x]$ and $\{x\}$ denote respectively the greatest integer less than equal to x and the fractional part of x, then

A. (a) $\lim\limits_{x \to 0^+} f(x) = 1$

B. (b) $\lim\limits_{x \to 0^-} f(x) = 1$

C. (c)$\cot^{-1}\left(\lim\limits_{x \to 0^-} f(x)\right)^2 = 1$

D. (d)None of the above

Answer: A

> ▶ Watch Video Solution

---

104. Given that the derivative $f'(a)$ exists. Indicate which of the following statements(s) is/are always true?

A. $f'(a) = \lim\limits_{h \to a} \dfrac{f(h) - f(a)}{h - a}$

B. $f'(a) = \lim\limits_{h \to 0} \dfrac{f(a) - f(a - h)}{h}$

C. $f'(a) = \lim\limits_{t \to 0} \dfrac{f(a + 2t) - f(a)}{t}$

D. $f'(a) = \lim\limits_{t \to 0} \dfrac{f(a + 2t) - t(a + t)}{2t}$

Answer: A::C::D

> ▶ Watch Video Solution

105. Let $f(x) = \lim\limits_{n \to \infty} \sum\limits_{r=0}^{n-1} \dfrac{x}{(rx + 1)\{(r + 1)x + 1\}}$. Then

A. $f(0) = 0$

B. $f(0) = x$

C. $f(0^+) = 1$

D. $f(0^-) = 1$

Answer: A

▶ Watch Video Solution

106. Find $f'(x)$ if $f(x) = e^{\{x^2\}}$

▶ Watch Video Solution

107. Find $\dfrac{dy}{dx}$ if $y = e^{5x}$

▶ Watch Video Solution

108. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23}\ldots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}\cdot a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that $a_{24} = 1, a_{42} = \dfrac{1}{8}$ and $a_{43} = \dfrac{3}{16}$

Let $S_n = \displaystyle\sum_{j=1}^{n} a_{4j}$, $\displaystyle\lim_{n \to \infty} \dfrac{S_n}{n^2}$ is equal to

A. $\dfrac{1}{4}$

B. $\dfrac{1}{8}$

C. $\dfrac{1}{16}$

D. $\dfrac{1}{32}$

Answer: D

▶ Watch Video Solution

109. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23}\ldots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}. a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical

column is a geometrical progression. It is known that each column in

geometric progression have the same common ratio. Given that

$a_{24} = 1, a_{42} = \dfrac{1}{8}$ and $a_{43} = \dfrac{3}{16}$

Let d_i be the common difference of the elements in with row then $\displaystyle\sum_{i=1}^{n} d_i$

is

A. n

B. $\dfrac{1}{2} - \dfrac{1}{2^{n+1}}$

C. $1 - \dfrac{1}{2^n}$

D. $\dfrac{n+1}{2^n}$

Answer: C

▶ **Watch Video Solution**

110. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23}\ldots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}.\,a_{nn} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical

column is a geometrical progression. It is known that each column in

geometric progression have the same common ratio. Given that

$a_{24} = 1, a_{42} = \dfrac{1}{8}$ and $a_{43} = \dfrac{3}{16}$

The value of $\displaystyle\lim_{n \to \infty} \sum_{i=1}^{n} a_{ii}$ is equal to

A. $\dfrac{1}{4}$

B. $\dfrac{1}{2}$

C. 1

D. 2

Answer: D

▶ **Watch Video Solution**

111. Find $\dfrac{dy}{dx}$ if $y = \dfrac{\sin x}{x}$

112. Statement 1 $\displaystyle\lim_{x \to \pi/2} \dfrac{\sin\left(\cot^2 x\right)}{\left(\pi - 2x\right)^2} = \dfrac{1}{2}$

Statement 2: $\displaystyle\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ and $\displaystyle\lim_{\theta \to 0} \dfrac{\tan\theta}{\theta} = 1$, where θ is measured in radians.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A::B::D

113. Statement 1 $\displaystyle\lim_{m \to \infty} \lim_{n \to \infty} \left\{ \sin^{2m} n!\pi n \right\} = 0 m, n \varepsilon N$,when x is rational.

Statement 2 When $n \to \infty$ and x is rational$n!x$ is integer.

A. (a)Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. (b)Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. (c)Statement 1 is true, Statement 2 is false

D. (d)Statement 1 is false, Statement 2 is true

Answer:

114. Statement 1: If $\displaystyle\lim_{x \to 0} \left\{ f(x) + \frac{\sin x}{x} \right\}$ does not exist then $\displaystyle\lim_{x \to 0} f(x)$ does not exist.

Statement 2: $\lim\limits_{x \to 0} \left(\dfrac{e^{1/x} - 1}{e^{1/x} + 1} \right)$ does not exist.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: B

▶ Watch Video Solution

115. Match the following

	Column I		Column II
(A)	$\lim\limits_{x \to \infty} \left(\dfrac{x}{1+x} \right)^x$ equals	(p)	e^2
(B)	$\lim\limits_{x \to \infty} \left(\sin\dfrac{1}{x} + \cos\dfrac{1}{x} \right)^x$ equals	(q)	$e^{-1/2}$
(C)	$\lim\limits_{x \to 0} (\cos x)^{\cot^2 x}$ equals	(r)	e
(D)	$\lim\limits_{x \to 0} \left[\tan\left(\dfrac{\pi}{4} + x \right) \right]^{1/x}$ equals	(s)	e^{-1}

> ▶ **Watch Video Solution**

116. Match the following

	Column I		Column II
(A)	$\lim\limits_{x \to \infty} (\sqrt{x + \sqrt{x}} - \sqrt{x - \sqrt{x}})$ equals	(p)	-2
(B)	The value of the $\lim\limits_{x \to 0} \dfrac{\sin 2x - 2\tan x}{\ln(1 + x^3)}$ is	(q)	-1
(C)	$\lim\limits_{x \to 0^+} (\ln \sin^3 x - \ln(x^4 + ex^3))$ equals	(r)	0
(D)	Let $\tan(2\pi \lvert \sin\theta \rvert) = \cot(2\pi \lvert \cos\theta \rvert)$, where $\theta \in R$ and $f(x) = (\lvert \sin\theta \rvert + \lvert \cos\theta \rvert)^x$. The value of $\lim\limits_{x \to \infty} \left[\dfrac{2}{f(x)} \right]$ equals (here, $[\cdot]$ represents greatest integer function)	(s)	1

> ▶ **Watch Video Solution**

117. If $(\lim)_{x \to 0} (x^{-3} \sin 3x + ax^{-2} + b)$ exists and is equal to 0, then

$a = -3 and b = \dfrac{9}{2}$ $a = 3 and b = \dfrac{9}{2}$ $a = -3 and b = -\dfrac{9}{2}$

$a = 3 and b = -\dfrac{9}{2}$

▶ Watch Video Solution

---

118. For a certain value of 'c' $\lim_{x \to \infty} \left[(x^5 + 7x^4 + 2)^c - x \right]$ is finite and

non-zero. Then the vlaue oimit is :

▶ Watch Video Solution

---

119. If $\lim_{x \to 0} \displaystyle\int_0^x \dfrac{2t\,dt}{(e^x - 1 - x)\sqrt{\frac{2a}{3} - \frac{t}{2} + 104}} = \dfrac{1}{19}$, then $\dfrac{a}{2010}$

equals.....

▶ Watch Video Solution

120. Evaluate $\lim\limits_{x \to 0^+} \log_{\sin x} \sin 2x$.

▶ Watch Video Solution

121.

$$\lim\limits_{x \to \infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 + cx^2 + 3x - d} \right) = 4$$

▶ Watch Video Solution

122. Let $f(x) = \lim\limits_{m \to \infty} \left\{ \lim\limits_{n \to \infty} \cos^{2m}(n!\pi x) \right\}$, where $x \in R$. Then prove that $f(x) = \{1,$ if x is rational and $0,$ if x is irrational

▶ Watch Video Solution

123. If $a_1 = 1$ and $a_{n+1} = \dfrac{4 + 3a_n}{3 + 2a_n}$, $n \geq 1$, show that $a_{n+2} \geq a_{n+1}$ and if a lim l as $n \to \infty$ the evaluate $\lim\limits_{n \to \infty} a_n$ `

▶ Watch Video Solution

124. Let a_1, a_2, a_n be sequence of real numbers with $a_{n+1} = a_n + \sqrt{1 + a_n^2}$ and $a_0 = 0$. Prove that $\lim\limits_{x \to \infty} \left(\dfrac{a_n}{2^{n-1}} \right) = \dfrac{2}{\pi}$

125. A square is inscribed in a circle of radius R, a circle is inscribed in this square then a square in this circle and so on n times. Find the limit of the sum of areas of all the squares as $n \to \infty$.

Exercise Single Option Correct Type Questions

1. $\lim\limits_{x \to 0} \dfrac{\sin\left(\pi \cos^2(\tan(\sin x))\right)}{x^2}$ is equal to

A. π

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{2}$

D. None of these

Answer: A

---

2. $\displaystyle\lim_{t \to 0} \dfrac{1 - (1 + t)^t}{In(1 + t) - t}$ is equal to

A. $\dfrac{1}{2}$

B. $-\dfrac{1}{2}$

C. 2

D. -2

Answer: C

3.   If $I_1 = \lim\limits_{x \to \infty} \left(\tan^{-1} \pi x - \tan^{-1} x \right) \cos x$ and

$I_2 = \lim\limits_{x \to 0} \left(\tan^{-1} \pi x - \tan^{-1} x \right) \cos x$ then (I_1, I_2) is

A. $(0, 0)$

B. $(0, 1)$

C. $(1, 0)$

D. None of the above

Answer: A

▶ Watch Video Solution

---

4. If $f(x) = 0$ is a quadratic equation such that $f(-\pi) = f(\pi) = 0$ and $f\left(\dfrac{\pi}{2}\right) = -\dfrac{3\pi^2}{4}$, then $\lim\limits_{x \to -\pi} \dfrac{f(x)}{\sin(\sin x)}$ is equal to (a)0 (b) π (c) 2π (d) none of these

A. 0

B. π

C. 2π

D. None of these

Answer: C

> ▶ **Watch Video Solution**

---

5. $\lim\limits_{x \to 0} \dfrac{\sqrt[3]{1 + \sin^2 x} - \sqrt[4]{1 - 2\tan x}}{\sin x + \tan^2 x}$ is equal to:

A. -1

B. 1

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{2}$

Answer: C

> ▶ **Watch Video Solution**

6. If $x_{n+1} = \sqrt{\dfrac{1 + x_n}{2}}$ and $|x_0| < 1, n \in W$ then $\displaystyle\lim_{n \to \infty} \dfrac{\sqrt{1 - x_0^2}}{x_1 x_2 x_3 ... x_{n+1}}$

=

A. -1

B. 1

C. $\cos^{-1}(x_0 + 1)$

D. $\cos^{-1}(x_0)$

Answer: D

▶ **Watch Video Solution**

7. For $n \in N$, let

$f_n(x) = \tan\dfrac{x}{2}(1 + \sec x)(1 + \sec 2x)(1 + \sec 4x)......(1 + \sec 2^n x)$,

the $\displaystyle\lim_{x \to 0} \dfrac{f_n(x)}{2x}$ is equal to :

A. 0

B. 2^n

C. 2^{n-1}

D. 2^{n+1}

Answer: C

8. Let $f(x)$ be a real valued function defined for all $x \geq 1$, satistying

$f(1) = 1$ and $f'(x) = \dfrac{1}{x^2 + f(x)}$, then $\lim\limits_{x \to \infty} f(x)$

A. (a) doesn't exist

B. exists and less than $\dfrac{\pi}{4}$

C. exists and less than $1 + \dfrac{\pi}{4}$

D. exists and equal to 0

Answer: C

9. The quadratic equation whose roots are the minimum value of $\sin^2 \theta - \sin \theta + \dfrac{1}{2}$ and $\lim\limits_{x \to \infty} \sqrt{(x + 1)(x + 2)} - x$ is

A. (a)$3x^2 - 7x + 3 = 0$

B. (b)$8x^2 - 14x + 3 = 0$

C. (c)$x^2 - 7x + 3 = 0$

D. (d)$2x^2 - 8x + 3 = 0$

Answer: B

▶ **Watch Video Solution**

10. If $x_1 = \sqrt{3}$ and $x_{n+1} = \dfrac{x_n}{1 + \sqrt{1 + x_n^2}}$, $\forall n \in N$ then $\lim\limits_{n \to \infty} 2^n x_n$ is equal to

A. $\dfrac{3}{2\pi}$

B. $\dfrac{2}{3\pi}$

C. $\dfrac{3\pi}{3}$

D. $\dfrac{3\pi}{2}$

Answer: C

> ▶ Watch Video Solution

11. $\displaystyle\lim_{x \to a^-} \dfrac{\sqrt{x-b}-\sqrt{a-b}}{(x^2-a^2)}, (a > b)$ is

A. $\dfrac{1}{4a}$

B. $\dfrac{1}{a\sqrt{a-b}}$

C. $\dfrac{1}{2a\sqrt{a-b}}$

D. $\dfrac{1}{4a\sqrt{a-b}}$

Answer: D

> ▶ Watch Video Solution

12. $\lim\limits_{n \to \infty} (\sin^n 1 + \cos^n 1)^n$ is equal to

13. The value of $\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \dfrac{n-k}{n^2} \cos\left(\dfrac{4k}{n}\right)$ equals to

A. (a) $\dfrac{1}{4}\sin 4 + \dfrac{1}{15}\cos 4 - \dfrac{1}{16}$

B. (b) $\dfrac{1}{4}\sin 4 - \dfrac{1}{16}\cos 4 + \dfrac{1}{16}$

C. (c) $\dfrac{1}{16}(1 - \sin 4)$

D. (d) $\dfrac{1}{16}(1 - \cos 4)$

Answer: D

14. about to only mathematics

A. 6

B. 7

C. 8

D. 9

Answer: A::B::D

15. $\int_{-1}^{2} \left[\dfrac{[x]}{1+x^2} \right] dx$, where [.] denotes the greatest integer function, is equal to

A. 0

B. 1

C. cot 1

D. not defined

Answer: A::B::D

16. If α and β are roots of $x^2 - \left(\sqrt{1 - \cos 2\theta}\right)x + \theta = 0$, where $0 < \theta < \dfrac{\pi}{2}$. Then $\displaystyle\lim_{\theta \to 0^+} \left(\dfrac{1}{\alpha} + \dfrac{1}{\beta}\right)$ is

A. (a) $\dfrac{1}{\sqrt{2}}$

B. (b) $-\sqrt{2}$

C. (c) $\sqrt{2}$

D. (d)None of these

Answer: C

▶ Watch Video Solution

17. If $f(x) = \dfrac{1}{3}\left(f(x+1) + \dfrac{5}{f(x+2)}\right)$ and $f(x) > 0, \forall x \varepsilon R$, then $\displaystyle\lim_{x \to \infty} f(x)$ is

A. 0

B. $\sqrt{\dfrac{2}{5}}$

C. $\sqrt{\dfrac{5}{2}}$

D. ∞

Answer: C

▶ Watch Video Solution

---

18. Let $f:(1,2)\overrightarrow{R}$ satisfies the inequality $\dfrac{\cos(2x-4)-33}{2} < f(x) < \dfrac{x^2|4x-8|}{x-2}\ \forall x \in (1,2).$ Then find $\lim\limits_{x \to 2^-} f(x).$

A. 16

B. -16

C. does'nt exist

D. θ

Answer: B

▶ Watch Video Solution

19. Let $f(x)$ be polynomial of degree 4 with roots 1,2,3,4 and leading coefficient 1 and $g(x)$ be the polynomial of degree 4 with roots $1, \dfrac{1}{2}, \dfrac{1}{3}$ and $\dfrac{1}{4}$ with leading coefficient 1. Then $\displaystyle\lim_{x \to 1} \dfrac{f(x)}{g(x)}$ equals

A. (a) $\dfrac{1}{24}$

B. (b) -24

C. (c) $\dfrac{1}{12}$

D. (d) $-\dfrac{1}{12}$

Answer: B

▶ Watch Video Solution

20. $\displaystyle\lim x \to \dfrac{\pi}{4} \dfrac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$ is equal to

A. $\sqrt{2}$

B. $3\sqrt{5}$

C. $5\sqrt{2}$

D. $-5\sqrt{2}$

Answer: C

21. If $\displaystyle\lim_{n\to\infty} \dfrac{n.3^n}{n(x-2)^n + n.3^{n+1} - 3^n} = \dfrac{1}{3}$, then the range of x is

(where $n \in N$)

A. 3

B. 4

C. 5

D. infinite

Answer: C

22. Let $f(x) = \lim\limits_{n \to \infty} \dfrac{1}{\left(\frac{3}{\pi}\tan^{-1} 2x\right)^{2n} + 5}$. Then the set of values of x

for which $f(x) = 0$ is

A. $|2x| > \sqrt{3}$

B. $|2x| < \sqrt{3}$

C. $|2x| \geq \sqrt{3}$

D. $|2x| \leq \sqrt{3}$

Answer: A::B::D

▶ **Watch Video Solution**

---

23. Find the integral value of n for which

$\lim\limits_{x \to 0} \dfrac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite nonzero number

A. 2

B. 3

C. 4

D. None of these

Answer: C

> ▶ Watch Video Solution

24. Find $\dfrac{dy}{dx}$ if $x^7 - e^x = \sin y$

> ▶ Watch Video Solution

25. $\displaystyle\lim_{x \to \frac{\pi}{2}} \dfrac{\left[\frac{x}{2}\right]}{\log_e (\sin x)}$ ([.] denotes greatest integer function)

A. 0

B. 1

C. -1

D. Doesn't exist

26. Let $a_1 = 1$, $a_n = n(a_{n-1} + 1)$ for $n = 2, 3, \ldots$ where

$$P_n = \left(1 + \frac{1}{a_1}\right)\left(1 + \frac{1}{a_2}\right)\left(1 + \frac{1}{a_3}\right)\ldots\left(1 + \frac{1}{a_n}\right)$$ then

$$\lim_{n \to \infty} P_n =$$

A. e

B. $\dfrac{e}{2}$

C. $2e$

D. $3e$

Answer: A::B::D

Watch Video Solution

27. If $f(x + y) = f(x) + f(y)$ for $x, y \in R$ and $f(1) = 1$, then find the value of $\lim\limits_{x \to 0} \dfrac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^2 \cdot f(\sin x)}$

A. $\log 2$

B. $\log 4$

C. $\log \sqrt{2}$

D. $\log 8$

Answer: C

▶ **Watch Video Solution**

---

28. Evaluate

$\lim\limits_{n \to \infty} n^{-n^2} \left[(n + 2^0)(n + 2^{-1})(n + 2^{-2})...(n + 2^{-n+1})\right]^n.$

A. e

B. e^2

C. e^3

D. e^4

Answer: B

29. If $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]} \end{cases}$, f or $[x] \neq 0, 0, f$ or $[x] = 0, where [x]$ denotes

the greatest integer less than or equal to x, then $(\lim)_{x \to 0} f(x)$ is (a) 1

(b) 0 (c) -1 (d) none of these

A. 1

B. 0

C. -1

D. does

Answer: D

30. Evaluate: $\left(\lim \right)_{x \to 2} \dfrac{x^3 - 6x^2 + 11x - 6}{x^2 - 6x + 8}$

A. 3

B. 2

C. 1

D. 4

Answer: A::B::D

▶ Watch Video Solution

---

31. Let r^{th} term t_r of a series if given by $t_c = \dfrac{r}{1 + r^2 + r^4}$. Then $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{n} t_r$ is equal to :

A. 2

B. $\dfrac{1}{2}$

C. 1

D. $\dfrac{1}{4}$

Answer: B

32. The value of $\displaystyle\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(\dfrac{r^3 - r + \frac{1}{r}}{2}\right)$ is

A. A. π

B. B. $\dfrac{\pi}{4}$

C. C. $\dfrac{\pi}{2}$

D. D. π

Answer: C

33. Let $x\tan\alpha + y\sin\alpha = \alpha$ and $x\alpha\cos ec\alpha + y\cos\alpha = 1$ be two variable straight lines, α being the parameter. Let P be the point of

intersection of the lines. In the limiting position when $a \to 0$, the point

P lies on the line :

A. $(2, \ -1)$

B. $(2, \ 1)$

C. $(-2, \ 1)$

D. $(-2, \ -1)$

Answer: A::B::D

▶ **Watch Video Solution**

---

34. The polynomial of least degree such that

$$\lim_{x \to 0} \left(1 + \frac{x^2 + f(x)}{x^2}\right)^{1/x} = e^2 \text{ is}$$

A. x^2

B. $x^3 + 2x^2$

C. $-x^2 + 2x^3$

D. None of these

Answer: C

▶ **Watch Video Solution**

---

35. If n is a non zero integer and [*] denotes the greatest integer function

then $\lim\limits_{x \to 0} \left[n\dfrac{\sin x}{x} \right] + \lim\limits_{x \to 0} \left[n\dfrac{\tan x}{x} \right]$ equals

A. $n + 1$

B. $2n$

C. $n - 1$

D. $2n - 1$

Answer: D

▶ **Watch Video Solution**

36. The value of $\lim\limits_{x \to a} \left[\sqrt{2-x} + \sqrt{1+x}\right]$, where $a \in \left[0, \dfrac{1}{2}\right]$ and $[.]$

denotes the greatest integer function is:

A. 1

B. 2

C. 3

D. 4

Answer: B

▶ **Watch Video Solution**

---

37. The value of

$$\lim_{x \to 0^+} \frac{-1 + \sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + }}}}{-1 + \sqrt{x^3 + \sqrt{x^3 + \sqrt{x^3 + \ldots \infty}}}}$$

is

A. a) $\dfrac{1}{2}$

B. b) $\dfrac{1}{4}$

C. c) $\dfrac{1}{8}$

D. d) 1

Answer: A::B::D

 Watch Video Solution

38. Evaluate: $\displaystyle\lim_{\theta \to 0} \dfrac{\cos^2\left(1 - \cos^2\left(1 - \cos^2\ldots.\ .\left(1 - \cos^2(\theta)\right)\right)\right)}{\sin\left(\pi \dfrac{\sqrt{\theta+4}-2}{\theta}\right)}$

A. (a) 2

B. (b) $\sqrt{2}$

C. (c) $\dfrac{1}{2}$

D. (d) $\dfrac{1}{\sqrt{2}}$

Answer: B

 Watch Video Solution

39. The value of $\lim\limits_{n \to \infty} a_n$ when $a_{n+1} = \sqrt{2 + a_n}$, $n = 1, 2, 3, \dots$ is

A. 1

B. 2

C. 3

D. 4

Answer: B

▶ **Watch Video Solution**

Exercise More Than One Correct Option Type Questions

1. If $\lim\limits_{x \to \infty} 4x \left(\dfrac{\pi}{4} - \tan^{-1} \left(\dfrac{x+1}{x+2} \right) \right) = y^2 + 4y + 5$ then y can be equal to

A. 1

B. -1

C. -4

D. -3

Answer: B::D

> ▶ Watch Video Solution

2. $\lim\limits_{x \to 0} \dfrac{1 - \cos\left(x^2\right)}{x^3(4^x - 1)}$ is equal to:

A. (a) $\dfrac{1}{2}\ln 2$

B. (b) $\ln 2$

C. (c) $\ln 4$

D. (d) $1 - \dfrac{1}{2}\ln\left(\dfrac{e^2}{4}\right)$

Answer: null

> ▶ Watch Video Solution

3. If $f(x) = e^{[\cot x]}$, where $[y]$ represents the greatest integer less than or equal to y then

A. $\lim\limits_{x \to \frac{\pi}{2}^+} f(x) = 1$

B. $\lim\limits_{x \to \frac{\pi}{2}^+} f(x) = \dfrac{1}{e}$

C. $\lim\limits_{x \to \frac{\pi}{2}^-} f(x) = \dfrac{1}{e}$

D. $\lim\limits_{x \to \frac{\pi}{2}^-} f(x) = 1$

Answer: B::D

▶ **Watch Video Solution**

---

4. $\lim\limits_{x \to 0} \left[m\dfrac{\sin x}{x} \right]$ is equal to (where $m \varepsilon I$ and $[.]$ denotes greatest integer function)

A. m if $m < 0$

B. $m - 1$ if $m > 0$

C. $m - 1$, if $m < 0$

D. m if $m > 0$

Answer: A::B

 Watch Video Solution

5. If $(\lim)_{x \to 0} \left(1 + ax + bx^2\right)^{\frac{2}{x}} = e^3,$ then find the value of a and b.
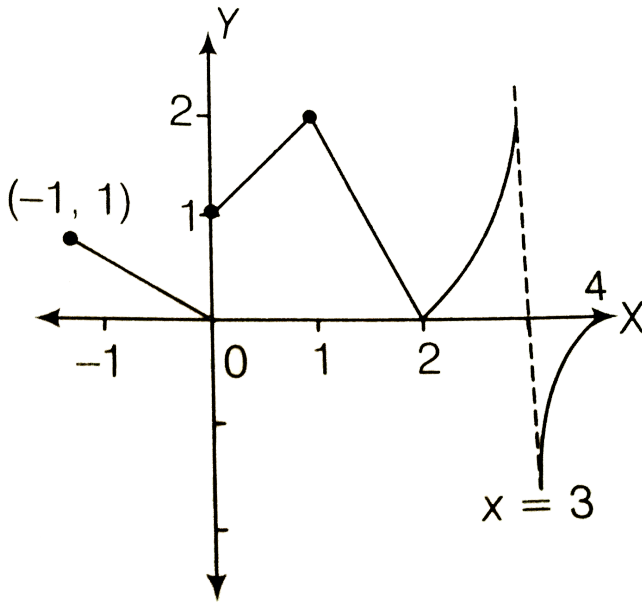
A. $a = 3, b = 0$

B. $a = \dfrac{3}{2}, b = 1$

C. $a = \dfrac{3}{2}, b = 4$

D. $a = 2, b = 3$

Answer: B::C

 Watch Video Solution

6. The graph of the function $y = f(x)$ is shown in the adjacent figure then correct statement is

A. $\lim\limits_{x \to 0^+} f(x) = 1$

B. $\lim\limits_{x \to 1} f(x) = 2$

C. $\lim\limits_{x \to 3} f(x) = $ does not exist

D. $\lim\limits_{x \to 4} f(x) = 0$

Answer: A::B::C::D

▶ Watch Video Solution

7. Find $\lim\limits_{x \to 0} \dfrac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$

8. If $l = \lim\limits_{x \to \infty} \left(\dfrac{x+1}{x-1}\right)^x$, the value of $\{l\}$ and $[l]$ are

A. 7

B. $7 - e^2$

C. -7

D. $e^2 - 7$

Answer: A::D

9. If $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]}, & f \text{ or } [x] \neq 0, 0, f \text{ or } [x] = 0, \end{cases} where [x]$ denotes

the greatest integer less than or equal to x, then $(\lim)_{x \to 0} f(x)$ is (a) 1

(b) 0 (c) -1 (d) none of these

A. $\lim\limits_{x \to 0^-} f(x) = \sin 1$

B. $\lim\limits_{x \to 0^+} f(x) = 0$

C. limt does not exist at $x = 0$

D. limt exists at $x = 0$

Answer: A::B::C::D

▶ **Watch Video Solution**

---

10. $\lim\limits_{x \to c} f(x)$ does not exist when

where $[.\,]$ and $\{.\,\}$ denotes greatest integer and fractional part of x

A. $f(x) = [x] - [2x - 1], c = 3$

B. $f(x) = [x] - x, c = 1$

C. $f(x) = \{x\}^2 - \{-x\}^2$, $c = 0$

D. $f(x) = \dfrac{\tan(sgnx)}{(sgnx)}$, $c = 0$

Answer: B::C

11. Identify the correct statement.

A. $\displaystyle\lim_{x \to \infty} \left[\sum_{r=1}^{n} \dfrac{1}{2^r} \right] = 1$

B. If $f(x) = (x - 1)\{x\}$, where $[.]$ and $\{.\}$ denotes greatest integer

function and fractional part of x respectively, the limit of $f(x)$ does

not exist at $x = 1$

C. $\displaystyle\lim_{x \to 0^+} \left[\dfrac{\tan x}{x} \right] = 1$

D. $\displaystyle\lim_{x \to 0^+} \left[\dfrac{\tan x}{x} \right] = 1$

Answer: C::D

12. For $a > 0$, let $l = \lim\limits_{x \to \frac{\pi}{2}} \dfrac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$ and

$m = \lim\limits_{x \to -\infty} \left(\sqrt{x^2 + ax}\right) - \left(\sqrt{x^2 - ax}\right)$ then solve it

▶ Watch Video Solution

---

13. Consider the function $f(x) = \left(\dfrac{ax + 1}{bx + 2}\right)^x$, where $a, b > 0$, the

$\lim\limits_{x \to \infty} f(x)$ is

    A. exists for all values of a and b

    B. zer for $a < b$

    C. non existent for $a > b$

    D. $e^{-(1/a)}$ or $e^{-(1/b)}$ if $a = b$

Answer: B::C::D

▶ Watch Video Solution

14. If $f(x) = \dfrac{x.2^x - x}{1 - \cos x}$ and $g(x) = 2^x . \sin\left(\dfrac{\log 2}{2^x}\right)$ then

A. $\lim\limits_{x \to 0} f(x) = \log 2$

B. $\lim\limits_{x \to \infty} g(x) = \log 4$

C. $\lim\limits_{x \to 0} f(x) = \log 4$

D. $\lim\limits_{x \to \infty} g(x) = \log 2$

Answer: C::D

> ▶ **Watch Video Solution**

---

15. If $\lim\limits_{x \to 3} \dfrac{x^3 + cx^2 + 5x + 12}{x^2 - 7x + 12} = l$ (finite real number), ten

A. $l = 4$

B. $c = -6$

C. $c = 4$

D. $x \varepsilon R$

Answer: A::B

Exercise Passage Based Questions

1. If $\lim\limits_{x \to a} f(x) = 1$ and $\lim\limits_{x \to a} g(x) = \infty$ then

$$\lim\limits_{x \to a} \{f(x)\}^{g(x)} = e^{\lim\limits_{x \to a} (f(x) - 1) g(x)}$$

$$\lim\limits_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$$ is equal to

A. (a) $1/e$

B. (b) $-1/e$

C. (c) e

D. (d) $-e$

Answer: A

2. If $\lim\limits_{x \to a} f(x) = 1$ and $\lim\limits_{x \to a} g(x) = \infty$ then

$\lim\limits_{x \to a} \{f(x)\}^{g(x)} = e^{\lim\limits_{x \to a} (f(x) - 1)g(x)}$ $\qquad$ $\lim\limits_{x \to 0} \left(\dfrac{x - 1 + \cos x}{x} \right)^{\frac{1}{x}}$ is

equal to

A. $(a)e^{1/2}$

B. $(b)e^{-1/2}$

C. $(c)e^{1}$

D. $(d)\dfrac{1}{e}$

Answer: B

> ▶ Watch Video Solution

3. If $\lim\limits_{x \to a} f(x) = 1$ and $\lim\limits_{x \to a} g(x) = \infty$ then

$\lim\limits_{x \to a} \{f(x)\}^{g(x)} = e^{\lim\limits_{x \to a} (f(x) - 1)g(x)}$ $\qquad$ $\lim\limits_{x \to 0} \left(\dfrac{a^x + b^x + c^x}{3} \right)^{\frac{2}{x}}$ is

equal to

A. $a^{2/3} + b^{2/3} + c^{2/3}$

B. abc

C. $(abc)^{2/3}$

D. 1

Answer: C

▶ Watch Video Solution

---

4. Find $\dfrac{dy}{dx}$ if $2x - 3y = \tan x$

▶ Watch Video Solution

---

5. Find |A| If $A = \begin{vmatrix} x^2 & 2x \\ x^4 & 5x \end{vmatrix}$

▶ Watch Video Solution

6. Let $f(x) = \lim\limits_{n \to \infty} \left(\cos \sqrt{\dfrac{x}{n}} \right)^n$, $g(x) = \lim\limits_{n \to \infty} \left(1 - x + x\sqrt[n]{e} \right)^n$

Now consider the function $y = h(x)$ where $h(x) = \tan^{-1}\left(g^{-1} f^{-1}(x) \right)$.

Range of the function $y = h(x)$ is

A. $\left(0, \dfrac{\pi}{2} \right)$

B. $\left(-\dfrac{\pi}{2}, 0 \right)$

C. R

D. $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2} \right)$

Answer: D

▶ Watch Video Solution

---

7. How many 4 digits numbers can be formed from the digits 1 to 10 if not digit can be repeated.

▶ Watch Video Solution

8. Find $\dfrac{dy}{dx}$ if $2y = e^y - 2x$

9. Let $f: N \to R$ and $g: N \to R$ be two functions and

$f(1) = 08, g(1) = 0.6,$

$f(n+1) = f(n)\cos(g(n)) - g(n)\sin(g(n))$ and $g(n+1) = f(n)\sin(g(n))$

for $n \geq 1.$ $\lim\limits_{n \to \infty} f(n)$ is equal to

    A. (a)— 1

    B. (b)0

    C. (c)1

    D. (d)does not exist

Answer: A

10. Let $f: N \to R$ and $g: N \to R$ be two functions and $f(1) = 0.8, g(1) = 0.6, f(n+1) = f(n)\cos(g(n)) - g(n)\sin(g(n))$ and $g(n+1) = f(n)\sin(g(n)) + g(n)\cos(g(n))$ for $n \geq 1$.

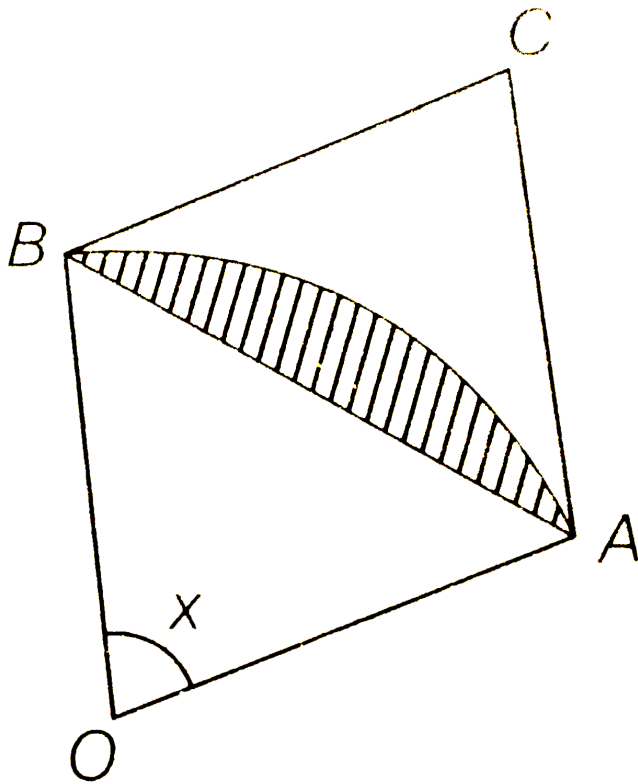$\lim\limits_{n \to \infty} f(n)$ is equal to

A. -1

B. 0

C. 1

D. does not exist

Answer: B

▶ Watch Video Solution

11. A circle arc of radius 1 subtends an angle of x radians as shown in figure. The centre of the circle is O and the point C is the intersection of two tangent lines at A and B. Let $T(x)$ be the area of $\triangle ABC$ and $S(x)$ be the area of shaded region.

$$\lim_{x \to 0} \frac{T(x)}{x^3} \text{ is}$$

A. $\dfrac{1}{2}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{8}$

Answer: D

12. A circle arc of radius 1 subtends an angle of x radians as shown in figure. The centre of the circle is O and the point C is the intersection of two tangent lines at A and B. Let $T(x)$ be the area of $\triangle ABC$ and $S(x)$ be the area of shaded region.
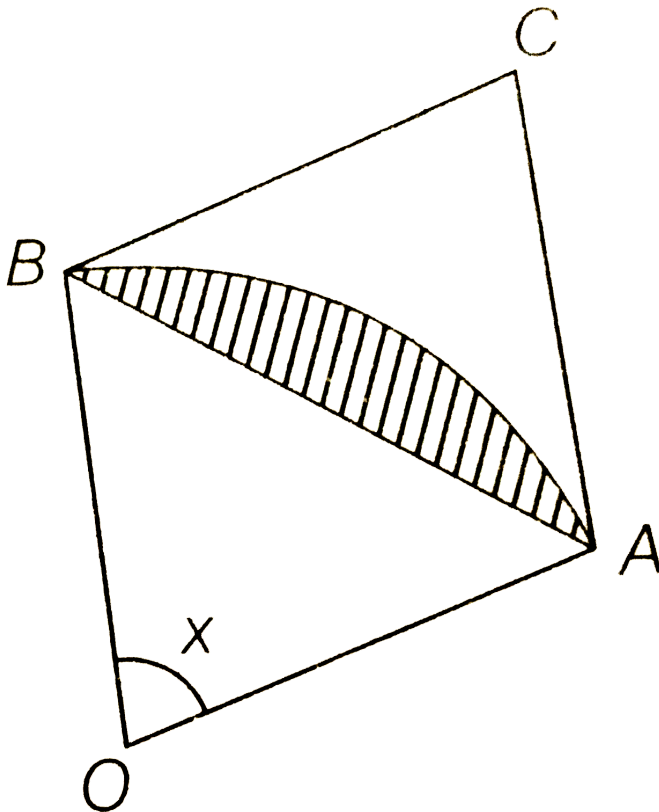

$$\lim_{x \to 0} \frac{S(x)}{x} \text{ is}$$

A. 0

B. $\dfrac{1}{2}$

C. 1

D. None of these

Answer: A

▶ Watch Video Solution

---

13. A circular are of radius 1 subtends an angle of x radians $0 < x < \dfrac{\pi}{2}$ as shown in the figure . the point r is the intersection of the two tangent line at P and Q. let $T(x)$ be the area of triangle PQR and $S(x)$ be the area of the shaded region then find $T(x) \& S(x) \& \lim\limits_{x \to 0} \dfrac{T(x)}{S(x)}$

A. $\dfrac{1}{4}$

B. $\dfrac{3}{4}$

C. $\dfrac{3}{2}$
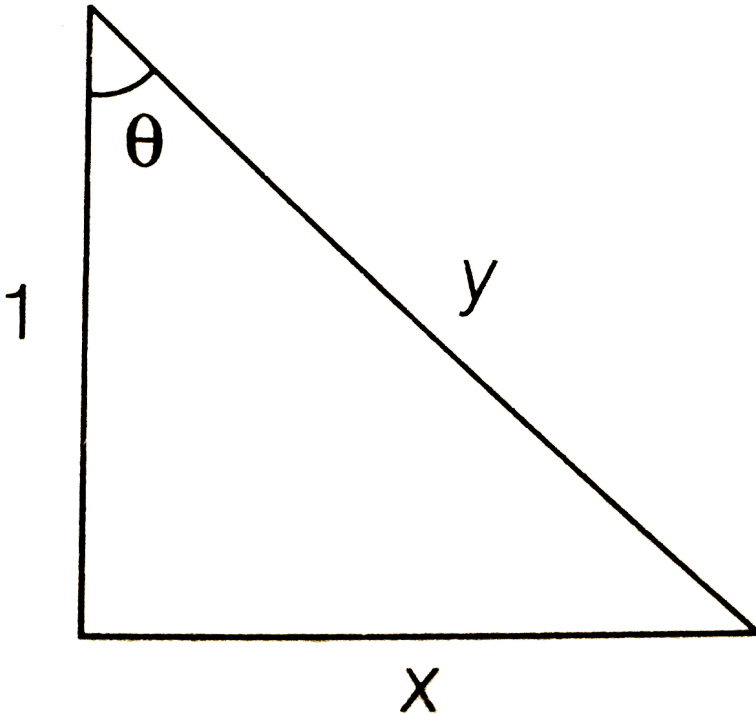
D. 0

Answer: C

Exercise Matching Type Questions

1. Match the statements of Column I with values of Column II.

Column I		Column II	
(A)	$\lim\limits_{x \to \frac{\pi^+}{2}} \tan^{-1}(\tan x)$	(p)	0
(B)	$\lim\limits_{n \to \infty} \left[\sum\limits_{r=1}^{n} \dfrac{1}{2^r} \right]$ ([·] denotes the greatest integer function)	(q)	Doesn't exist
(C)	$\lim\limits_{x \to \infty} \sec^{-1}\left(\dfrac{x}{x+1} \right)$	(r)	$-\dfrac{\pi}{2}$
(D)	$\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos x}{(1 - \sin x)^{2/3}}$	(s)	$\dfrac{\pi}{2}$

2. A right angled triangle has legs 1 and x. The hypotenuse is y and angle opposite to the side x is θ. Shown as

	Column I		Column II
(A)	$\lim\limits_{\theta \to \frac{\pi}{2}} (\sqrt{y} - \sqrt{x})$	(p)	0
(B)	$\lim\limits_{\theta \to \frac{\pi}{2}} (y - x)$	(q)	$\dfrac{1}{2}$
(C)	$\lim\limits_{\theta \to \frac{\pi}{2}} (y^2 - x^2)$	(r)	1
(D)	$\lim\limits_{\theta \to \frac{\pi}{2}} (y^3 - x^3)$	(s)	∞

Watch Video Solution

3. Find $\dfrac{dy}{dx}$ if $2y = x - \cos 2x$

▶ **Watch Video Solution**

Matching Type Questions

1. Find $\dfrac{dy}{dx}$ if $1 - y = x$

▶ **Watch Video Solution**

Exercise Single Integer Answer Type Questions

1. Let $L = \lim\limits_{x \to \infty} \left(x \log x + 2x \cdot \log \sin \left(\dfrac{1}{\sqrt{x}} \right) \right)$, then value of $\left(-\dfrac{2}{L} \right)$ is .....

▶ **Watch Video Solution**

2. For $n\varepsilon N$ let x_n be defined as $\left(1 + \dfrac{1}{n}\right)^{(n+x_n)} = e$ then $\displaystyle\lim_{n \to \infty} (2x_n)$ equals.....

▶ Watch Video Solution

3. Let $f(x) = x^4 - x^3$ then find f '(2)

▶ Watch Video Solution

4. If the arithmetic mean of the product of all pairs of positive integers whose sum is n is A_n then $\displaystyle\lim_{n \to \infty} \dfrac{n^2}{A_n}$ equals to

▶ Watch Video Solution

5. The value of $\displaystyle\lim_{n \to \infty} \sum_{k=1}^{n} \dfrac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)}$ is equal to

▶ Watch Video Solution

6. The value of $\lim\limits_{x \to \frac{\pi}{2}} \sqrt{\dfrac{\tan x - \sin\{\tan^{-1}(\tan x)\}}{\tan x + \cos^2(\tan x)}}$ is ....

> ▶ Watch Video Solution

7. Express in the form of complax number if $z = i^5$

> ▶ Watch Video Solution

8. If the two $AB: \left(\displaystyle\int_0^{2t} \left(\dfrac{\sin x}{x} + 1 \right) dx \right) x + y = 3t$ and

$AC: 2tx + y = 0$ intersect at a point A the x-coordinate of a point A as

$t \to 0$, is equal to $\dfrac{p}{q}$ (p and q are in their lowest form) the $(p + q)$ is

....

> ▶ Watch Video Solution

9. Find $\dfrac{dy}{dx}$ if $e^x = \log y - \sin x$

10. If $L = \lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{\cos \tan^{-1}(\tan x)}{x - \pi/2}$ then $\cos(2\pi L)$ is

11. Number of solutions of the equation $\csc\theta = k$ in $[0, \pi]$ where

$k = \lim\limits_{n \to \infty} \prod\limits_{r=2}^{n} \left(\dfrac{r^3 - 1}{r^3 + 1}\right)$ is ____

12. If C satisfies the equation $\lim\limits_{x \to \infty} \left(\dfrac{x + c}{x - c}\right)^x = 4$ then $\left|\dfrac{e^c}{2}\right|$ is ....

13. If $\displaystyle\lim_{x \to -\infty} \dfrac{\left(3x^4 + 2x^2\right) \cdot \sin\left(\frac{1}{x}\right) + |x|^3 + 5}{|x|^3 + |x^2| + |x| + 1} = k$ then $\left|\dfrac{k}{2}\right|$ is ....

14. If $f(x) = \displaystyle\lim_{t \to 0} \left[\dfrac{2x}{\pi} \cdot \tan^{-1}\left(\dfrac{x}{t^2}\right)\right]$, then $f(1)$ is ....

15. Differentiate $x^3 - 5\sin x$ w.r.t x

16. If $l = \displaystyle\lim_{x \to 1^+} 2^{-2^{\frac{1}{1-x}}}$ and $m = \displaystyle\lim_{x \to 1^+} \dfrac{x\sin(x - [x])}{x - 1}$ (where $[.]$ denotes greatest integer function). Then $(l + m)$ is ....

17. The value of $\lim\limits_{x \to 0} \left[\dfrac{\sin x . \tan x}{x^2} \right]$ is ....

(where [.] denotes greatest integer function).

> ▶ Watch Video Solution

18. $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{n} \dfrac{r}{1 \times 3 \times 5 \times 7 \times 9 \times \ldots \times (2r + 1)}$ is equal to

> ▶ Watch Video Solution

19. Find $\dfrac{dy}{dx}$ if $y = \sin^4 x$

> ▶ Watch Video Solution

20. If $f(x + y + z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \varepsilon R$ the value of $\lim\limits_{x \to \infty} \sum\limits_{r=1}^{n} \dfrac{(4r)f(3r)}{n^3}$ is ....

> ▶ Watch Video Solution

21. If $\lim\limits_{n \to \infty} \dfrac{n.3^n}{n(x-2)^n + n.3^{n+1} - 3^n} = \dfrac{1}{3}$, then the range of x is (where $n \in N$)

22. The value of

$\lim\limits_{x \to \infty} \left(\left((x-1)(x-2)(x+3)(x+10)(x+15) \right)^{\frac{1}{5}} - x \right)$ is ....

23. If $\lim\limits_{x \to \infty} \left([f(x)] + x^2 \right)\{f(x)\} = k$, where $f(x) = \dfrac{\tan x}{x}$ and $[.\,]$, $\{.\,\}$

denote geatest integer and fractional part of x respectively, the value of

$[k/e]$ is ....

24. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim\limits_{x \to 0} \left(1 + \dfrac{p(x)}{x^2} \right) = 2$. Then find the value of $p(2)$.

▶ Watch Video Solution

25. If α is the number of solution of $|x| = \log(x - [x])$, (where [.] denotes greatest integer function) and

$\lim\limits_{x \to \alpha} \dfrac{x e^{ax} - b \sin x}{x^3}$ is finite, the value of $(a - b)$ is

▶ Watch Video Solution

26. Suppose $x_1 = \tan^{-1} 2 > x_2 > x_3 > \ldots$ are the real numbers satisfying $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \cdot \sin x_n \cdot \sin x_{n+1}$ for all $n > 1$ and $l = \lim\limits_{x \to \infty} x_n$, the value of $[4l]$ is...where [t] denotes greatest integer function.

▶ Watch Video Solution

1. Let $\alpha, \beta \in R$ such that $\lim\limits_{x \to 0} \dfrac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

▶ Watch Video Solution

2. about to only mathematics

▶ Watch Video Solution

3. Let $f : R \overrightarrow{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \dfrac{1}{2}$. Suppose that $F(x) = \displaystyle\int_{-1}^{x} f(t) dt f$ or $allx \in [-1, 2] and G(x) = \displaystyle\int_{-1}^{x} t|f(f(t))| dt$ f or $allx \in [-1, 2] I \dfrac{f(\lim)_{x\,1} \overrightarrow{}(F(x))}{G(x)} = \dfrac{1}{14}$, Then the value of $f\left(\dfrac{1}{2}\right)$ is

▶ Watch Video Solution

4. The largest value of non negative integer for which

$$\lim_{x \to 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

5. Let $L = (\lim)_{x \to 0} \dfrac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$, $a > 0$. $If\, Li \in ite$, then $a = 2$

(b) $a = 1\ L = \dfrac{1}{64}$ (d) $L = \dfrac{1}{32}$

A. $a = 2$

B. $a = 1$

C. $L = \dfrac{1}{64}$

D. $L = \dfrac{1}{32}$

Answer: C

6. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

A. $\dfrac{1}{4}$

B. $\dfrac{1}{24}$

C. $\dfrac{1}{16}$

D. $\dfrac{1}{8}$

Answer: C

▶ **Watch Video Solution**

7. Let $p = \lim\limits_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$. Then $\log_e p$ is equal to

A. 2

B. 1

C. $\dfrac{1}{2}$

D. $\dfrac{1}{4}$

Answer: B

8. $\lim\limits_{n \to \infty} \left(\dfrac{(n+1)(n+2)(n+3)\ldots\ldots 2n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to

A. $\dfrac{18}{e^4}$

B. $\dfrac{27}{e^2}$

C. $\dfrac{9}{e^2}$

D. $3\log 3 - 2$

Answer: D

9. $\lim\limits_{x \to 0} \dfrac{\sin\left(\pi \cos^2 x\right)}{x^2}$ is equal to

A. $\dfrac{\pi}{2}$

B. 1

C. $-\pi$

D. π

Answer: D

> ▶ Watch Video Solution

---

10. $\lim\limits_{x \to 0} \dfrac{(1 - \cos x)(3 + \cos x)}{x \tan 4x}$ is equal to

A. 4

B. 3

C. 2

D. $\dfrac{1}{2}$

Answer: B

> ▶ Watch Video Solution

11. If $\lim_{x \to \infty} \left(\dfrac{x^2 + x + 1}{x + 1} - ax - b \right) = 4,$ **then**

A. $a = 1, b = 4$

B. $a = 1, b = -4$

C. $a = 2, b = -3$

D. $a = 2, b = 3$

Answer: D

▶ **Watch Video Solution**

12. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$\left((1 + a)^{\frac{1}{3}} - 1 \right) x^2 + \left((1 + a)^{\frac{1}{2}} - 1 \right) x + \left((1 + a)^{\frac{1}{6}} - 1 \right) = 0$ where

$a > -1$ then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$

A. $-\dfrac{5}{2}$ and 1

B. $-\dfrac{1}{2}$ and -1

C. $-\dfrac{7}{2}$ and 2

D. $-\dfrac{9}{2}$ and 3

Answer: C

---

13. If $(\lim)_{x \to 0}\left[1 + x1n\left(1 + b^2\right)\right]^{\frac{1}{x}} = 2b\sin^2\theta, b > 0, sm f\theta \in (-\pi, \pi],$ then the value of θ is $\pm\dfrac{\pi}{4}$ (b) $\pm\dfrac{\pi}{3}$ (c) $\pm\dfrac{\pi}{6}$ (d) $\pm\dfrac{\pi}{2}$

A. $\pm\dfrac{\pi}{4}$

B. $\pm\dfrac{\pi}{3}$

C. $\pm\dfrac{\pi}{6}$

D. $\pm\dfrac{\pi}{2}$

Answer: D

14. The value of $\lim\limits_{x \to 0} \left((\sin x)^{\frac{1}{x}} + \left(\dfrac{1}{x} \right)^{\sin x} \right)$, where $x > 0$, is (a)0 (b) -1 (c) 1 (d) 2

A. 0

B. -1

C. 1

D. 2

Answer: D

▶ Watch Video Solution

15. $\lim\limits_{h \to 0} \dfrac{f\left(2h + 2 + h^2\right) - f(2)}{f(h - h^2 + 1) - f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$ then (a) limit does not exist (b) is equal to $-\dfrac{3}{2}$ (c) is equal to $\dfrac{3}{2}$ (d) is equal to 3

A. does not exist

B. is equal to $-3/2$

C. is equal to $3/2$

D. is equal to 3

Answer: C

> ▶ Watch Video Solution

---

16. $\lim\limits_{x \to 0} \dfrac{\sin(nx)((a - n)nx - \tan x)}{x^2} = 0$, when n is a non-zero positive

integer, then a is equal to

A. 0

B. $\dfrac{n + 1}{n}$

C. n

D. $n + \dfrac{1}{n}$

Answer: C

> ▶ Watch Video Solution

17. The integer n for which $\lim\limits_{x \to 0}\left((\cos x - 1)\dfrac{\cos x - e\widehat{x}}{x^n}\right)$ is finite nonzero number is _____

A. 1

B. 2

C. 3

D. 4

Answer: C

▶ Watch Video Solution

---

18. Let $f: R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then
$\lim\limits_{x \to 0}\left(\dfrac{f(1 + x)}{f(1)}\right)^{1/x} =$ (a)1 (b) $e^{\frac{1}{2}}$ (c) e^2 (d) e^3

A. 1

B. $e^{\frac{1}{2}}$

C. e^2

D. None of these

Answer: C

▶ Watch Video Solution

19. $\lim\limits_{x \to \infty} \left(\dfrac{x - 3}{x + 2} \right)$ is equal to :

A. e

B. e^{-1}

C. e^{-5}

D. e^5

Answer: C

▶ Watch Video Solution

1. If $\quad f(a) = 2 \quad f'(a) = 1 \quad g(a) = -1 \quad g'(a) = 2 \quad$ then

$\lim\limits_{x \to a} \dfrac{g(x)f(a) - g(a)f(x)}{x - a}$ is

A. -5

B. 3

C. -3

D. 5

Answer: C

▶ **Watch Video Solution**

2. The value of $\lim\limits_{x \to 0} \dfrac{x \cos x - \log(1 + x)}{x^2}$ is

A. 1

B. $\dfrac{1}{4}$

C. $\dfrac{1}{2}$

D. None of these

Answer: C

▶ **Watch Video Solution**

---

3. Evaluate $\displaystyle \lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

A. $\dfrac{3}{2}$

B. $-\dfrac{3}{2}$

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{2}$

Answer: C

▶ **Watch Video Solution**

4. The value of $\lim\limits_{x \to a} \dfrac{\cos x - \cos a}{\cot x - \cot a}$ is

A. $-\sin^3 a$

B. $\cos^2 a$

C. $\sin^3 a$

D. $\cot a$

Answer: A

▶ Watch Video Solution

5. The value of $\lim\limits_{x \to 0} \left(\dfrac{1}{x^2} - \cot x \right)$ is

A. 0

B. 1

C. $\dfrac{1}{4}$

D. None of these

Answer: C

6. Evaluate $\lim\limits_{x \to \infty} \left[\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1} \right]$.

A. $\dfrac{1}{2}$

B. $-\dfrac{1}{2}$

C. does'nt exist

D. None of these

Answer: A

7. The value of $\lim\limits_{n \to \infty} \dfrac{1^3 + 2^3 + 3^3 + \ldots\ldots + n^3}{(n^2 + 1)^2}$

A. $\dfrac{1}{4}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{2\sqrt{2}}$

D. None of these

Answer: A

8. The value of $\displaystyle\lim_{n \to \infty} \dfrac{1.\,n + 2.\,(n-1) + 3.\,(n-2) + \ldots + n.1}{1^2 + 2^2 + \ldots + n^2}$

A. 1

B. -1

C. $\dfrac{1}{\sqrt{2}}$

D. $\dfrac{1}{2}$

Answer:

9. The value of $\lim\limits_{n \to \infty} \dfrac{a^n + b^n}{a^n - b^n}$, (where $a > b >$ is

A. 1

B. -1

C. $\dfrac{1}{2}$

D. $\dfrac{1}{\sqrt{2}}$

Answer:

▶ Watch Video Solution

Exercise For Session 2

1. If $(\lim)_{x \to 0} \left(x^{-3} \sin 3x + ax^{-2} + b \right)$ exists and is equal to 0, then

$a = -3 \text{ and } b = \dfrac{9}{2}$ $a = 3 \text{ and } b = \dfrac{9}{2}$ $a = -3 \text{ and } b = -\dfrac{9}{2}$

$a = 3 \text{ and } b = -\dfrac{9}{2}$

A. $a = -3, b = \dfrac{9}{2}$

B. $a = 3, b = \dfrac{9}{2}$

C. $a = -3, b = \dfrac{-9}{2}$

D. None of these

Answer: D

▶ **Watch Video Solution**

---

2. The value of $\displaystyle\lim_{x \to a} \dfrac{x \sin a - a \sin x}{x - a}$ is

A. $a \sin a - \cos a$

B. $\sin a - a \cos a$

C. $\cos a + a \sin a$

D. $\sin a + a \cos a$

Answer: B

▶ **Watch Video Solution**

3. Evaluate the following limits:

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

A. $\dfrac{1}{4}$

B. $\dfrac{1}{2}$

C. 2

D. Doesn't exist

Answer: A

▶ **Watch Video Solution**

---

4. evaluate $\lim\limits_{\theta \to \frac{\pi}{4}} \dfrac{\sqrt{2} - \cos \theta - \sin \theta}{(4\theta - \pi)^2}$

A. $\dfrac{1}{16\sqrt{2}}$

B. $\dfrac{1}{16}$

C. $\dfrac{1}{8\sqrt{2}}$

D. $\dfrac{1}{2\sqrt{2}}$

Answer: D

5. The value of $\displaystyle\lim_{x \to \pi/4} \dfrac{(\cos x + \sin x)^3 - 2\sqrt{2}}{1 - \sin 2x}$ is

A. $\dfrac{\sqrt{3}}{2}$

B. $-\dfrac{3}{\sqrt{2}}$

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{\sqrt{2}}$

Answer: B

Exercise For Session 3

1. Evaluate $\lim\limits_{x \to 0} \dfrac{e^x - e^{x \cos x}}{x + \sin x}$.

A. 0

B. 1

C. -1

D. None of these

Answer: A

▶ Watch Video Solution

2. The value of $\lim\limits_{x \to y} \dfrac{x^y - y^x}{x^x - y^y}$ is:

A. (a) $\dfrac{1 - \log x}{1 + \log x}$

B. (b) $\dfrac{1 - \log y}{1 + \log y}$

C. (c) $\dfrac{\log x - \log y}{\log x + \log y}$

D. (d) None of these

Answer: B

3. The value of $\lim\limits_{x \to 0} \dfrac{p^x - q^x}{r^x - s^x}$ is

A. $\dfrac{1 - \log p}{1 + \log p}$

B. $\dfrac{\log p - \log q}{\log r - \log s}$

C. $\dfrac{\log p.\ \log q}{\log r.\ \log s}$

D. None of these

Answer: B

4. The value of $\lim\limits_{x \to \infty} (x + 2)\tan^{-1}(x + 2) - \left(x \tan^{-1} x\right)$ is

A. $\dfrac{\pi}{2}$

B. Doesn't exist

C. $\dfrac{\pi}{4}$

D. None of these

Answer: D

▶ Watch Video Solution

5. The value of $\displaystyle\lim_{x \to 4} \dfrac{(\cos\alpha)^x - (\sin\alpha)^x - \cos 2\alpha}{x - 4}$, $\alpha\varepsilon\left(0, \dfrac{\pi}{2}\right)$ is

A. $\log(\cos\alpha) + (\sin\alpha)^4 \log(\sin\alpha)$

B. $\left(\cos^4\alpha\right)\log(\cos\alpha) - (\sin\alpha)^4 \log(\sin\alpha)$

C. $\left(\cos^4\alpha\right)\log(\cos\alpha)$

D. None of these

Answer: B

▶ Watch Video Solution

6. The value of $\lim\limits_{x \to 0} \left(\dfrac{1^x + 2^x + 3^x + \ldots\ldots\ldots\ldots + n^x}{n} \right)^{a/x}$ is

A. $(n!)^{a/n}$

B. $n!$

C. $a^{n!}$

D. Doesn't exist

Answer:

▶ Watch Video Solution

---

7. If $(\lim)_{x \, 0} \left(1 + ax + bx^2 \right)^{\frac{2}{x}} = e^3$, then find the value of a and b.

A. $a = \dfrac{3}{2}, b \varepsilon R$

B. $a = \dfrac{1}{2}, b \varepsilon R$

C. $a \varepsilon R, b \varepsilon R$

D. None of these

Answer:

---

8. If α an β are roots of $ax^2 + bx + c = 0$ the value for $\lim\limits_{x \to \alpha} \left(1 + ax^2 + bx + c\right)^{2/x-\alpha}$ is

A. $e^{2a(\alpha - \beta)}$

B. $e^{a(\alpha - \beta)}$

C. $e^{\frac{2a}{3}(\alpha - \beta)}$

D. None of these

Answer:

---

9. The value of $\lim (n \to \infty)\left((1.5)^n + \left[(1 + 0.0001)^{10000}\right]^n\right)^{\frac{1}{n}}$, where

[.] denotes the greatest integer function is:

A. 1

B. $\dfrac{1}{2}$

C. does'nt exist

D. 2

Answer:

▶ **Watch Video Solution**

10. The value of $\lim\limits_{x \to 0} |x|^{[\cos x]}$, [.] denotes greatest integer function is

A. 0

B. Doesn't exist

C. 1

D. None of these

Answer:

▶ **Watch Video Solution**

1. Evaluate $\lim\limits_{x \to 0^+} (\sin x)^x$

▶ Watch Video Solution

2. Evaluate $\lim\limits_{x \to 0^+} (\sin x)^{\tan x}$

▶ Watch Video Solution

3. evaluate $\lim\limits_{n \to \infty} \left(\dfrac{e^n}{\pi}\right)^{\frac{1}{n}}$

▶ Watch Video Solution

4. The value of $\lim\limits_{x \to 0} \dfrac{e - (1+x)^{1/x}}{\tan x}$ is

A. e

B. $\dfrac{11e}{24}$

C. $\dfrac{e}{2}$

D. None of these

Answer: D

▶ **Watch Video Solution**

Exercise For Session 5

1. The value of $\lim\limits_{x \to 1} (\{1 - x + [x - 1] + [1 - x]\}$ (where [.] donetes the greatest integral function) is

A. -1

B. Doesn't exist

C. 1

D. None of these

Answer:

2. The value of $\lim\limits_{x \to 0} \dfrac{\sin[x]}{[x]}$ (where [.] denotes the greatest integer function) is The value of $\lim\limits_{x \to 0} \left[\dfrac{\sin x}{x}\right]$ (where [.] denotes the greatest integer function) is

A. 1

B. $\sin 1$

C. does'nt exist

D. None of these

Answer:

3. The value of $\lim\limits_{x \to 0} \sin^{-1}\{x\}$ (where {.} denotes fractional part of x) is

A. 0

B. $\dfrac{\pi}{2}$

C. does'nt exist

D. None of these

Answer:

▶ Watch Video Solution

---

4. If [.] denotes the greatest integer function then $\lim\limits_{x \to 0} \left[\dfrac{x^2}{\tan x . \sin x} \right] =$

A. 0

B. 1

C. does'nt exist

D. None of these

Answer:

▶ Watch Video Solution

1. The value o
$$\lim_{x \to \infty} \left(\frac{1}{n^3}\right)\left([1^2x + 1^2] + [2^2x + 2^2] + \dots . + [n^2x + n^2]\right)$$ is

where [.] denotes the greatest integer function.

A. $\dfrac{x}{3}$

B. $x + \dfrac{1}{3}$

C. $\dfrac{x}{3} + \dfrac{1}{3}$

D. None of these

Answer:

▶ Watch Video Solution

2. The value of

$$\lim_{x \to \infty} \left\{ \lim_{n \to \infty} \frac{\left[1^2(\sin x)^x\right] + \left[2^2(\sin x)^x\right] + \cdots \cdots + \left[n^2(\sin x)^x\right]}{n^3} \right\}$$

is (wehre [.] denotes the greatest integer function)

A. A. $\dfrac{x}{3} + \dfrac{\sin x}{3}$

B. B. $\dfrac{x}{3} + (\sin x)^x$

C. C. $\dfrac{1}{3}$

D. D. 0

Answer: option 4

▶ **Watch Video Solution**

---

3. The value of $\lim\limits_{x \to 1^+} \dfrac{\int_1^x |t - 1| dt}{\sin(x - 1)}$ is:

A. (a) 0

B. (b) 1

C. (c) doesn't exist

D. (d) None of these

Answer:

4. The value of $\lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \log\left(1 + \dfrac{k}{n}\right)^{\frac{1}{n}}$,is

A. $\log_e\left(\dfrac{e}{4}\right)$

B. $\log_e\left(\dfrac{4}{e}\right)$

C. $\log_e 4$

D. None of these

Answer:

5. Evaluate: $\left(\lim\right)_{n\to\infty} n\left[\dfrac{1}{na} + \dfrac{1}{na+1} + \dfrac{1}{na+2} + + \dfrac{1}{nb}\right]$

A. $\log\left(\dfrac{a}{b}\right)$

B. $\log\left(\dfrac{b}{a}\right)$

C. $\log(ab)$

D. None of these

Answer:

▶ **Watch Video Solution**