



MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

MATHEMATICAL INDUCTION



1. Prove that
$$:\!1^2+2^2+3^2+\ +n^3=igg\{rac{n(n+1)}{2}igg\}^2$$
 .

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2. Using the principle of mathematical induction, prove that : $1.2.3+2.3.4++n(n+1)(n+2)=rac{n(n+1)(n+2)(n+3)}{4}$ for all $n\in N$.

3. Using the principle of mathematical induction, prove that : $1.2.3+2.3.4++n(n+1)(n+2)=rac{n(n+1)(n+2)(n+3)}{4}$ for all $n\in N$.

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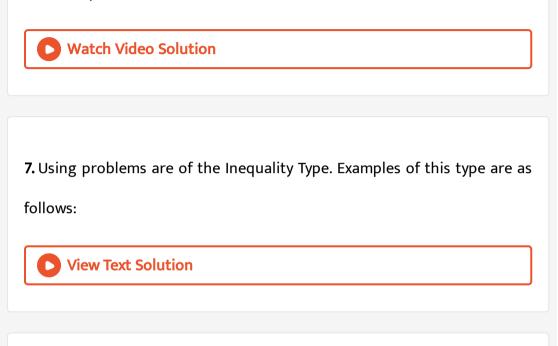
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4. Prove by mathematical induction that $\sum_{r=0}^n r^n C_r = n.2^{n-1}, \ orall n \in N.$

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5. Use the principle of mathematical induction to show that $5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$ divisible by 19 for all natural numbers n.

6. Use the principle of mathematical induction to show that $a^n - b^n$) is divisble by a - b for all natural numbers n.



8. Using mathematical induction prove that $n^3 - 7n + 3$ is divisible by

 $3,\,\forall n\in N$



9. if a + b = c + d and $a^2 + b^2 = c^2 + d^2$, then show by

mathematical induction $a^n + b^n = c^n + d^n$



10. Let
$$I_m = \int_0^\pi igg(rac{1-\cos mx}{1-\cos x}igg) dx$$
 use mathematical induction to prove

that $l_m=m\pi, m=0,1,2$

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11. Given that $u_{n+1}=3u_n-2u_{n-1},$ and $u_0=2,u_1=3,$ then prove

that $u_n = 2^n + 1$ for all positive integer of n

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12. Let
$$u_1 = 1$$
, $u_2 = 2$, $u_3 = \frac{7}{2}$ and $u_{n+3} = 3u_{n+2} - \left(\frac{3}{2}\right)u_{n+1} - u_n$.
Use the principle of mathematical induction to show that
 $u_n = \frac{1}{3} \left[2^n + \left(\frac{1+\sqrt{3}}{2}\right)^n + \left(\frac{1-\sqrt{3}}{2}\right)^n \right] \forall n \ge 1.$

/ - \

13. If p is a natural number, then prove that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $p^2 + p + 1$ for every positive integer n.

A. P B. $P^2 + P$ C. $P^2 + P + 1$ D. $P^2 - 1$

Answer:

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14. Let P(n) denote the statement that n^2+n is odd . It is seen that $P(n) \Rightarrow P(n+1), P(n)$ is true for all

A. n > 1

 $\mathsf{B.}\,n$

 $\mathsf{C}.\,n>2$

D. None of these

Answer:



15. For a positive integer
$$n$$
 let
 $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{(2^n) - 1}$. Then $a(100) \le 100$ b.
 $a(100) > 100$ c. $a(200) \le 100$ d. $a(200) \le 100$

A. a(100) > 100

B. a(100) < 200

 $\mathsf{C.}\,a(200) \leq 100$

D.a(200) > 100

Answer: D

16. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$. Then which of the following is true ?

A. Principle of mathematical induction can be used to prove the

formula

 $\mathsf{B}.\,S(k) \Rightarrow S(k+1)$

 $\mathsf{C.}\,S(k) \not\nearrow S(k+1)$

D. S(1) is correct

Answer:

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17.
$$10^n+3ig(4^{n+2}ig)+5$$
 is divisible by $(n\in N)$

A. 7

B. 5

C. 9

Answer:

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- 18. Statement-1 For all natural number n, $1+2+....+n<(2n+1)^2$ Statement -2 For all natural numbers , $(2n+3)^2-7(n+1)<(2n+3)^3$.
 - A. Statement -1 is true, Statement -2 is true Statement -2 is correct

explanation for Statement -1.

B. Statement -1 is true, Statement -2 is true, Statement -2 is not the

correct explanation for Statement -1

- C. Statement-1 is true, Statement-2 is false
- D. Statement-1 is false, Statement -2 is true.

Answer: B



19. Prove the following by the principle of mathematical induction: $7+77+777++777++\ddot{n}-digits7=rac{7}{81}ig(10^{n+1}-9n-10ig)$ for all $n\in NB$.

A. Statement -1 is true, Statement -2 is true Statement -2 is correct

explanation for Statement -1.

B. Statement -1 is true, Statement -2 is true, Statement -2 is not the

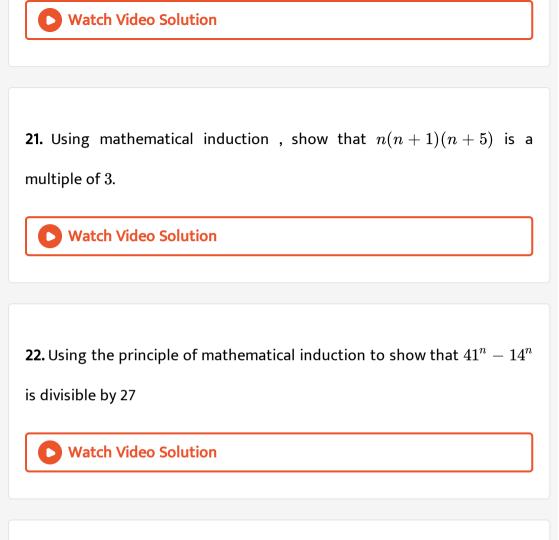
correct explanation for Statement -2

- C. Statement-1 is true, Statement-2 is false
- D. Statement-1 is false, Statement -2 is true.

Answer: C

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20. Prove by induction that $41^n - 14^n$ is divisible by 27



23. Use the principle of mathematical induction to prove that for all

 $n \in N$

$$\sqrt{2 + \sqrt{2 + \sqrt{2 + ... + ... + \sqrt{2}}}} = 2\cos{\left(rac{\pi}{2^{n+1}}
ight)}$$

when the LHS contains \boldsymbol{n} radical signs.

24. Prove by mathematical induction that $10^{2n-1} + 1$ is divisible by 11



25. Using the principle of mathematical induction to prove that $\int_{0}^{\pi/2} \frac{\sin^2 nx}{\sin x} dx = 1 + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2n-1}$ Watch Video Solution

26. Use induction to show that for all $n \in N$.

$$\sqrt{a+\sqrt{a+\sqrt{a+....}+\sqrt{a}}} < rac{1+\sqrt{(4a+1)}}{2}$$

where'a' is fixed positive number and n radical signs are taken on LHS.

27. Prove that
$$\sum_{r=0}^n \ \ n C_r \sin r x \cos(n-r) x = 2^{n-1} \sin(nx) \cdot$$

Mathematical Induction Exercise 1 Single Option Correct Tpye Questions

1. If
$$a_n = \sqrt{7 + \sqrt{7 + \sqrt{7} + \dots}}$$

having n radical signs then by

methods of mathematical induction which is true

A.
$$a_n>7,~orall n\geq 1$$

B. $n_n>3,~orall n\geq 1$
C. $a_n<4,~orall n\geq 1$
D. $a_n<3,~orall n\geq 1$

Answer:

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2. Prove by the mathematical induction $x^{2n} - y^{2n}$ is divisible by x + y

A. all $n \in N$ B. C.

Answer:

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3. Show by using the principle of mathematical induction that for all natural number $n>2, 2^n>2n+1$

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Exercise Statement I And Ii Type Questions

1. If $a_1 = 1, a_2 = 5$ and $a_{n+2} = 5a_{n+1} - 6a_n, n \ge 1$, show by using

mathematical induction that $a_n = 3^n - 2^n$

A. Statement -1 is true, Statement -2 is true, Statement -2 is correct

explanation for Statement -1

B. Statement -1 is true , Statement -2 is true , Statement -2 is not

correct explanation for Staement -1

C. Statement -1 is true, Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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2. Statement -1 for all natural numbers n , $2.7^n+3.5^n-5$ is divisible by

24.

Statement -2 if f(x) is divisible by x, then f(x+1) - f(x) is divisible by $x+1, \ \forall x \in N.$

A. Statement -1 is true, Statement -2 is true, Statement -2 is correct

explanation for Statement -2

B. Statement -1 is true, Statement -2 is true, Statement -2 is not

correct explanation for Staement -2

C. Statement -1 is true, Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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3. Statement -1 For all natural numbers n , $0.5 + 0.55 + 0.555 + \dots$ upto n terms $= \frac{5}{9} \left\{ n - \frac{1}{9} \left(1 - \frac{1}{10^n} \right) \right\}$, Statement-2 $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{(1 - r)}$, for 0 < r < 1.

A. (a)Statement -1 is true, Statement -2 is true, Statement -2 is correct

explanation for Statement -1

B. (b)Statement -1 is true, Statement -2 is true, Statement -2 is not

correct explanation for Staement -1

C. (c)Statement -1 is true , Statement -2 is false

D. (d)Statement -1 is false , Statement - 2 is true.

Answer:

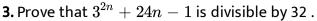
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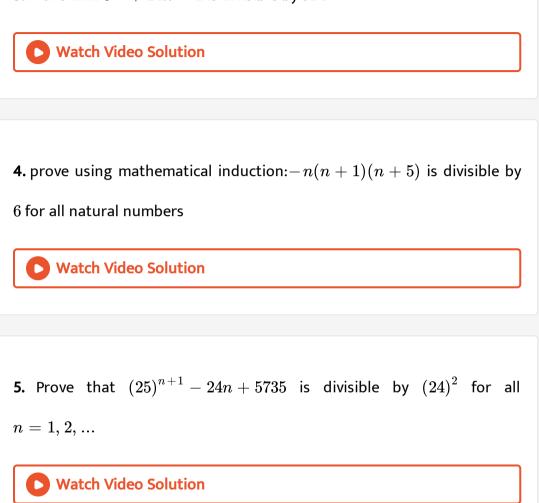
Exercise Subjective Type Questions

1. Prove the following by the principle of mathematical induction: $11^{n+2}+12^{2n+1}$ is divisible 133 for all $n\in N$.

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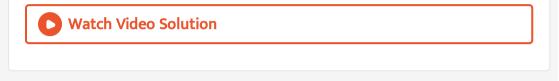
2. $n^7 - n$ is divisible by 42 .





6. Prove the following by the principle of mathematical induction: $x^{2n-1}+y^{2n-1}$ is divisible by x+y for all $n\in N$.

7. Prove by induction that if n is a positive integer not divisible by 3, then $3^{2n}+3^n+1$ is divisible by 13.



8. Prove that the product of three consecutive positive integers is divisible by 6.

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9. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.



10. When the square of any odd number, greater than 1, is divided by 8, it always leaves remainder (a)1 (b) 6 (c) 8 (d) Cannot be determined



11. Prove the following by using induction for all
$$n \in N$$
. $1+2+3+\ldots + n = rac{n(n+1)}{2}$

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12. Prove the following by the principle of mathematical induction: $1^2+2^2+3^2+\ +n^2=rac{n(n+1)(2n+1)}{6}$

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13. Prove the following by the principle of mathematical induction: $1.3 + 2.4 + 3.5 + + (2n - 1)(2n + 1) = \frac{n(4n^2 + 6n - 1)}{3}$ 14. Prove the following by the principle of mathematical induction:

$$rac{1}{2.5} + rac{1}{5.8} + rac{1}{8.11} + + rac{1}{(3n-1)(3n+2)} = rac{n}{6n+4}$$

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15. Prove
$$1.4.7 + 2.5.8 + 3.6.9 + \dots$$
 upto n terms $= \frac{n}{4}(n+1)(n+6)(n+7)$

16.
$$\frac{1^2}{1.3} + \frac{2^2}{3.5} + \frac{3^2}{5.7} + \dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{(n)(n+1)}{(2(2n+1))}$$

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17. Let $a_0=2, a_1=5$ and for $n\geq 2, a_n=5a_{n-1}-6a_{n-2}.$ Then prove

by induction that $a_n=2^n+3^n\,orall\,n\in Z^+.$

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18. If $a_1=1, a_{n+1}=rac{1}{n+1}a_n, a\geq 1$, then prove by induction that $a_{n+1}=rac{1}{(n+1)!}n\in N.$

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19. if a,b,c,d,e and f are six real numbers such that a+b+c=d+e+f

 $a^2 + b^2 + c^2 = d^2 + e^2 + f^2$ and $a^3 + b^3 + c^3 = d^3 + e^3 + f^3$, prove

by mathematical induction that $a^n+b^n+c^n=d^n+e^n+f^n$ $orall n\in N.$

20. Prove that
$$\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \dots + \tan^{-1}\left(\frac{1}{n^2 + n + 1}\right)$$

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Exercise Questions Asked In Previous 13 Years Exam

1. Statement-1: For every natural number $n \geq 2$,

$$rac{1}{\sqrt{1}} + rac{1}{\sqrt{2}} + rac{1}{\sqrt{3}} + ... + rac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-2: For every natural number $n \ge 2$,

$$\sqrt{n(n+1)} < n+1$$

A. Statement-1 is true, Statement-2 is true, Statement-2 is correct

explanation for Statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a

correct explanation for Statement-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer:



2. Statement -1 For each natural number $n, \left(n+1
ight)^7 - n^7 - 1$ is divisible by 7.

Statement -2 For each natural number $n, n^7 - n$ is divisible by 7.

A. Statement-1 is false , Statement-2 is true

B. Statement-1 is true, Statement-2 is true, Statement-2 is correct

explanation for Statement-1

C. Statement-1 is true, Statement-2 is true, Statement-2 is not a

correct explanation for Statement-1

D. Statement-1 is true, Statement-2 is false

Answer:



