MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

MATHEMATICAL INDUCTION

Examples

1. Prove that :$1^2+2^2+3^2+\ +n^3=\left\{\frac{n(n+1)}{2}\right\}^2$.

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2. Using the principle of mathematical induction, prove that : $1.2.3+2.3.4+\ +n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$ for all $n\in N$.

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3. Using the principle of mathematical induction, prove that : $1.2.3+2.3.4+\ +n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$ for all $n \in N$.

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4. Prove by mathematical induction that $\sum_{r=0}^{n} r^n C_r = n.2^{n-1},\ \forall n \in N.$

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5. Use the principle of mathematical induction to show that $5^{2n+1}+3^{n+2}.2^{n-1}$ divisible by 19 for all natural numbers n.

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6. Use the principle of mathematical induction to show that $a^n - b^n)$ is divisble by $a - b$ for all natural numbers n.

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7. Using problems are of the Inequality Type. Examples of this type are as follows:

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8. Using mathematical induction prove that $n^3 - 7n + 3$ is divisible by 3, $\forall n \in N$

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9. if $a + b = c + d$ and $a^2 + b^2 = c^2 + d^2$, then show by mathematical induction $a^n + b^n = c^n + d^n$

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10. Let $I_m = \int_0^{\pi} \left(\frac{1 - \cos mx}{1 - \cos x} \right) dx$ use mathematical induction to prove that $l_m = m\pi, m = 0, 1, 2.....$

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11. Given that $u_{n+1} = 3u_n - 2u_{n-1}$, and $u_0 = 2, u_1 = 3$, then prove that $u_n = 2^n + 1$ for all positive integer of n

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12. Let $u_1 = 1, u_2 = 2, u_3 = \frac{7}{2}$ and $u_{n+3} = 3u_{n+2} - \left(\frac{3}{2} \right) u_{n+1} - u_n.$

Use the principle of mathematical induction to show that $u_n = \frac{1}{3} \left[2^n + \left(\frac{1 + \sqrt{3}}{2} \right)^n + \left(\frac{1 - \sqrt{3}}{2} \right)^n \right] \forall n \geq 1.$

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13. If p is a natural number, then prove that $p^{n+1}+(p+1)^{2n-1}$ is divisible by p^2+p+1 for every positive integer n.

A. P

B. P^2+P

C. P^2+P+1

D. P^2-1

Answer:

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14. Let P(n) denote the statement that n^2+n is odd . It is seen that $P(n) \Rightarrow P(n+1)$, $P(n)$ is true for all

A. $n>1$

B. n

C. $n>2$

D. None of these

Answer:

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15. For a positive integer n let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots . + \frac{1}{(2^n) - 1}$. Then $a(100) \leq 100$ b. $a(100) > 100$ c. $a(200) \leq 100$ d. $a(200) \leq 100$

A. $a(100) > 100$

B. $a(100) < 200$

C. $a(200) \leq 100$

D. $a(200) > 100$

Answer: D

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16. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true ?

A. Principle of mathematical induction can be used to prove the formula

B. $S(k) \Rightarrow S(k + 1)$

C. $S(k) \not\Rightarrow S(k + 1)$

D. S(1) is correct

Answer:

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17. $10^n + 3\left(4^{n+2}\right) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 7

Answer:

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18. Statement-1 For all natural number n, $1+2+\ldots.+n<(2n+1)^2$ Statement -2 For all natural numbers , $(2n+3)^2-7(n+1)<(2n+3)^3$.

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -1

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: B

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19. Prove the following by the principle of mathematical induction: $7+77+777+\ +777+\ +\ \ddot{n}-digits7=\frac{7}{81}\left(10^{n+1}-9n-10\right)$ for all $n\in NB$.

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -2

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: C

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20. Prove by induction that 41^n-14^n is divisible by 27

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21. Using mathematical induction , show that $n(n+1)(n+5)$ is a multiple of 3.

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22. Using the principle of mathematical induction to show that $41^n - 14^n$ is divisible by 27

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23. Use the principle of mathematical induction to prove that for all $n \in N$

$$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\ldots+\sqrt{2}}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

when the LHS contains n radical signs.

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24. Prove by mathematical induction that $10^{2n-1}+1$ is divisible by 11

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25. Using the principle of mathematical induction to prove that $\int_0^{\pi/2} \frac{\sin^2 nx}{\sin x} dx = 1 + \frac{1}{3} + \frac{1}{5} + \ldots . + \frac{1}{2n-1}$

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26. Use induction to show that for all $n \in N$.

$$\sqrt{a+\sqrt{a+\sqrt{a+\ldots.+\sqrt{a}}}} < \frac{1+\sqrt{(4a+1)}}{2}$$

where'a' is fixed positive number and n radical signs are taken on LHS.

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27. Prove that $\sum_{r=0}^{n} \hat{} \, nC_r \sin rx \cos(n-r)x = 2^{n-1}\sin(nx)\cdot$

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Mathematical Induction Exercise 1 Single Option Correct Tpye Questions

1. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7} + ...}}$ having n radical signs then by methods of mathematical induction which is true

A. $a_n > 7,\ \forall n \geq 1$

B. $n_n > 3,\ \forall n \geq 1$

C. $a_n < 4,\ \forall n \geq 1$

D. $a_n < 3,\ \forall n \geq 1$

Answer:

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2. Prove by the mathematical induction$x^{2n} - y^{2n}$ is divisible by$x + y$

A. all $n \in N$

B.

C.

D.

Answer:

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3. Show by using the principle of mathematical induction that for all natural number $n > 2, 2^n > 2n + 1$

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Exercise Statement I And Ii Type Questions

1. If $a_1 = 1, a_2 = 5$ and $a_{n+2} = 5a_{n+1} - 6a_n, n \geq 1$, show by using mathematical induction that $a_n = 3^n - 2^n$

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -1

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -1

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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2. Statement -1 for all natural numbers n , $2.7^n + 3.5^n - 5$ is divisible by 24.

Statement -2 if f(x) is divisible by x, then $f(x+1) - f(x)$ is divisible by $x+1$, $\forall x \in N$.

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -2

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -2

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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3. Statement -1 For all natural numbers n , $0.5 + 0.55 + 0.555 + ...$ upto n terms $= \frac{5}{9}\left\{n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right\}$,

Statement-2 $a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1 - r^n)}{(1 - r)}$, for $0 < r < 1$.

A. (a)Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -1

B. (b)Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -1

C. (c)Statement -1 is true , Statement -2 is false

D. (d)Statement -1 is false , Statement - 2 is true.

Answer:

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Exercise Subjective Type Questions

1. Prove the following by the principle of mathematical induction: $11^{n+2}+12^{2n+1}$ is divisible 133 for all $n \in N$.

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2. n^7-n is divisible by 42 .

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3. Prove that $3^{2n} + 24n - 1$ is divisible by 32 .

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4. prove using mathematical induction:$-n(n+1)(n+5)$ is divisible by 6 for all natural numbers

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5. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2, \ldots$

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6. Prove the following by the principle of mathematical induction: $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$ for all $n \in N$.

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7. Prove by induction that if n is a positive integer not divisible by 3, then $3^{2n} + 3^n + 1$ is divisible by 13.

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8. Prove that the product of three consecutive positive integers is divisible by 6.

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9. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

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10. When the square of any odd number, greater than 1, is divided by 8, it always leaves remainder (a)1 (b) 6 (c) 8 (d) Cannot be determined

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11. Prove the following by using induction for all $n \in N$. $1+2+3+\ldots\ldots+n=\frac{n(n+1)}{2}$

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12. Prove the following by the principle of mathematical induction: $1^2+2^2+3^2+\ +n^2=\frac{n(n+1)(2n+1)}{6}$

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13. Prove the following by the principle of mathematical induction: $1.3+2.4+3.5+\ +(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$

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14. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}++\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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15. Prove $1.4.7+2.5.8+3.6.9+\ldots\ldots$ upto n terms $=\frac{n}{4}(n+1)(n+6)(n+7)$

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16. $\frac{1^2}{1.3}+\frac{2^2}{3.5}+\frac{3^2}{5.7}+\ldots..+\frac{n^2}{(2n-1)(2n+1)}=\frac{(n)(n+1)}{(2(2n+1))}$

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17. Let $a_0 = 2, a_1 = 5$ and for $n \geq 2, a_n = 5a_{n-1} - 6a_{n-2}$. Then prove by induction that $a_n = 2^n + 3^n \forall n \in Z^+$.

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18. If $a_1 = 1, a_{n+1} = \frac{1}{n+1} a_n, a \geq 1$, then prove by induction that $a_{n+1} = \frac{1}{(n+1)!} n \in N$.

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19. if a,b,c,d,e and f are six real numbers such that $a + b + c = d + e + f$ $a^2 + b^2 + c^2 = d^2 + e^2 + f^2$ and $a^3 + b^3 + c^3 = d^3 + e^3 + f^3$, prove by mathematical induction that $a^n + b^n + c^n = d^n + e^n + f^n \forall n \in N$.

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20. Prove that

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{13}\right)+\ldots\ldots\ldots+\tan^{-1}\left(\frac{1}{n^2+n+}\right.$$

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Exercise Questions Asked In Previous 13 Years Exam

1. Statement-1: For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}>\sqrt{n}$$

Statement-2: For every natural number $n \geq 2$,

$$\sqrt{n(n+1)}<n+1$$

A. Statement-1 is true , Statement-2 is true, Statement-2 is correct explanation for Statement-1

B. Statement-1 is true , Statement-2 is true , Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer:

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2. Statement -1 For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7.

Statement -2 For each natural number n, $n^7 - n$ is divisible by 7.

A. Statement-1 is false , Statement-2 is true

B. Statement-1 is true , Statement-2 is true , Statement-2 is correct explanation for Statement-1

C. Statement-1 is true , Statement-2 is true , Statement-2 is not a correct explanation for Statement-1

D. Statement-1 is true , Statement-2 is false

Answer:

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