MATHS

BOOKS - ARIHANT MATHS (ENGLISH)

MONOTONICITY MAXIMA AND MINIMA

Examples

1. Find the interval in which

$f(x) = 2x^3 + 3x^2 - 12x + 1$ is increasing.

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2. Find the interval in which

$f(x) = x^3 - 3x^2 - 9x + 20$ is strictly increasing or strictly decreasing.

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3. Show that the function $f(x) = x^2$ is a strictly increasing function on $(0, \infty)$.

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4. Find the interval of increase or decrease of the $f(x) = \int_{-1}^{x} (t^2 + 2t)(t^2 - 1)\,dt$

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5. The function $f(x) = \sin^4 x + \cos^4 x$ increasing if \`0

A. $0 < x < \pi/8$

B. $\pi/4 < x < 3\pi/8$

C. $3\pi/8 < x < 5\pi/8$

D. $5\pi/8 < x < 3\pi/4$

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6. Let $f(x) = \int_0^x e^t (t-1)(t-2)dt.$ Then, f decreases in the interval

A. $(-\infty, -2)$

B. $(-2, -1)$

C. $[1, 2]$

D. $(2, \infty)$

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7. If $f(x) = x.\, e^{x(1-x)}$, then f(x) is

A. increasing on $\left[-\frac{1}{2}, 1\right]$

B. decreasing on R

C. increasing on R

D. decreasing on $\left[-\frac{1}{2}, 1\right]$

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8. Find the interval for which $f(x) = x - \sin x$ is increasing or decreasing.

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9. If $H(x_0)$=0 for some x=$x_0$and $\frac{d}{dx}H(x) > 2cxH(x)$ for all $x \geq x_0$ where $c > 0$ then

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10. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)} + \frac{y^2}{f(k+11)} = 1.$ If $f(x)$ is a positive decr4easing function, then the set of values of k for which the major axis is the x-axis is $(-3, 2)$. the set of values of k for which the major axis is the y-axis is $(-\infty, 2)$. the set of values of k for which the major axis is the y-axis is $(-\infty, -3) \cup (2, \infty)$ the set of values of k for which the major axis is the y-axis is $(-3, -\infty,)$

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11. Let f(x)=3x-5, then show that f(x) is strictly increasing.

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12. Let $\phi(x) = \sin(\cos x),$ then check whether it is increasing or decreasing in $[0, \pi/2]$.

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13. Let $\phi(x) = \cos(\cos x)$, then check whether it is increasing or decreasing in $[0, \pi/2]$.

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14. Let $f(x) = \begin{cases} xe^{ax}, & x \leq 0 \\ x + ax^2 - x^3, & x > 0 \end{cases}$ where a is postive constant . Find the interval in which f'(X) is increasing.

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15. If $a < 0$ and $f(x) = e^{ax} + e^{-ax}$ is monotonically decreasing . Find the interval to which x belongs.

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16. If $0 < \alpha < \frac{\pi}{6}$, then the value of $(\alpha \cos ec\alpha)$ is

A. less than $\frac{\pi}{3}$

B. more than $\frac{\pi}{3}$

C. less than $\frac{\pi}{6}$

D. more than $\frac{\pi}{6}$

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17. If f: R rightarrow R, f(x) is a differentiable bijective function , then which of the following may be true?

A. $(f(x) - x)f''(x) < 0, \forall x \in R$

B. $(f(x) - x)f''(x) > 0, \forall \times \in R$

C. If $(f(x) - x)f''(x) > 0,$ then $f(x) = f^{-1}$ has no solution

D. If $(f(x) - x)f''(x) > 0, \quad \text{then} \quad f(x) = f^{-1}(x)$

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18. If $f(x) and g(x)$ are two positive and increasing functions, then which of the following is not always true? (a) $[f(x)]^{g(x)}$ is always increasing (b) $[f(x)]^{g(x)}$ is decreasing, when $f(x) < 1$ (c) $[f(x)]^{g(x)}$ is increasing, then $f(x) > 1$. (d) If $f(x) > 1, then [f(x)]^{g(x)}$ is increasing.

A. $(f(x))^{g(x)}$ is always incrasing

B. if $(f(x))^{g(x)}$ is increasing then $f(x) < 1$

C. if $(f(x))^{gx}$) is increasing then $f(x) > 1$

D. if $f(x) > 1$ then $(f(x))^{g(x)}$ is increasing

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19. If the function $y = \sin(f(x))$ is monotonic for all values of x [where f(x) is continuous], then the maximum value of the difference between the maximum and the minumum value of f(x) is

A. π

B. 2π

C. $\frac{\pi}{2}$

D. None of the above

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20. If f"(x) > 0 and f(1) = 0 such that $g(x) = f(\cot^2 x + 2\cot x + 2) where 0 < x < \pi$, then g'(x) decreasing in (a, b). where $a + b + \frac{\pi}{4}$...

A. $(0, \pi)$

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(\frac{3\pi}{4}, \pi\right)$

D. $\left(0, \frac{3\pi}{4}\right)$

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21. Find the critical points(s) and stationary points (s) of the function $f(x) = (x-2)^{2/3}(2x+1)$

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22. The integral value of 'b' for which the function $f(x) = \left(b^2 - 3b + 2\right)\left(\cos^2 x - \sin^2 x\right) + (b-1)x + \sin\left(b^2 + b + 1\right)$ does not possesses any stationary point is

A. $[1, \infty]$

B. $(0, 1) \cup (1, 4)$

C. $\left(\frac{3}{2}, \frac{5}{2}\right)$

D. None of these

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23. The set of critical pionts of the fuction f(x) given by

$$f(x) = x - \log_e x + \frac{1}{t} - 2 - 2\cos 4t dt is$$
x
2

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24. Using calculus, find the order relation between x and $\tan^{-1}$ x when $x \in [0, \infty)$.

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25. Using calculus, find the order relation between x and $\tan^{-1}$ x when $x \in [0, \infty)$.

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26. For all $x \in (0, 1)$ (a) $e^x < 1 + x$ (b) $(\log)_e(1 + x) < x$ (c) $\sin x > x$ (d) $(\log)_e x > x$

A. $e^x < 1 + x$

B. $\log_e(1 + x) < x$

C. $\sin x > x$

D. $\log_e x > x$

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27. If f'(x) changes from positive to negative at x_0 while moving from left to right,

i.e. $f'(x) > 0, x < x_0$

$f'(x) < 0, x > x_0,$ then f(x)has local maximum value at $x = x_0$

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28. If f'(x) changes from negative to positive at x_0 while moving from left to right, i.e. $f'(x) < 0, x < x_0$

$f'(x) > 0, x > x_0,$

then f(x) has local minimum value at $x = x_0$

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29. If sign of f'(x) doesn't change at $x_0,$

while moving from left to right, then f(x) has neither a maximum nor a minimum at $x_0.$

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30. Let $f(x) = x^3 - 3x^2 + 6$ find the point at which f(x) assumes local maximum and local minimum.

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31. Let $f(x) = x + \frac{1}{x}, x \neq 0.$ Discuss the maximum and minimum value of f(x).

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32. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 dt$ has a local minimum at $x =$ 0 (b) 1 (c) 2 (d) 3

A. 0

B. 1

C. 2

D. 3

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33. Find the local maximum and local minimumof $f(x) = x^3 - 3x$ in [-2,4].

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34. Show that the function

$f(x) = \begin{cases} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{cases}$ is continuous at x = 2

A. f(x) is increasing on [-1,2]

B. f(x) is continuos on [-1,3]

C. f'(x) does not exist at x=2

D. f(x) has the maximum value at x=2

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35. Let $f(x) = \sin x - x$ on$[0, \pi/2]$ find local maximum and local minimum.

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36. Let $f(x) = x(x-1)^2$, find the point at which f(x) assumes maximum and minimum.

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37. Let $f(x) = (x-1)^4$ discuss the point at which f(x) assumes maximum or minimum value.

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38. Discuss the function

$f(x) = x^6 - 3x^4 + 3x^2 - 5$, and plot the graph.

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39. Discuss the function

$f(x) = \frac{1}{2}\sin 2x + \cos x.$, and plot its graph.

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40. Discuss the function

$y = x + In(x^2 - 1)$ and plot its graph.

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41. Let $f(x) = 2x^3 - 9x^2 + 12x + 6.$ Discuss the global maxima and minima of $f(x) \in [0, 2] and (1, 3)$ and, hence, find the range of $f(x)$ for corresponding intervals.

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42. Let $f(x) = 2x^3 - 9x^2 + 12x + 6$. Discuss the global maxima and minima of $f(x) \in [0, 2] and (1, 3)$ and, hence, find the range of $f(x)$ for corresponding intervals.

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43. Discuss the minima of $f(x) = \{x\}$,

(where{,} denotes the fractional part of x)for x=6.

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44. Let $f(x) = \begin{cases} |x-2| + a^2 - 9a - 9, & \text{if } x < 2 \\ 2x - 3, & \text{if } x \geq 2 \end{cases}$

Then, find the value of 'a' for which f(x) has local minimum at x=2

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45. Let $f(x) = \begin{cases} 6, x \leq 1 \\ 7 - x, x > 1 \end{cases}$ 'then for f(x) at x=1 discuss maxima and minima.

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46. Find the values of 'a' for which,

$$f(x) = \begin{cases} 4x - x^3 + \log\left(a^2 - 3a + 3\right), & 0 \le x < 3 \\ x - 18, & x \ge 3 \end{cases}$$

, f(x) as a local minima at x=3 is

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47. Let $-1 \le p \le 1$. Show that the equation $4x^3 - 3x - p = 0$ has a unique root in the interval [1/2,1] and identify it.

A. $\dfrac{\cos^{-1} p}{3}$

B. $\cos\left(\dfrac{1}{3}\cos^{-1} p\right)$

C. $\cos\left(\cos^{-1} p\right)$

D. None of these

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48. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1$ = 0 is ________.

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49. The values of parameter a for which the point of minimum of the function $f(x) = 1 + a^2x - x^3$ satisfies the inequality $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0 are$ (a) $(2\sqrt{3}, 3\sqrt{3})$ (b) $-3\sqrt{3}, -2\sqrt{3})$ (c) $(-2\sqrt{3}, 3\sqrt{3})$ (d) $(-2\sqrt{2}, 2\sqrt{3})$

A. $(-3\sqrt{3}, \infty)$

B. $(-3\sqrt{3}, -2\sqrt{3}) \cup (0, \infty)$

C. $(-3\sqrt{3}, -2\sqrt{3}) \cup (2\sqrt{3}, 3\sqrt{3})$

D. $(0, \infty)$

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50. The values of a and b for which all the extrema of the function, $f(x) = a^2x^3 - 0.5ax^2 - 2x - b$, is positive and the minima is at the point $x_0 = \frac{1}{3}$, are

A. when $a = -2 \Rightarrow b < -\frac{11}{27}$ and when a $= 3 \Rightarrow b < -\frac{1}{2}$

B. when$a = 3 \Rightarrow b < -\frac{11}{27}$and when a $= 2 \Rightarrow b < -\frac{1}{2}$

C. when $a = -2 \Rightarrow b < -\frac{1}{2}$ and when a $= 3 \Rightarrow b < -\frac{11}{27}$

D. None of the above

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51. If $f''(x) + f'(x) + f^2(x) = x^2$ be the differentiable equation of a curve and let p be the point of maxima then number of tangents which can be drawn from p to $x^2 - y^2 = a^2$ is/are........ .

A. 2

B. 1

C. 0

D. either 1 or 2

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52. Let $f(x)=\int_0^x \cos\left(\frac{t^2+2t+1}{5}\right)dt,\ o>x>2,$ then

A. increases monotonically

B. decreasing montonically

C. has one point of local maximum

D. has one point of local minima

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53. As 'x' ranges over the interval (o, ∞), the function

$f(x) = \sqrt{9x^2 + 173x + 900} - \sqrt{9x^2 + 77x + 900}$, ranges over

A. (o,4)

B. (0,8)

C. (0,12)

D. (0,16)

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54. Let $g : [1, 6] \to [0, \ )$ be a real valued differentiable function satisfying $g'(x) = \dfrac{2}{x + g(x)}$ and $g(1) = 0$, then the maximum value of g cannot exceed $\ln 2$ (b) $\ln 6$ $6 \ln 2$ (d) $2 \ln 6$

A. log2

B. log 6

C. 6 log 2

D. 2 log 6

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55. The minimum value of the function,

$f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$. For all permissible real values of x is

A. -10

B. -6

C. -7

D. -8

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56. The least natural number a for which $x + ax^{-2} > 2 \forall x \in (0, \infty)$ is 1

(b) 2 (c) 5 (d) none of these

A. 1

B. 2

C. 5

D. None of these

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57. If k $\sin^2 x + \frac{1}{k}\cos ec^2 x = 2,\ x \in \left(0, \frac{\pi}{2}\right)$,

then $\cos^2 x + 5\sin x \cos x + 6\sin^2 x$ is equal to

A. $\dfrac{k^2 + 5k + 6}{k^2}$

B. $\dfrac{k^2 - 5k + 6}{k^2}$

C. 6

D. None of these

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58. Find the least value of the expression

$x^2 + 4y^2 + 3z^2 - 2x - 12y - 6z + 14$

A. 0

B. 1

C. no least value

D. None of the above

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59. STATEMENT 1 : On the interval $\left[\frac{5\pi}{4}, \frac{4\pi}{3}\right]$. the least value of the function $f(x) = \int_{\frac{5x}{4}}^{x} (3\sin t + 4\cos t) dt is 0$ STATEMENT 2 : If $f(x)$ is a decreasing function on the interval $[a, b]$, then the least value of $f(x)$ is $f(b)$.

A. $\frac{3}{2} + \frac{1}{\sqrt{2}} - 2\sqrt{3}$

B. $\frac{3}{2} - \frac{1}{\sqrt{2}} + 2\sqrt{3}$

C. $\frac{3}{2} - \frac{1}{\sqrt{2}} - 2\sqrt{3}$

D. None of these

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60. For any the real $\theta$the maximum value of$\cos^2(\cos\theta) + \sin^2(\sin\theta)$ is

A. 1

B. $1 + \sin^2 1$

C. $1 + \cos^2 1$

D. does not exist

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61. If $\sin\theta + \cos\theta = 1,$ then the minimum value of $(1 + \cos ec\theta)(1 + \sec\theta)$ is

A. 3

B. 4

C. 6

D. 9

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62. The coordinates of the point on the curve $x^3 = y(x - a)^2$ where the ordinate is minimum is

A. $(2a, 8a)$

B. $\left(-2a, \frac{-8a}{9}\right)$

C. $\left(3a, \frac{27a}{4}\right)$

D. $\left(-3a, \frac{-27a}{16}\right)$

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63. If $a, b \in R$ distinct numbers satisfying $|a-1|+|b-1|=|a|+|b|=|a+1|+|b+1|$, then the minimum value of $|a-b|$ is

A. 3

B. 0

C. 1

D. 2

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64. Statement 1: The maximum value of $\left(\sqrt{-3+4x-x^2}+4\right)^2+(x-5)^2 (where 1 \leq x \leq 3) is 36.$ Statement 2: The maximum distance between the point $(5,\,-4)$ and the point on the circle $(x-2)^2+y^2=1$ is 6

A. 34

B. 36

C. 32

D. 20

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65. If $a>b>0$ and $f(\theta)=\dfrac{\left(a^2-b^2\right)\cos\theta}{a-b\sin\theta}$, then the maximum value of $f(\theta)$, is

A. $2\sqrt{a^2+b^2}$

B. $\sqrt{a^2+b^2}$

C. $\sqrt{a^2 - b^2}$

D. $\sqrt{b^2 - a^2}$

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66. If composite function $f_1(f_2(f_3((f_n(x))))n$ timesis an decreasing function and if $'r'$ functions out of total $'n'$ functions are decreasing function while rest are increasing, then the maximum value of $r(n-r)$ is

(a) $\frac{n^2-4}{4}$, when n is of the form 4k (b) $\frac{n^2}{4}$, when n is an even number (c) $\frac{n^2-1}{4}$, when n is an odd number (d) $\frac{n^2}{4}$, when n is of the form 4k+2

A. $\frac{n^2-1}{4}$ when n is an even number

B. $\frac{n^2}{4}$ when n is an odd number

C. $\frac{n^2-1}{4}$ when n is odd number

D. None of these

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67. Let $f(x) = \sin x + ax + b$. Then which of the following is/are true?

(a) $f(x) = 0$ has only one real root which is positive if $a > 1, b < 0$. (b) $f(x) = 0$ has only one real root which is negative if $a > 1, b < 0$. (c) $f(x) = 0$ has only one real root which is negative if $a > 1, b > 0$. (d) none of these

A. only one real root which is positive, if $a > 1, b < 0$

B. only one real root which is negative, if $a > 1, b > 0$

C. only one real root which is negative, if $a < -1, b < 0$

D. None of the above

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68. The function $f(x)=\int_0^x \sqrt{1-t^4}dt$ is such that: (A) t is defined in the interval $[-1, 1]$ (B) $f(x)$ is increasing dunction (C) f is an odd function (D) the point $(0, 0)$ is the point of inflexion

A. it is defined on the interval [-1,1]

B. it is an increasing function

C. it is an odd function

D. the point (0,0) is the point of inflection

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69. The function $\dfrac{\sin(x+a)}{\sin(x+b)}$ has no maxima or minima if

A. $b-a=n\pi, n\in 1$

B. $b-a=(2n+1)\pi, n\in 1$

C. $b-a=2n\pi, n\in 1$

D. None of these

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70. Let $F(x) = 1 + f(x) + (f(x))^2 + (f(x))^3$ where $f(x)$ is an increasing differentiable function and $F(x) = 0$ hasa positive root, then

A. F(x) is an increasing function

B. $F(0) \leq 0$

C. $f(0) \leq -1$

D. $F(0) > 0$

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71. The extremum values of the function $f(x) = \dfrac{1}{\sin x + 4} - \dfrac{1}{\cos x - 4}$, where $x \in R$

A. $\frac{4}{8-\sqrt{2}}$

B. $\frac{2\sqrt{2}}{8-\sqrt{2}}$

C. $\frac{2\sqrt{2}}{4\sqrt{2}+1}$

D. $\frac{4\sqrt{2}}{8+\sqrt{2}}$

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72. The function $f(x)=x^{\frac{1}{3}}(x-1)$ has two inflection points has one point of extremum is non-differentiable has range $\left[-3x2^{-\frac{8}{3}}, \infty\right)$

A. has 2 inflection points

B. is strictly increasing for $x>\frac{1}{4}$ and strictly decreasing for $x<\frac{1}{4}$

C. is concave down in $\left(-\frac{1}{2}, 0\right)$

D. area increased by the curve lying in the fourth quadrant is $\frac{9}{28}$

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73. Assume that inverse of the function f is denoted by g, then which of the

folllowing statement hold good?

A. If f is increasing, then g is also increasing

B. If f is decreasing, then g is increasing

C. The function g is injective

D. The function g is onto

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74. Statement I :Among all the rectangles of the given perimeter, the square has the largest area. Also among all the rectangles of given area, the square has the least perimeter.

Statement II :For $x > 0, y > 0, \quad \text{if} \quad x + y$= constant, then xy will be maximum for y=x and if xy=constant, then x+y will be minimum for y=x.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statemetn I is true, Statement II is also true, Statement II is not correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

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75. Statement I :The function $f(x) = \left(x^3 + 3x - 4\right)\left(x^2 + 4x - 5\right)$ has local extremum at x=1.

Statement II :f(x) is continuos and differentiable and f'(1)=0.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statemetn I is true, Statement II is also true, Statement II is not correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

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76. Statement I :If f(x) is increasing function with upward concavity, then concavity of $f^{-1}(x)$ is also upwards.

Statement II : If f(x) is decreasing function with upwards concavity, then concavity of $f^{-1}(x)$ is alo upwards.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statemetn I is true, Statement II is also true, Statement II is not correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

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77. Let $f : R \overrightarrow{R}$ be differentiable and strictly increasing function throughout its domain. Statement 1: If $|f(x)|$ is also strictly increasing function, then $f(x) = 0$ has no real roots. Statement 2: When $x \overrightarrow{\infty}$ or $\overrightarrow{-} \infty, f(x) \overrightarrow{0}$, but cannot be equal to zero.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statemetn I is true, Statement II is also true, Statement II is not correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

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78. Let x_1, x_2, x_3, x_4 be the roots (real or complex) of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. If $x_1 + x_2 = x_3 + x_4$ and a,b,c,$d \in R$, then

If $b < 0$, then how many different values of a, we may have

A. -1

B. 1

C. -2

D. 2

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79. If x_1, x_2, x_3, x_4 be the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. If $x_1 + x_2 = x_3 + x_4$ and $a, b, c, d \in R$,then (i) If a =2, then the value of b-c (ii) $b < 0$, then how many different values of a, we may have

A. 3

B. 2

C. 1

D. 0

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80. Let x_1, x_2, x_3, x_4 be the roots (real or complex) of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$. If $x_1 + x_2 = x_3 + x_4$ and a,b,c,$d \in R$, then

If $b < 0$, then how many different values of a, we may have

A. $\left(-\infty, \frac{1}{4}\right)$

B. $(-\infty, 3)$

C. $(-\infty, 1)$

D. $(-\infty, 4)$

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81. Let $f(x)=ax^2+bx+C, a, b, c \in R$.It is given $|f(x)| \leq 1, |x| \leq 1$ The possible value of $|a+c|$,if $\frac{8}{3}a^2+2b^2$ is maximum, is given by

A. 1

B. 0

C. 2

D. 3

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82. The absolute maximum and minimum values of functions can be found by their monotonic and asymptotic behaviour provided they exist. We may agree that finite limits values may be reagarded as absolute maximum or minimum. For instance the absolute maximum value of $\frac{1}{1+x^2}$ is unity. It is attained at x=0 while absolute maximum value of the same functions is zero which is a limiting value of $(x \to \infty \text{ or } x \to -\infty)$

The function $x^4 - 4x + 1$ will have.

A. absolute maximum value

B. absolute minimum value

C. both absolute maximum and minimum values

D. None of these

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83. The absolute maximum and minimum values of functions can be found by their monotonic and asymptotic behaviour provided they exist. We may agree that finite limits values may be reagarded as absolute maximum or minimum. For instance the absolute maximum value of $\frac{1}{1+x^2}$ is unity. It is attained at x=0 while absolute maximum value of the same functions is zero which is a limiting value of $(x \to \infty \text{ or } x \to -\infty)$

The absolute minimum value of the function $\frac{x-2}{\sqrt{x^2+1}}$ is

A. -1

B. $\frac{1}{2}$

C. $-\sqrt{5}$

D. None of these

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84. The absolute maximum and minimum values of functions can be found by their monotonic and asymptotic behaviour provided they exist. We may agree that finite limits values may be reagarded as absolute maximum or minimum. For instance the absolute maximum value of $\frac{1}{1+x^2}$ is unity. It is attained at x=0 while absolute maximum value of the same functions is zero which is a limiting value of $(x \to \infty \text{ or } x \to \ -\infty)$

The absolute minimum and maximum values of the function $\frac{x^2-x+1}{x^2+x+1}$ is

A. 1 and 3

B. $\frac{1}{2}$ and 3

C. $\frac{1}{3}$ and 3

D. None of these

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85. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \frac{1}{2}\right)$ any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable ,$\forall x \in$ R through $(0, 1)$ Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X- axists

The number of solutions $f(x) = 2ex$ is equal to

A. 0

B. 1

C. 2

D. None of these

Answer: B

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86. We are given the curves $y = \int_{-\infty}^{x} f(t)$ dt through the point $\left(0, \frac{1}{2}\right)$ and y=f(X), where $f(x) > 0$ and f(x) is differentiable, $\forall x \in R$ through (0,1). If tangents drawn to both the curves at the point wiht equal abscissae intersect on the point on the X-axis, then

$$\int_{x \to \infty} (f(x))^{f(-x)}$$ is

A. (a)3

B. (b)6

C. (c)1

D. (d)None of these

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87. We are given the curves $y = \int_{-\infty}^{x} f(t)$ dt through the point $\left(0, \frac{1}{2}\right)$ and y=f(X), where $f(x) > 0$ and f(x) is differentiable, $\forall x \in R$ through (0,1). If tangents drawn to both the curves at the point wiht equal abscissae intersect on the point on the X-axis, then

The function f(x) is

A. increasing for all x

B. non-monotonic

C. decreasing for all x

D. None of these

Answer: A

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88. Let $f(x)=\left(1+\frac{1}{x}\right)^{x}(x>0)$ and

$$g(x)=\begin{cases}x\ln\left(1+\frac{1}{x}\right), & \text{if } 0<x\leq 1\\ 0, & \text{if } x=0\end{cases}$$

$$\lim_{x\to 0^{+}} g(x)$$

A. (a) is equal to 0

B. (b) is equal to 1

C. (c) is equal to e

D. (d) is non-existent

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89. Let $f(x)=\left(1+\frac{1}{x}\right)^{x}(x>0)$ and

$$g(x)=\begin{cases} x\ln\left(1+\frac{1}{x}\right), & \text{if } \ 0<x\leq 1 \\ 0, & \text{if } \ x=0 \end{cases}$$

$$\lim_{x\to 0^+} \ g(x)$$

A. has a maxima but non minima

B. has a minima but not maxima

C. has both of maxima and minima

D. is a monotonic

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90. Consider the cubic $f(x) = 8x^3 + 4ax^2 + 2bx + a$ where $a, b \in R$.

For $a = 1$ if $y = f(x)$ is strictly increasing $\forall x \in R$ then maximum range of values of b is:

A. (a) $\left(-\infty, \frac{1}{3}\right]$

B. (b) $\left(\frac{1}{3}, \infty\right)$

C. (c) $\left[\frac{1}{3}, \infty\right)$

D. (d) $(-\infty, \infty)$

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91. For $b = 1$, if $y = f(x) = 8x^3 + 4ax^2 + 2bx + 1$ is non monotonic then the sum of all the integral values of $a \in [1, 100]$, is

A. 4950

B. 5049

C. 5050

D. 5047

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92. If the sum of the base 2 logarithms of the roots of the cubic $f(x) = 8x^3 + 4ax^2 + 2x + a = 0$ is 5 then the value of $'a'$ is

A. a) -64

B. b) -8

C. c) -128

D. d) -256

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93. If $\sin x + x \geq |k|x^2, \ \forall x \in \left[0, \frac{\pi}{2}\right]$, then the greatest value of k is

A. $\frac{-2(2+\pi)}{\pi^2}$

B. $\frac{2(2+\pi)}{\pi^2}$

C. can't be determined finitely

D. zeero

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94. Consider a twice differentiable function f(x) of degree four symmetrical to line x = 1 defined as $f: R \to R$ and $f''(2) = 0$. (A) The Sum of the roots is

A. 0

B. 1

C. 2

D. 5

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95. Consider a twice differentiable function f(x) of degree four symmetrical to line x = 1 defined as $f: R \to R$ and $f''(2) = 0.$ if f(1)=0, f(2)=1 then the value of f(3) is

A. $6/7$

B. $7/5$

C. $8/5$

D. $9/5$

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96. The function $S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt$ has two critical points in the interval $[1, 2.4]$. One of the critical points is a local minimum and the other is a local maximum .

The local maximum occurs at x equals ____

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97. The radius of a right circular cylinder increases at a constant rate. Its altitude is a linear function of the radius and increases three times as fast as the radius when the radius is $1\ cm$ and the altitude is $6\ cm$. When the radius is $6cm$, the volume is increasing at the rate of $1\ \frac{cm^3}{s}$. When the radius is $36cm$, the volume is increasing at a rate of $n\ \frac{cm^3}{s}$. What is the value of n?

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98. The graphs $y = 2x^3 - 4x + 2$ and $y = x^3 + 2x - 1$ intersect in exactly 3 distinct points. Then find the slope of the line passing through two of these points.

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99. The sets of the value of 'a' for which the equation $x^4 + 4x^3 + ax^2 + 4x + 1 = 0$ + has all its roots real given by $(a_1, a_2) \cup \{a_3\}$. then $|a_3 + a_2|$ is

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100. Consider a polynomial $P(x)$ of the least degree that has a maximum equal to 6 at $x = 1$ and a minimum equal to 2 at $x = 3$. Then the value of $P(2) + P(0) - 7$ is

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101. Let $g(x) > 0$ and $f'(x) < 0, \forall x \in R,$ then show

$g(f(x+1)) < g(f(x-1))$

$f(g(x+1)) < f(g(x-1))$

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102. Let

$f'(\sin x) < 0$ and $f''(\sin x) > 0, \forall x \in \left(0, \frac{\pi}{2}\right)$ and $g(x) = f(\sin x) + f$

then find the interval in which g(x) is increasing and decreasing.

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103. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, $where$ $0 < x \leq 1,$ then in this interval

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104. Let f:[0,,∞) $\rightarrow$ [0,,∞)and g:[0,,∞) $\rightarrow$ [0,,∞) be non increasing and non decreasing functions respectively and h(x) =g(f(x)).

If h(0)=0 .Then show h(x) is always identically zero.

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105. A cubic function $f(x)$ vanishes at $x = -2$ and has relative minimum/maximum at $x = -1 and x = \frac{1}{3}$ if $\int_{-1}^{1} f(x)dx = \frac{14}{3}$. Find the cubic function $f(x)$.

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106. Given that $S = \left|\sqrt{x^2+4x+5} - \sqrt{x^2+2x+5}\right|$ for all real x, then find the maximum value of S^4

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107. Find the maximum value of

$$f(x) = \frac{40}{3x^4 + 8x^3 - 18x^2 + 60}$$

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108. Use the function $f(x) = x^{\frac{1}{x}}, x > 0,$ to determine the bigger of the two numbers $e^{\pi} and \pi^{e}.$

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110. Using the relation $2(1 - \cos x) < x^2$ $,x = 0$ or prove that $\sin(\tan x) \geq x, \forall \epsilon \left[0, \frac{\pi}{4}\right]$

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111. Prove that for $x \in \left[0, \frac{\pi}{2}\right]$, $\sin x + 2x \geq \frac{3x(x+1)}{\pi}$.

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112. Find a point M on the curve $y = \frac{3}{\sqrt{2}} x \ln x, \; x \in \left(e^{-1.5}, \infty\right)$ such that the segment of the tangent at M intercepted between M and the Y-axis is shortest.

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113. Sohan has x children by his first wife. Geeta has $(x+1)$ children by her first husband. The marry and have children of their own. The whole family has 24 children. Assuming that two children of the same parents do not fight, prove that the maximum possible number of fights that can take place is 191.

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114. Let P be the point on curve $4x^2 + \alpha^2 y^2 = 4\alpha^2$,$0 < \alpha^2 < 8$ whose distance from Q (0,-2) is greatest if R is the reflection of P in the X axis then find the least distance of R from the line 3x-4y+7=0 is

A. 1

B. 2

C. 3

D. 4

A. 1

B. 2

C. 3

D. 4

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115. What normal to the curve $y = x^2$ forms the shortest chord?

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116. Let $f(x)$=$\sin^3 x + \lambda \sin^2 x$,$\frac{\pi}{2}$<x<$\frac{\pi}{2}$. Find the intervals in which λ should lie in order that $f(x)$ has exactly one minimum and exactly one maximum.

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117. Determine the point of maxima and minima of the function $f(x) = \frac{1}{8}(\log)_e x - bx + x^2, x > 0,$ where $b \geq 0$ is constant.

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118. Find the points on the curve $ax^2 + 2bxy + ay^2 = c,$

$0 < a < b < c,$ whose distance from the origin is minimum.

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119. The function $f(x) = (x^2 - 4)^n (x^2 - x + 1), n \in N,$ assumes a local minimum value at $x = 2$. Then find the possible values of n

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120. For what values of a, the function $f(x) = \left\{ \left(\frac{\sqrt{a+4}}{1-a} \right) x^5 - 3x + \log(5) \right.$ decreases for all real x.

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121. Let $a + b = 4, wherea < 2, andletg(x)$ be a differentiable function. If $\frac{dg}{dx} > 0$ for all x, prove that $\int_0^a g(x)dx + \int_0^b g(x)dx \in crerasesas(b - a) \in crerases.$

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122. Let $g(x) = 2f\left(\frac{x}{2}\right) + f(2 - x)$ and $f''(x) < 0 \forall x \in (0, 2)$. If $g(x)$ increases in (a, b) and decreases in (c, d), then the value of $a + b + c + d - \frac{2}{3}$ is

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123. Let $f'(x) > 0$ and $f''(x) > 0$ where $x_1 < x_2$.

Then show $f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + (x_2)}{2}$.

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124. Let $f'(x) > 0$ and $f''(x) > 0$ where $x_1 < x_2$.

Then show $f\left(\frac{x_1 + x_2}{2}\right) < \frac{f(x_1) + (x_2)}{2}$.

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125. If f(x) is monotonically increasing function for all $x \in R,$ such that $f''(x) > 0$ and $f^{-1}(x)$ exists, then prove that

$$\frac{f^{-1}(x_1) + f^{-1}(x_2) + f^{-1}(x_3)}{3} < \left(\frac{f^{-1}(x_1 + x_2 + x_3)}{3}\right)$$

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126. A box of maximum volume with top open is to be made by cutting out four equal squares from four corners of a square tin sheet of side length a feet and then folding up the flaps. Find the side of the square cut-off.

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127. find the volume of the greatest right circular cone that can be described by the revolution about a side of a right angled triangle of hypotenuse 1 ft.

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128. A window of perimeter P (including the base of the arch) is in the form of a rectangle surrounded by a semi-circle. The semi-circular portion is fitted with the colored glass while the rectangular part is fitted with the clear glass that transmits three times as much light per square meter as the colored glass does. What is the ratio for the sides of the rectangle so that the window transmits the maximum light?

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129. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S. If a, b, c and d denote the lengths of sides of the quadrilateral, prove that $2 \leq a_2 + b_2 + c_2 + d_2 \leq 4$

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131. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

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132. Let $A(p^2, -p), B(q^2, q), C(r^2, -r)$ be the vertices of triangle ABC. A parallelogram AFDE is drawn with D,E, and F on the line segments BC, CA and AB, respectively. Using calculus, show that the maximum area of such a parallelogram is $\frac{1}{2}(p+q)(q+r)(p-r)$.

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133. LL' is the latus rectum of the parabola y^2= 4ax and PP' is a double ordinate drawn between the vertex and the latus rectum. Show that the area of the trapezium $PP'LL'$ is maximum when the distance PP' from the vertex is $a/9$.

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134. The circle $x^2 + y^2 = 1$ cuts the x-axis at $PandQ\cdot$ Another circle with center at Q and variable radius intersects the first circle at R above the x-axis and the line segment PQ at S. Find the maximum area of triangle QSR.

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135. Find the intervals in which $f(x) = (x-1)^3(x-2)^2$ is increasing or decreasing.

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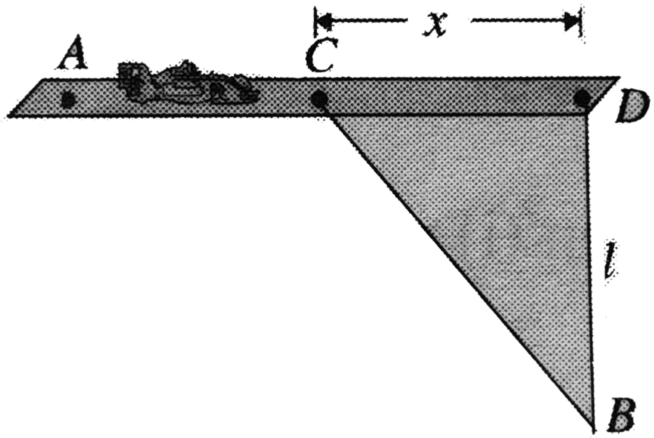
136. From point A located on a highway as shown in figure, one has to get by car as soon as possible to point B located in the field at a distance l from the highway. It is known that the car moves in the field time slower on the highway. At what distance from point D one must turn off the highway?



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137. The function $f(x) = x^2 - x + 1$ is increasing and decreasing in the intervals

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138. A boat moves relative to water with a velocity with a velocity v is n times less than the river flow u. At what angle to the stream direction must the boat move to minimize drifting ?

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139. Consider a square with vertices at $(1, 1), (-1, 1), (-1, -1), and (1, -1)$. Set S be the region consisting of all points inside the square which are nearer to the origin than to any edge. Sketch the region S and find its area.

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140. The interval on which the function f(x) $= 2x^3 + 9x^2 + 12x - 1$ is decreasing is

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EXAMPLE

1. Discuss the nature of following functions, graphically.

A. $f(x) = x^3$

B. $f(x) = \frac{1}{|x|}$

C. $f(x) = e^x$

D. $f(x) = [x]$

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2. Let $f(x) = x^3$ find the point at which f(x) assumes local maximum and local minimum.

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3. If $x^2 + y^2 + z^2 = 1 \text{for} \quad x, y, z \in R,$ then the maximum value of $x^3 + y^3 + z^3 - 3xyz$ is

A. $\frac{1}{2}$

B. 1

C. 2

D. 3

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4. A solid cylinder of height H has a conical portion of same height and radius $1/3rd$ of height removed from it.

Rain water is accumulating in it, at the rate equal to π times the instaneous radius of the water surface inside the hole, the time after which hole will filled with water is

A. $\frac{H^2}{3}$

B. H^2

C. $\dfrac{H^2}{6}$

D. $\dfrac{H^2}{4}$

Answer: c

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5. Statement 1: $f(x) = x + \cos x$ is increasing $\forall x \in R$. Statement 2: If $f(x)$ is increasing, then $f'(x)$ may vanish at some finite number of points.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statemetn I is true, Statement II is also true, Statement II is not correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

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6. Consider a ΔOAB formed by the point $O(0,0), A(2,0), B\left(1, \sqrt{3}\right)$. $P(x, y)$ is an arbitrary interior point of triangle moving in such a way that $d(P, OA) + d(P, AB) + d(P, OB) = \sqrt{3}$, where $d(P, OA), d(P, AB), d(P, OB)$ represent the distance of P from the sides OA,AB and OB respectively

Area of region reperesenting all possible position of point P is equal to

A. $2\sqrt{3}$

B. $\sqrt{6}$

C. $\sqrt{3}$

D. None of these

Answer: A

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7. Let $f(x) = ax^2 + bx + c, ab, c \in R$. It is given $|f(x)| \le 1, |x| \le 1$

The possible value of $|a + c|$, $\quad$ if $\frac{8}{3}a^2 + 2ab^2$ is maximum is given by

A. 1

B. 0

C. 2

D. 3

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8. Let $f(x) = ax^2 + bx + c, ab, c \in R$. It is given $|f(x)| \le 1, |x| \le 1$

The possible value of $|a + c|$, $\quad$ if $\frac{8}{3}a^2 + 2ab^2$ is maximum is given by

A. 32

B. $\frac{32}{3}$

C. $\frac{2}{3}$

D. $\frac{16}{3}$

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9. The absolute maximum and minimum values of functions can be found by their monotonic and asymptotic behaviour provided they exist. We may agree that finite limits values may be reagarded as absolute maximum or minimum. For instance the absolute maximum value of $\frac{1}{1+x^2}$ is unity. It is attained at x=0 while absolute maximum value of the same functions is zero which is a limiting value of $(x \to \infty \text{ or } x \to -\infty)$

The function $x^4 - 4x + 1$ will have.

A. have absolute maximum value $-\frac{1}{2}$

B. has absolute minimum value $-\frac{25}{2}$

C. not lie between $-\frac{25}{2}$ and $-\frac{1}{2}$

D. always be negative

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10. The absolute maximum and minimum values of functions can be found by their monotonic and asymptotic behaviour provided they exist. We may agree that finite limits values may be reagarded as absolute maximum or minimum. For instance the absolute maximum value of $\frac{1}{1+x^2}$ is unity. It is attained at x=0 while absolute maximum value of the same functions is zero which is a limiting value of $(x \to \infty \text{ or } x \to -\infty)$

The function $x^4 - 4x + 1$ will have.

A. cot(sinx)

B. tan(logx)

C. $x^{2005} - x^{1947} + 1$

D. $x^{2006} + x^{1947} + 1$

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11. Let $f(x)=\begin{cases}\max\{t^3-t^2+t+1, 0\le t\le x\}, & 0\le x\le 1\\ \min\{3-t, 1<t\le x\}, & 1<x\le 2\end{cases}$ and

$g(x)=\begin{cases}\max\{3/8t^4+1/2t^3-3/2t^2+1, 0\le t\le x\}, 0\le x\le 1\\ \min\{3/8t+1/32\sin^2\pi t+5/8, 1\le t\le x\}1, \le x\le 2\end{cases}$

The function $f(x), \forall x\in[0,2]$ is

A. continuous and differentiable

B. continuous but not differentiable

C. discontinuous

D. None of the above

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12. The graph of derivative of a function f(x) is given (i.e. y=f'(x)). Analyse the graph in the given domain and answer the following questions , if it is given that f(0)=0

The function f(x) for $-a \leq x \leq a,$ is

A. a) always decreasing

B. b) always increasing

C. c) increasing for (-a,0) and decreasing for (0,a)

D. d) increasing for (0,a) and decreasing for (-a,0)

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13. If a function (continuos and twice differentiable) is always concave upward in an interval, then its graph lies always below the segment joining extremities of the graph in that interval and vice-versa.

Let $f: R^+ \to R^+$ is such that $f(x) \geq 0 \forall x \in [a, b]$. Then value of $\int_a^b f(x)dx$ cannot exceed:

A. $\dfrac{(f(a) + f(b))(b-a)}{3}$

B. $\dfrac{(f(b) - f(a))(b-a)}{2}$

C. $\dfrac{(f(b) + f(a))(b-a)}{2}$

D. None of the above

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14. If $f(x) = \left\{\frac{1}{x}\right\}$ and $g(x) = \{x^2\}$, then the number of positive roots satisfying the equations f(x)=g(x) such that $2 < x^2 < 3$

A. 1

B. 0

C. 3

D. 2

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15. Match the Statements of Column I with values of Column II.

	Column I		Column II
(A)	The sides of a triangle vary slightly in such a way that its circumradius remains constant, if $\frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C}+1=\lvert m\rvert$, then the value of m is	(p)	1
(B)	If the length of subtangent to the curve $x^2y^2=16$ at the point $(-2, 2)$ is $\lvert k\rvert$, then the value of k is	(q)	-1
(C)	If the curve $y=2e^{2x}$ intersects the Y-axis at an angle $\cot^{-1}\lvert(8n-4)/3\rvert$, then the value of n is	(r)	2
(D)	If the area of a triangle formed by normal at the point $(1, 0)$ on the curve $x=e^{\sin y}$ with axes is $\lvert 2t+1\rvert/6$ sq units, then the value of t is	(s)	-2

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16. Match the Statements of Column I with values of Column II.

	Column I		Column II
(A)	The sides of a triangle vary slightly in such a way that its circumradius remains constant, if $\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} + 1 = \lvert m \rvert$, then the value of m is	(p)	1
(B)	If the length of subtangent to the curve $x^2 y^2 = 16$ at the point $(-2, 2)$ is $\lvert k \rvert$, then the value of k is	(q)	-1
(C)	If the curve $y = 2e^{2x}$ intersects the Y-axis at an angle $\cot^{-1} \lvert (8n-4)/3 \rvert$, then the value of n is	(r)	2
(D)	If the area of a triangle formed by normal at the point $(1, 0)$ on the curve $x = e^{\sin y}$ with axes is $\lvert 2t+1 \rvert/6$ sq units, then the value of t is	(s)	-2

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17. The set of all points where $f(x)$ is increasing is $(a, b) \cup (c, \infty)$. Find $[a + b + c]$ (where [.] denotes the greatest integre function) given that

$$f(x) = 2f\left(\frac{x^2}{2}\right) + f(6 - x^2),\ \forall x \in R$$

and $f''(x) > 0,\ \forall x \in R$.

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18. Let $f(x)$ be a cubic polynomial defined by

$f(x) = \frac{x^3}{3} + (a-3)x^2 + x - 13$. Then the sum of all possible values(s) of a for which $f(x)$ has negative point of local minimum in the interval $[1, 5]$ is

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19. If $f(x) = \max |2\sin y - x|$, (where $y \in R$), then find the minimum value of f(x).

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20. Let $f(x) = \sin^{-1}\left(\frac{2\phi(x)}{1+\phi^2(x)}\right)$. Find the interval in which $f(x)$ is increasing or decreasing.

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21. Find the minimum value of

$$f(x) = |x+2| + |x-2| + |x|.$$

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22. The interval to which b may belong so that the functions.

$$f(x) = \left(1 - \frac{\sqrt{21 - 4b - b^2}}{b+1}\right)x^3 + 5x + \sqrt{16},$$

increases for all x.

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23. One corner of a long rectangular sheet of paper of width 1 unit is folded over so as to reach the opposite edge of the sheet. Find the minimum length of the crease.

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Exercise For Session 1

1. The curvey y=f(x) which satisfies the condition $f'(x) > 0$ and $f''(x) < 0$ for all real x, is

A.

B.

C.

D.

Answer: D

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2. The interval in which $f(x) = \cot^{-1} x + x$ increases , is

A. a) R

B. b) $(0, \infty)$

C. c) $R - \{n\pi\}$

D. d) None of these

Answer: C

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3. The interval in which $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3$ increases or decreases in $(0, \pi)$

A. decreases on $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and increases on $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$

B. decreases on $\left(\frac{\pi}{2}, \pi\right)$and increases on $\left(0, \frac{\pi}{2}\right)$

C. decreases on $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$ and increases on $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

D. decreases on $\left(0, \frac{\pi}{2}\right)$ and increases on $\left(\frac{\pi}{2}, \pi\right)$

Answer: C

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4. The interval in which $f(x) = \int_0^x \{(t+1)(e^t - 1)(t-2)(t-4)\}$ dt increases and decreases

A. increases on $(-\infty, -4) \cup (-10) \cup (2, \infty)$ and decreases on $(-4, -1) \cup (0, 2)$

B. increases on $(-\infty, -4) \cup (-12)$ and decreases on $(-4, -1) \cup (2, \infty)$

C. increases on $(-\infty, -4) \cup (2, \infty)$ and decreases on $(-4, 2)$

D. increases on $(-4, -1) \cup (0, 2)$ and decreases on $(-\infty, -4) \cup (-10) \cup (2, \infty)$

Answer: A

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5. The interval of monotonicity of the function $f(x) = \frac{x}{\log_e x}$, is

A. a) increases when $x \in (e, \infty)$ and decreases when $x \in (0, e)$

B. b) increases when $x \in (e, \infty)$ and decreases when $x \in (0, e) - \{1\}$

C. c) increases when $x \in (0, e)$ and decreases when $x \in (e, \infty)$

D. d) increases when $x \in (0, e) - \{1\}$ and decreases when $x \in (e, \infty)$

Answer: B

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6. Let $f(x) = x^3 + ax^2 + bx + 5\sin^2 x$ be an increasing function on the set R. Then find the condition on a and b.

A. $a^2 - 3b + 15 > 0$

B. $a^2 - 3b + 5 < 0$

C. $a^2 - 3b + 15 < 0$

D. $a^2 - 3b + 5 > 0$

Answer: C

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7. Let $g(x) = f(x) + f(1-x)$ and $f''(x) < 0$, when $x \in (0, 1)$. Then $f(x)$ is

A. a) increasing on $\left(0, \frac{1}{2}\right)$ and decreasing on $\left(\frac{1}{2}, 1\right)$

B. b) increasing on $\left(\frac{1}{2}, 1\right)$ and decreasing on $\left(0, \frac{1}{2}\right)$

C. c) increasing on $(0, 1)$

D. d) decreasing on $(0, 1)$

Answer: B

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Exercise For Session 2

1. Determine all the critical points for the function $f(x) = 6x^5 + 33x^4 - 30x^3 + 100$

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2. Find the critical points of $f(x) = x^{2/3}(2x - 1)$

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3. Determine all the critical points for the function : $f(x) = xe^{x^2}$

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4. The number of critical points of $f(x) = \max\{\sin x, \cos x\}, \forall x \in (-2\pi, 2\pi)$, is

A. 5

B. 6

C. 7

D. 8

Answer: C

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Exercise For Session 3

1. Show that $\sin x < x < \tan x$ for $0 < x < \pi/2$.

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2. prove that $\frac{x}{1+x} < \log(1+x) < x$, for all $x > 0$

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3. Show that : $x - \frac{x^3}{6} < \sin x$ for $0 < x < \frac{\pi}{2}$

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4. If $ax^2 + \frac{b}{x} \geq c$ for all positive x where $a > 0$ and $b > 0$, show that $27ab^2 \geq 4c^3$.

A. $27ab^2 \geq 4c^3$

B. $27ab^2 < 4c^3$

C. $4ab^2 \geq 27c^3$

D. None of these

Answer: A

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5. If $ax + \frac{b}{x} \geq c$ for all positive x where $a, b, > 0$, then $ab < \frac{c^2}{4}$ (b) $\geq \frac{c^2}{4}$ (c) $ab \geq \frac{c}{4}$ (d) none of these

A. $ab < \frac{c^2}{4}$

B. $ab \geq \frac{c^2}{4}$

C. $ab \geq \frac{c}{4}$

D. None of these

Answer: B

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Exercise For Session 4

1. The minimum value of x^x is attained when x is equal to

A. e

B. e^{-1}

C. 1

D. e^2

Answer: B

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2. The function $'f'$ is defined by $f(x) = x^p(1-x)^q$ for all $x \in R$, where $p, \ q$ are positive integers, has a maximum value, for x equal to : $\frac{pq}{p+q}$ (b) 1 (c) 0 (d) $\frac{p}{p+q}$

A. $\frac{pq}{p+q}$

B. 1

C. 0

D. $\frac{p}{p+q}$

Answer: D

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3. The least area of a circle circumscribing any right triangle of area S is:

A. πS

B. $2\pi S$

C. $\sqrt{2}\pi S$

D. $4\pi S$

Answer: A

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4. The coordinate of the point on the curve $x^2 = 4y$ which is atleast distance from the line y=x-4 is

A. (a)(2,1)

B. (b)(-2,1)

C. (c)(-2,-1)

D. (d)None of these

Answer: A

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5. The largest area of a rectangle which has one side on the x-axis and the two vertices on the curve $y = e^{-x^2}$ is

A. $\sqrt{2}e^{-1/2}$

B. $2e^{-1/2}$

C. $e^{-1/2}$

D. None of these

Answer: A

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6. Let $f(x) = \log(2x - x^2) + \frac{\sin(\pi x)}{2}$. Then which of the following is/are true? Graph of f is symmetrical about the line $x = 1$ Maximum value of $fis1$. Absolute minimum value of f does not exist. none of these

A. gaph of f is symmetrical about the line x=1

B. graph of f is symmetrical about the line x=2

C. minimum value of f is 1

D. minimum value of f does not exist

Answer: D

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7. The sum of the legs of a right triangle is 9 cm. When the triangle rotates about one of the legs, a cone result which has the maximum volume. Then

A. (a)slant heigth of such a cone is $3\sqrt{5}$

B. (b)maximum value of the cone is 32π

C. (c)curved surface of the cone is $18\sqrt{5}\pi$

D. (d)semi vertical angle of cone is $\tan^{-1}\sqrt{2}$

Answer: A::C

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8. Least value of the function , $f(x)=2^{x^2}-1+\frac{2}{2^{x^2}+1}$ is :

A. 0

B. $\frac{3}{2}$

C. $\frac{2}{3}$

D. 1

Answer: D

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9. The greatest and the least value of the function, $f(x)=\sqrt{1-2x+x^2}-\sqrt{1+2x+x^2}, x\in(-\infty,\infty)$ are

A. 2,-2

B. 2,-1

C. 2,0

D. none

Answer: A

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10. The minimum value of the polynimial $x(x+1)(x+2)(x+3)$ is

A. a) 0

B. b) $\frac{9}{16}$

C. c) -1

D. d) $-\frac{3}{2}$

Answer: C

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11. The difference between the greatest and least value of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$ is

A. $\frac{4}{3}$

B. 1

C. $\frac{9}{4}$

D. $\frac{1}{6}$

Answer: C

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12. Find the point at which the slope of the tangent of the function $f(x) = e^x \cos x$ attains maxima, when $x \in [-\pi, \pi]$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. π

Answer: D

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13. If λ, μ are real numbers such that , $x^3 - \lambda x^2 + \mu x - 6 = 0$ has its real roots and positive, then the minimum value of $\mu,$ is

A. a) $3(6)^{1/3}$

B. b) $3(6)^{2/3}$

C. c) $(6)^{1/3}$

D. d) $(6)^{2/3}$

Answer: B

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14. Investigate for the maxima and minima of the function

$$f(x)=\int_1^x\left[2(t-1)(t-2)^3+3(t-1)^2(t-2)^2\right]dt$$

A. maximum when $x=\frac{7}{5}$ and minimum when x=1

B. maximum when x=1 and minimum when x=0

C. maximum when x=1 and minimum when x=2

D. maximum when x=1 and minimum when $x=\frac{7}{5}$

Answer: D

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15. The set of value(s) of a for which the function $f(x)=\frac{ax^3}{3}+(a+2)x^2+(a-1)x+2$ possesses a negative point of inflection is (a) $(-\infty,-2)\cup(0,\infty)$ (b) $\left\{-\frac{4}{5}\right\}$ (c) $(-2,0)$ (d) empty set

A. $(-\infty,2)\cup(0,\infty)$

B. $\{-4/5\}$

C. (-2,0)

D. empty set

Answer: A

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Exercise For Session 5

1. Let

$f(x) = \{x^3 - x^2 + 10x - 5, x \leq 1, \quad -2x + (\log)_2(b^2 - 2), x > 1$

Find the values of b for which $f(x)$ has the greatest value at $x = 1$.

A. $1 < b \leq 2$

B. $b = \{12\}$

C. $b \in (-\infty, -1)$

D. $[-\sqrt{130} - \sqrt{2}] \cup (\sqrt{2}, (\sqrt{130})$

Answer: D

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2. Solution(s) of the equation. $3x^2 - 2x^3 = \log_2(x^2 + 1) - \log_2 x$ is/are

A. 1

B. 2

C. 3

D. None of these

Answer: A

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3. Let $f(x) = \cos 2\pi x + x - [x]([\cdot]$ denotes the greatest integer function). Then number of points in $[0, 10]$ at which f(x) assumes its local maximum value, is

A. (a)0

B. (b)10

C. (c)9

D. (d)infinite

Answer: B

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4. If $f(x) = |x| + |x - 1| - |x - 2|$, then $f(x)$

A. a) has minima at $x = 1$

B. b) has maxima at $x = 0$

C. c) has neither maxima nor minima at $x = 3$

D. d) none of these

Answer: C

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5. $f(x) = 1 + [\cos x]x, \text{ in } \ 0 < x \leq \frac{\pi}{2}$

A. has a minimum value 0

B. has a maximum value 2

C. is continuos in $\left[0, \frac{\pi}{2}\right]$

D. is not differentiable at $x = \frac{\pi}{2}$

Answer: C: D

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6. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ ([.] denotes the greates integer function) and f(x) is non-constant continuous function, then

A. $\lim_{x \to a} f(x)$ is irrational

B. $\lim_{x \to a}$ f(x) is non-integer

C. f(x) has local maxima at x=a

D. f(x) has local minima at x=a

Answer: D

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7. Find the value of a if $x^3 - 3x + a = 0$ has three distinct real roots.

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8. Prove that there exist exactly two non-similar isosceles triangles ABC such that $\tan A + \tan B + \tan C = 100$.

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Exercise (Single Option Correct Type Questions)

1. If $f:[1, 10] \rightarrow [1, 10]$ is a non-decreasing function and $g:[1, 10] \rightarrow [1, 10]$ is a non-increasing function. Let $h(x) = f(g(x))$ with $h(1) = 1,$ then $h(2)$

A. lies in (1,2)

B. is more than two

C. is equal to one

D. is not defined

Answer: C

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2. P is a variable point on the curve y= f(x) and A is a fixed point in the plane not lying on the curve. If PA^2 is minimum, then the angle between PA and the tangent at P is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. None of these

Answer: C

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3. Let $f(x)\begin{cases} 1+\sin x, & x<0 \\ x^2-x+1\geq 0 \end{cases}$ Then

A. f has a local maximum at x=0

B. f has a local minimum at x=0

C. f is increasing everywhere

D. f is decreasing everywhere

Answer: A

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4. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}(t-b)^{2m+1}dt, a \ne b$, then

A. (a)x=b is a point of local minimum

B. (b)x=b is a point of local maximum

C. (c)x=a is a point of local minimum

D. (d)x=a is a point of local maximum.

Answer: A

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5. Find the intervals in which the following function is increasing and decreasing $f(x) = x^2 - 6x + 7$

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6. If f is twice differentiable such that $f''(x) = -f(x)$, $f'(x) = g(x)$, $h'(x) = [f(x)]^2 + [g(x)]^2$ and $h(0) = 2$, $h(1) = 4$, then the equation $y = h(x)$ represents.

A. a) a straight line with slope 2

B. b) a straight line with y-intercept 1

C. c) a straight line with x-intercept 2

D. d) None of the above

Answer: D

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7. If f(x) = $\begin{cases} 2x^2 + \frac{2}{x^2}, & 0 < |x| \leq 2 \\ 3, & x > 2 \end{cases}$ then

A. (a) $x = 1, -1$ are the points of global minima

B. (b) x=1,-1 are the points of local minima

C. (c) x=0 is the point of local minima

D. (d) $x = 0$ is the point of local minimum

Answer: B

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8. $\sin x + \cos x = y^2 - y + a$ has no value of x for any value of y if a belongs to (a) $(0, \sqrt{3})$ (b) $(-\sqrt{3}, 0)$ (c) $(-\infty, -\sqrt{3})$ (d) $(\sqrt{3}, \infty)$

A. $(0, \sqrt{3})$

B. $(-\sqrt{3}, 0)$

C. $(-\infty, -\sqrt{3})$

D. $(\sqrt{3}, \infty)$

Answer: D

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9. $f: R \to R$ is defined by $f(x) = \dfrac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ is :

A. f(x) is an increasing function

B. f(x) is a decreasing function

C. f(x) is a onto

D. None of the above

Answer: D

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10. Suppose that $f(x)$ is a quadratic expresson positive for all real x. If $g(x) = f(x) + f'(x) + f''(x)$, then for any real $x (where f'(x) and f''(x)$ represent 1st and 2nd derivative, respectively).

a. $g(x) < 0$ b. $g(x) > 0$ c. $g(x) = 0$ d. $g(x) \geq 0$

A. $g(x) > 0$

B. $g(x) \leq 0$

C. $g(x) \geq 0$

D. $g(x) < 0$

Answer: A

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11. Let $f(x) = \min\{1, \cos x, 1 - \sin x\}, \; -\pi \leq x \leq \pi$, Then, f(x) is

A. f(x) is differentiable at 0

B. f(x) is differentiable at $\frac{\pi}{2}$

C. f(x) has local maxima at=0

D. none of the above

Answer: A B

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12.

$f(x) = \{2 - |x^2 + 5x + 6|, x \neq 2a^2 + 1, x = -2 Thentheran \geq ofa,$

so that $f(x)$ has maxima at $x = -2,$ is $|a| \geq 1$ (b) $|a| < 1$ $a > 1$ (d) $a < 1$

A. $|a| \geq 1$

B. $|a| < 1$

C. $a > 1$

D. $a < 1$

Answer: A

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13. Maximum number of real solution for the equation $ax^n + x^2 + bx + c = 0$, where $a, b, c \in R$ and n is an even positive number, is

A. 2

B. 3

C. 4

D. infinite

Answer: D

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14. Maximum number area of rectangle whose two sides are $x = x_0, x = \pi - x_0$ and which is inscribed in a region bounded by y=sin x and X-axis is obtained when $x_0 \in$

A. $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

B. $\left(\frac{\pi - 1}{2}, \frac{\pi}{2}\right)$

C. $\left(o, \frac{\pi}{6}\right)$

D. None of these

Answer: B

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15. f(x)=-1+kx+k neither touches nor intecepts the curve f(x)= log x, then minimum value of k $\in$

A. $\left(\frac{1}{e}, \frac{1}{\sqrt{e}}\right)$

B. (e, e^2)

C. $\left(\frac{1}{\sqrt{e}}, e\right)$

D. None of these

Answer: A

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16. f(x) is polynomial of degree 4 with real coefficients such that f(x)=0 satisfied by x=1, 2, 3 only then $f'(1)f'(2)f'(3)$ is equal to -

A. positive

B. negative

C. 0

D. inadequate data

Answer: C

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17. A curve whose concavity is directly proportional to the logarithm of its x-coordinates at any of the curve, is given by

A. $c_1 . x^2(2\log x - 3) + c_2 x + c_3$

B. $c_1 x^2(2\log x + 3) + c_2 x + c_3$

C. $c_1 x^2(2\log x) + c_2$

D. none of the above

Answer: A

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18. $f(x) = 4\tan x - \tan^2 x + \tan^3 x, x \neq n\pi + \dfrac{\pi}{2}$

A. a) $f(x)$ is increasing for all $x \in R$

B. b) $f(x)$ is decreasing for all $x \in R$

C. c) $f(x)$ is increasing in its domain

D. d) none of the above

Answer: C

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19. $f(x) = \begin{cases} 3 + |x-k|, & x \leq k \\ a^2 - 2 + \dfrac{\sin(x-k)}{(x-k)}, & x > k \end{cases}$ has minimum at $x = k$, then:

A. $a \in R$

B. $|a| < 2$

C. $|a| > 2$

D. $1 < |a| < 2$

Answer: C

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20. Let f(x) be linear functions with the properties that $f(1) \leq f(2), f(3) \geq f(4)$ and $f(5) = 5$. Which one of the following statements is true?

A. $f(0) < 0$

B. $f(0) = 0$

C. $f(1) < f(0) < f(-1)$

D. $f(0) = 5$

Answer: D

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21. If P(x) is polynomial satisfying $P(x^2) = x^2 P(x)$ and $P(0) = -2, P'(3/2) = 0$ and $P(1) = 0$.

The maximum value of P(x) is

A. (a) $-\frac{1}{3}$

B. (b) $\frac{1}{4}$

C. (c) $-\frac{1}{2}$

D. (d) none of the above

Answer: B

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22. Find the vertex and length of latus rectum of the parabola $x^2 = -4(y-a)$.

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23. Let $f(x) = x^2 - 2x$ and $g(x) = f(f(x) - 1) + f(5 - f(x))$, then

A. $g(x) < 0, \forall x \in R$

B. $g(x) < 0, \text{for some } x \in R$

C. $g(x) \geq 0, \text{for some } x \in R$

D. $g(x) \geq 0, \forall x \in R$

Answer: D

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24. Let $f: N \to N$ in such that $f(n+1) > f(f(n))$ for all $n \in N$ then

A. (a) $f(n) = n^2 - n + 1$

B. (b) $f(n) = n - 1$

C. (c) $f(n) = n^2 + 1$

D. (d) none of the above

Answer: D

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25. The equation $|2ax-3|+|ax+1|+|5-ax|=\frac{1}{2}$ possesses

A. (a) infinite number of real solution for some $a \in R$

B. (b) finitely many real solution for some $a \in R$

C. (c) no real solutions for some $a \in R$

D. (d) no real solutions for all $a \in R$

Answer: D

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26. Let a_1, a_2, a_n be sequence of real numbers with $a_{n+1}=a_n+\sqrt{1+a_n^2}$ and $a_0=0$. Prove that $\lim_{x \to \infty}\left(\frac{a_n}{2^{n-1}}\right)=\frac{2}{\pi}$

A. $\pi / 4$

B. $4 / \pi$

C. π

D. $\pi / 2$

Answer: B

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27. A function f is defined by $f(x) = |x|^m |x-1|^n \forall x \in R$. The local maximum value of the function is $(m, n \in N)$, 1 (b) $m^{\cap}$ ˆ m $\frac{m^m n^n}{(m+n)^{m+n}}$ (d) $\frac{(mn)^{mn}}{(m+n)^{m+n}}$

A. 1

B. $m^n . n^m$

C. $\frac{m^m . n^n}{(m+n)^{m+n}}$

D. $\frac{(mn)^{mn}}{(m+n)^{m+n}}$

Answer: C

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Exercise (More Than One Correct Option Type Questions)

1. Which of the following is/are true?

(you may use $f(x) = \dfrac{\ln((\ln x))}{\ln x}$

A. $(In2.1)^{In2.2} > (In2.2)^{In2.1}$

B. $(In4)^{In5} > (In5)^{In4}$

C. $(In30)^{In31} > (In31)^{In30}$

D. $(In28)^{30} < (In30)^{In28}$

Answer: B::C

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2. If $\lim_{x \to a} f(x) = \lim_{x \to a} [f(x)]$ ([.] denotes the greates integer function) and f(x) is non-constant continuous function, then

A. $\lim_{x \to a} f(x)$ is an integer

B. $\lim_{x \to a} f(x)$ is non-integer

C. f(x) has local maximum at x=a

D. f(x) has local minimum at x=a

Answer: A::D

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3. Let S be the set of real values of parameter λ for which the equation f(x) = $2x^3 - 3(2+\lambda)x^2 + 12\lambda$ x has exactly one local maximum and exactly one local minimum. Then S is a subset of

A. $(-4, \infty)$

B. $(-3, 3)$

C. $(3, 8)$

D. $(-\infty, -1)$

Answer: C::D

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4. $h(x) = 3f\left(\frac{x^2}{3}\right) + f(3 - x^2)\ \forall x \in (-3, 4)$ where $f''(x) > 0\ \forall x \in (-3, 4)$, then $h(x)$ is

A. a) increasing in $\left(\frac{3}{2}, 4\right)$

B. b) increasing in $\left(-\frac{3}{2}, 0\right)$

C. c) decreasing in $\left(-3, -\frac{3}{2}\right)$

D. d) decreasing in $\left(0, \frac{3}{2}\right)$

Answer: A::B::C::D

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5. Let $f(x) = \log(2x - x^2) + \frac{\sin(\pi x)}{2}$. Then which of the following is/are true? Graph of f is symmetrical about the line $x = 1$ Maximum value of $fis1$. Absolute minimum value of f does not exist. none of these

A. graph of f is symmetrical about the line x=1

B. graph of f is symmetrical about the line x=2

C. maximum value of f is 1

D. minimum value of f does not exist

Answer: A::C::D

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6. Show that f(x) $= \tan^{-1}(\sin x + \cos x)$ is an increasing function in $\left(0, \frac{\pi}{4}\right)$.

A. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

B. $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

C. $\left(\frac{5\pi}{4}, \frac{3\pi}{2}\right)$

D. $\left(-2\pi, -\frac{7\pi}{4}\right)$

Answer: A::B::C::D

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7. If the maximum and minimum values of the determinant

$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α and β, then

A. $\alpha + \beta^{99} = 4$

B. $\alpha^3 - \beta^{17} = 26$

C. $(\alpha^{2n} - \beta^{2n})$ is always an even integer for $n \in N$

D. a triangle can be drawn having it's sides as α, β and $\alpha - \beta$

Answer: A::B::C

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8. Let $f(x) = \begin{cases} x^2 + 4x, & -3 \leq x \leq 0 \\ -\sin x, & 0 < x \leq \frac{\pi}{2} \\ -\cos x - 1, & \frac{\pi}{2} < x \leq \pi \end{cases}$ then

A. x=-2 is the point of global minima

B. x=π is the point of global maxima

C. f(x) is non-differentiable at $x = \frac{\pi}{2}$

D. f(x) is dicontinuos at x=0

Answer: A::B::C

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9. Let $f(x) = ab\sin x + b\sqrt{1 - a^2}\cos x + c$, where $|a| < 1, b > 0$ then

A. maximum value of f(x) is b, if c = 0

B. difference of maximum and minimum value of f(x) is 2b

C. $f(x) = c, \quad \text{if} \quad x = -\cos^{-1} a$

D. $f(x) = c, \quad \text{if} \quad x = \cos^{-1} a$

Answer: A::B::C

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10. If $f(x) = \int_{x^m}^{x^n} \frac{dt}{\ln t}, x > 0 \text{ and } n > m,$ then

A. $f'(x) = \frac{x^{m-1}(x-1)}{\ln x}$

B. f(x) is decreasing for $x > 1$

C. f(x) is increasing in (0,1)

D. f(x) is increasing for $x > 1$

Answer: C::D

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11. $f(x) = \sqrt{x-1} + \sqrt{2-x}$ and $g(x) = x^2 + bx + c$ are two given functions such that $f(x)$ and $g(x)$ attain their maximum and minimum values respectively for same value of x, then

A. a) $f(x)$ extreme point at $x = \frac{1}{2}$

B. b) $f(x)$ extreme point at $x = \frac{3}{2}$

C. c) $b = 3$

D. d) $b = -3$

Answer: B::D

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12. Find the intervals in which $f(x) = 6x^2 - 24x + 1$ increases and decreases

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13. A function f is defined by $f(x) = \int_0^\pi \cos t \cos(x - t) dt, 0 \leq x \leq 2\pi$ then the minimum value of $f(x)$ is

A. f(x) is continuos but not differentiable in $(0, 2\pi)$

B. Maximum value of f is π

C. There exists atleast one $c \in (0, 2\pi) \quad \text{if} \quad f'(c) = 0$

D. Minimum value of f is $-\frac{\pi}{2}$

Answer: A::B

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14. Let $f(x)$ be a differentiable function on the interval $(-\infty, 0)$ such that $f(1) = 5$ and $\lim_{a \to x} \frac{af(x) - xf(a)}{a - x} = 2, \forall x \in R.$

Then which of the following alternatives is/are correct?

A. $f(x)$ has an inflection point

B. $f'(x) = 3, \forall x \in R$

C. $\int_0^2 f(x)dx = -10$

D. Area bounded by $f(x)$ with coordinate axes is $\frac{2}{3}$

Answer: B::C::D

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15. If $f: R \to R,$ f (x) is differentiable bijective function then which of the following is ture ?

A. $(f(x) - x)f''(x) < 0, \ \forall x \in R$

B. $(f(x) - x)f''(x) > 0, \ \forall x \in R$

C. If $f(x) - x)f''(x) > 0, then f(x) = f^{-1}(x)$ has no solution

D. If $f(x) - x)f''(x) > 0, then f(x) = f^{-1}(x)$ has atleast a real solution

Answer: B::C

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16. Let $f: (0, \infty) \to (0, \infty)$ be a derivable function and F(x) is the primitive of f(x) such that $2(F(x) - f(x)) = f^2(x)$ for any real positive x

A. f is strictly increasing

B. $\lim_{x \to \infty} \frac{f(x)}{x} = 1$

C. f is strictly decreasing

D. f is non-monotonic

Answer: A::B

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Exercise (Statement I And Ii Type Questions)

1. Statement I The equation $3x^2 + 4ax + b = 0$ has atleast one root in (o,1), if 3+4a=0.

Statement II $f(x) = 3x^2 + 4x + b$ is continuos and differentiable in (0,1)

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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2. Statement I For the function

$f(x) = \begin{cases} 15 - x & x < 2 \\ 2x - 3 & x \geq 2 \end{cases} x = 2$ has neither a maximum nor a minimum point.

Statament II ff'(x) does not exist at x=2.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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3. Statement I $\phi(x) = \int_0^x (3\sin t + 4\cos t)dt, \left[\frac{\pi}{6}, \frac{\pi}{3}\right] \phi(x) -$ attains its maximum value at $x = \frac{\pi}{3}$.

Statement II $\phi(x) = \int_0^x (3\sin t + 4\cos t)dt, \phi(x)$ is increasing function in $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: A

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4. Let f(x) a twice differentiable function in [a,b], given that f(x) and f''(x) has same sign in [a,b].

Statement I f'(x)=0 has at the most real root in [a,b].

Statement II An increasing function can intersect the X-axis at the most once.

A. (a)Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. (b)Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. (c)Statement I is true, Statement II is false

D. (d)Statement I is false, Statement II is true

Answer: A

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5. Let $u = \sqrt{c+1} - \sqrt{c}$ and $v = \sqrt{c} - \sqrt{c-1}, c > 1$and let $f(x) = In(1+x), \forall x \in (-1, \infty)$.

Statement I $f(u) > f(v), \forall c > 1$ because

Statement II f(x) is increasing ffunction, hence for $u > v, f(u) > f(v)$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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6. Let $f(0) = 0, f\left(\frac{\pi}{2}\right) = 1, f\left(\frac{3\pi}{2}\right) = -1$ be a continuos and twice differentiable function.

Statement I $|f''(x)| \leq 1$ for atleast one $x \in \left(0, \frac{3\pi}{2}\right)$ because

Statement II According to Rolle's theorem, if y=g(x) is continuos and differentiable, $\forall x \in [a, b]$ and $g(a) = g(b)$, then there exists atleast one such that g'(c)=0.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: A

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7. Statement I For any ΔABC.

$$\sin\left(\frac{A+B+C}{3}\right) \geq \frac{\sin A+\sin B+\sin C}{3}$$

Statement II y= sin x is concave downward for $x \in (0, \pi]$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: B

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8. $f(x)$ is a polynomial of degree 3 passing through the origin having local extrema at $x = \pm 2$ Statement 1 : Ratio of areas in which $f(x)$ cuts the circle $x^2+y^2=36 is 1:1$. Statement 2 : Both $y=f(x)$ and the circle are symmetric about the origin.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: A

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Exercise (Passage Based Questions)

1. Let $f(x)=\frac{1}{1+x^2}$, let m be the slope, a be the x-intercept and b be they y-intercept of a tangent to y=f(x).

Absicca of the point of contact of the tangent for which m is greatest, is

A. $\frac{1}{\sqrt{3}}$

B. 1

C. -1

D. $-\frac{1}{\sqrt{3}}$

Answer: D

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2. Let $f(x)=\frac{1}{1+x^2}$, let m be the slope, a be the x-intercept and b be they y-intercept of a tangent to y=f(x).

Value of b for the tangent drawn to the curve y=f(x) whose slope is greatest, is

A. $\frac{9}{8}$

B. $\frac{3}{8}$

C. $\frac{1}{8}$

D. $\frac{5}{8}$

Answer: A

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3. Let $f(x)=\frac{1}{1+x^2}$, let m be the slope, a be the x-intercept and b be they y-intercept of a tangent to y=f(x).

Value of a for the tangent drawn to the curve y=f(x) whose slope is greatest, is

A. $-\sqrt{3}$

B. 1

C. -1

D. $\sqrt{3}$

Answer: A

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4. Consider the function $f(x) = \max.$ $\left[x^2 \quad (1-x)^2 \quad 2x(1-x)\right], x \in [0, 1]$ The interval in which f(x) is increasing, is

A. $\left(\frac{1}{3}, \frac{2}{3}\right)$

B. $\left(\frac{1}{3}, \frac{1}{2}\right)$

C. $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

D. $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

Answer: D

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5. Let f(x) = Max. $\left\{x^2, (1-x)^2, 2x(1-x)\right\}$ where $x \in [0, 1]$ If Rolle's theorem is applicable for f(x) on largestpossible interval [a, b] then the value of $2(a+b+c)$ when $c \in [a, b]$ such that f'(c) = 0, is

A. (a) $\frac{2}{3}$

B. (b) $\frac{1}{3}$

C. (c) $\frac{1}{2}$

D. (d) $\frac{3}{2}$

Answer: D

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6. Consider the function $f(x) = \max x^2, \left[(1-x)^2 \quad 2x(1-x)\right], x \in [0, 1]$

The interval in which f(x) is increasing, is

A. $\left(\frac{1}{3}, \frac{2}{3}\right)$

B. $\left(\frac{1}{3}, \frac{1}{2}\right)$

C. $\left(0, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

D. $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

Answer: C

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7. $f(x), g(x)$ and $h(x)$ are all continuous and differentiable functions in $[a, b]$. Also $a < c < b$ and $f(a) = g(a) = h(a)$. Point of intersection of the tangent at $x = c$ with chord joining $x = a$ and $x = b$ is on the left of c in $y = f(x)$ and on the right in $y = h(x)$. And tangent at $x = c$ is parallel to the chord in case of $y = g(x)$. Now answer the following questions.

If $c = \dfrac{a+b}{2}$ for each b, then:

A. $g(x) = Ax^2 + Bx + c$

B. $g(x) = \log x$

C. $g(x) = \sin x$

D. $g(x) = e^x$

Answer: A

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8. In the non-decreasing sequence of odd integers $(a_1, a_2, a_3, \ldots.) = \{1, 3, 3, 3, 5, 5, 5, 5, 5\ldots.\}$ each positive odd integer k appears k times. It is a fact that there are integers b,c and d such that for all positive integer n, $a_n = b[\sqrt{n+c}] + d$ (where [.] denotes greatest integer function). The possible vaue of b+c+d is

A. (a)0

B. (b)1

C. (c)2

D. (d)4

Answer: C

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9. In the non-decreasing sequence of odd integers $(a_1, a_2, a_3, \ldots.) = \{1, 3, 3, 3, 5, 5, 5, 5, 5\ldots.\}$ each positive odd integer k appears k times. It is a fact that there are integers b,c and d such that for all positive integer n, $a_n = b[\sqrt{n+c}] + d$ (where [.] denotes greatest integer function). The possible value of $\frac{b-2d}{8}$ is

A. (a)0

B. (b)1

C. (c)2

D. (d)4

Answer: A

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10. In the non-decreasing sequence of odd integers $(a_1, a_2, a_3, \dots) = \{1, 3, 3, 3, 5, 5, 5, 5, 5 \dots\}$ each positive odd integer k appears k times. It is a fact that there are integers b,c and d such that for all positive integer n, $a_n = b[\sqrt{n+c}] + d$ (where [.] denotes greatest integer function). The possible value of $\frac{c+d}{2b}$ is

A. (a)0

B. (b)1

C. (c)2

D. (d)4

Answer: A

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11. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, f(x) has its non-zero local minimum and maximum values at -3 and 3, respectively. If $a_3 \in$ the domain of the function

$$h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right).$$

The value of $a_1 + a_2$ is

A. 30

B. -30

C. 27

D. -27

Answer: D

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12. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, f(x) has its non-zero local minimum and maximum values at -3 and 3, respectively. If $a_3 \in$ the domain of the function

$h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$.

The value of a_0 is

A. equal to 50

B. greater than 54

C. less than 54

D. less than 50

Answer: B

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13. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, f(x) has its non-zero local minimum and maximum values at -3 and 3, respectively. If $a_3 \in$ the domain of the function

$$h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right).$$

The value of a_0 is

A. $a_0 > 730$

B. $a_0 > 830$

C. $a_0 = 830$

D. none of the above

Answer: A

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14. f: $D \to R$, $f(x) =$ (x^2 +bx+c)/$\left(x^2 + b_1x + c_1\right)$ whereα, β are the roots of the equation $x^2 + bx + c = 0$ and α_1, β_1 are the roots of $x^2 + b_1x + c_1 = 0$. Now answer the following questions for f(x). A combination of graphical and analytical approach may be helpful in solving these problems. (If α_1 and β_1 are real, then f(x) has vertical asymptote at x = α_1, β_1

A. f(x) is increasing in (α_1, β_1)

B. f(x) is decreasing in (α, β)

C. f(x) is decreasing in (β_1, β)

D. f(x) is decreasing in $(-\infty, \alpha)$

Answer: A

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15. f: $D \to R, f(x) =$ (x^2 +bx+c)/$\left(x^2 + b_1 x + c_1\right)$ whereα, β are the roots of the equation $x^2 + bx + c = 0$ and α_1, β_1 are the roots of $x^2 + b_1 x + c_1 = 0$. Now answer the following questions for f(x). A combination of graphical and analytical approach may be helpful in solving these problems. (If α_1 and β_1 are real, then f(x) has vertical asymptote at x = α_1, β_1

A. f(x) has a maxima in $[\alpha_1, \beta_1]$ and a minima is $[\alpha, \beta]$

B. f(x) has a minima in (α_1, β_1) and a maxima in (α, β)

C. $f'(x) > 0$ where ever defined

D. $f'(x) < 0$ where ever defined

Answer: A

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16. f: $D \to R, f(x) =$ (x^2 +bx+c)/$\left(x^2 + b_1 x + c_1\right)$ whereα, β are the roots of the equation $x^2 + bx + c = 0$ and α_1, β_1 are the roots of $x^2 + b_1 x + c_1 = 0$. Now answer the following questions for f(x). A combination of graphical and analytical approach may be helpful in solving these problems. (If α_1 and β_1 are real, then f(x) has vertical asymptote at x = α_1, β_1

A. f'(x)=0 has real and distinct roots

B. f'(x)=0 has real and equal roots

C. f'(x)= 0 has imaginary roots

D. nothing can be said

Answer: A

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17. f: $D \rightarrow R, f(x) =$ (x^2 +bx+c)/$\left(x^2 + b_1 x + c_1\right)$ whereα, β are the roots of the equation $x^2 + bx + c = 0$ and α_1, β_1 are the roots of $x^2 + b_1 x + c_1 = 0.$ Now answer the following questions for f(x). A combination of graphical and analytical approach may be helpful in solving these problems. (If α_1 and β_1 are real, then f(x) has vertical asymptote at x = α_1, β_1

A. 1

B. 0

C. -1

D. does not exist

Answer: B

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18. f: $D \to R$, $f(x) =$ (x^2 +bx+c)/$\left(x^2 + b_1 x + c_1\right)$ whereα, β are the roots of the equation $x^2 + bx + c = 0$ and α_1, β_1 are the roots of $x^2 + b_1 x + c_1 = 0$. Now answer the following questions for f(x). A combination of graphical and analytical approach may be helpful in solving these problems. (If α_1 and β_1 are real, then f(x) has vertical asymptote at x = α_1, β_1

A. x-coordinate of point of minima is greater than the x-coordinate of point of maxima

B. x-coordinate of point of minima is less than x-coordinate of point of maxima

C. it also depends upon c and c_1

D. nothing can be said

Answer: B

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19. consider the function $f(x) = \frac{x^2}{x^2 - 1}$

The interval in which f is increasing is

A. (-1,1)

B. $(-\infty, -1) \cup (-1, 0)$

C. $(-\infty, -\infty) - \{-1, 1\}$

D. $(0, 1) \cup (1, \infty)$

Answer: B

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20. consider the function $f(x) = \frac{x^2}{x^2 - 1}$

If f is defined from $R - (-1, 1) \rightarrow R$ then f is

A. injective but not surjective

B. surjective but not inective

C. injective as well as surjective

D. neither injective nor surjective

Answer: D

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21. consider the function $f(x) = \frac{x^2}{x^2 - 1}$

f has

A. local maxima but not local minima

B. local minima but not local maxima

C. both local maxima and local minima

D. neither local maxima nor local minima

Answer: A

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22. Let $f(x)=e^{(P+1)x}-e^{x}$ for real number $P>0,$ then

The value of $x=S_p$ for which f(x) is minimum, is

A. $\dfrac{-\log_{e(P+1)}}{P}$

B. $-\log_{e(P+1)}$

C. $-\log_{eP}$

D. $\log_e\left(\dfrac{P+1}{P}\right)$

Answer: A

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23. Let $f(x)=e^{(P+1)x}-e^{x}$ for real number $P>0,$ then

Use the fact that $1+\dfrac{P}{2}\leq\dfrac{e^{P}-1}{P}\leq 1+\dfrac{P}{2}+P^{2}(0<P\leq 1),$ the value of $\lim_{P\to 0^{+}}(S_P-t_P)$ is

A. $-\log_e \cdot \frac{\left(e^{P-1}\right)}{P}$

B. $-\frac{1}{P}\log_e\left(\frac{e^{P-1}}{P}\right)$

C. $-\frac{1}{P}\log_e \cdot \left(\frac{(P+1)\left(e^{P-1}\right)}{P}\right)$

D. $-\log_e\left((P+1)\left(e^{P}-1\right)\right)$

Answer: C

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24. Let $f(x) = e^{(P+1)x} - e^{x}$ for real number $P > 0,$ then

Let $g(t) = \int_t^{t+1} f(x)e^{t-x}dx.$ The value of $t = t_P,$ for which g(t) is minimum, is

A. 0

B. $\frac{1}{2}$

C. 1

D. non-existent

Answer: B

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25. Consider f, g and h be three real valued function defined on R. Let $f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$. Then,

The length of a longest interval in which the function h(x) is increasing, is

A. $\pi/8$

B. $\pi/4$

C. $\pi/6$

D. $\pi/2$

Answer: B

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26. Consider f, g and h be three real valued function defined on R.

Let $f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$

Q. General solution of the equation $h(x) = 4$, is :

[where $n \in I$]

A. $(4n+1)\pi/8$

B. $(8n+1)\pi/8$

C. $(2n+1)\pi/4$

D. $(7n+1)\pi/4$

Answer: A

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27. Consider f, g and h be three real valued function defined on R. Let $f(x) = \sin 3x + \cos x, g(x) = \cos 3x + \sin x$ and $h(x) = f^2(x) + g^2(x)$. Then, Number of point (s) where the graphs of the two function, y=f(x) and y=g(x) intersects in $[0, \pi]$, is

A. 2

B. 3

C. 4

D. 5

Answer: C

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28. Consider f,g and h be three real valued functions defined on R. Let $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > o \end{cases}$,$g(x)\left(1 - x^2\right)$ and $h(x)$be such that h''(x)=6x-4.

Also, h(x) has local minimum value 5 at x=1

The equation of tangent at m(2,7) to the curve y=h(x), is

A. 5x+y=17

B. x+5y=37

C. x-5y+33=0

D. 5x-y=3

Answer: D

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29. Consider f,g and h be three real valued functions defined on R. Let $f(x) = \{(-1, \quad x < 0), (0, \qquad x = 0)$,$(1, x > o)$"g(x)(1-x^(2))andh(x) "be such that" h''(x)=6x-4. Also, h(x) has local minimum value 5 at x=1

The area bounded by y=h(x),y=g(f(x))between x=0 and x=2 equals

A. $23/2$

B. $20/3$

C. $32/3$

D. $40/3$

Answer: C

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30. Consider f,g and h be three real valued functions defined on R. Let
$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0, g(x)\left(1 - x^2\right) \text{ and } h(x) \text{be such that } \\ 1, & x > o \end{cases}$$ h''(x)=6x-4.

Also, h(x) has local minimum value 5 at x=1

Range of function $\sin^{-1}\sqrt{(fog(x))}$ is

A. $(0, \pi/2)$

B. $\{0, \pi/2\}$

C. $\{-[\pi/2, 0, \pi/2\}$

D. $\{\pi/2\}$

Answer: B

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31. Consider f,g and h be three real valued differentiable functions defined on R. Let $g(x) = x^3 + g''(1)x^3 + (3g'(1) - g''(1) - 1)x + 3g'(1)$

$f(x) = xg(x) - 12x + 1$

and $f(x) = (h(x))^2$, where $g(0) = 1$

The function y=f(x) has

A. (a)Exactly one local minima and no local maxima

B. (b)Exactly one local maxima and no local minima

C. (c)Exactly one local maxima and two local minima

D. (d)Exactly two local maxima and no local minima

Answer: C

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32. Find the intervals in which $f(x) = (x-1)^3(x-2)^2$ is increasing or decreasing.

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33. Consider f,g and h be three real valued differentiable functions defined on R. Let $g(x) = x^3 + g''(1)x^2 + (3g'(1) - g''(1) - 1)x + 3g'(1)$

$f(x) = xg(x) - 12x + 1$ and $f(x) = (h(x))^2$, where $g(0) = 1$ Which one of the following does not hold good for y=h(x)

A. Exactly one critical point

B. No point of inflexion

C. Exactly one real zero in (0,3)

D. Exactly one tangent parallel to y-axis

Answer: C

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Exercise (Single Integer Answer Type Questions)

1. A particular substance is being cooled by a stream of cold air (temperature of the air is constant and is $5^\circ C$) where rate of cooling is directly proportional to square of difference of temperature of the substance and the air.

If the substance is cooled from $40^\circ C$ to $30^\circ C$ in 15 min and temperature after 1 hour is $T^\circ C$, then find the value of $[T]/2$, where [.] represents the greatest integer function.

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2. The minimum value of $\dfrac{\tan\left(x+\frac{\pi}{6}\right)}{\tan x}, x \in \left(0, \dfrac{\pi}{3}\right)$, is

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3. Number of positive integral value(s) of a for which the curve $y = a^x$ intersects the line $y = x$ is:

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4. The least value of a for which the equation $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ has at least one solution in the interval $\left(0, \frac{\pi}{2}\right)$ 9 (b) 4 (c) 8 (d) 1

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5. $Legf(x)=\left\{x^{\frac{3}{5}}, \quad \text{if } \quad x \leq 1-(x-2)^{3}, \quad \text{if } \quad x>1\right.$ Then the number of critical points on the graph of the function is___

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6. Number of critical points of the function.

$f(x)=\frac{2}{3}\sqrt{x^{3}}-\frac{x}{2}+\int_{1}^{x}\left(\frac{1}{2}+\frac{1}{2}\cos 2t-\sqrt{t}\right)$ dt which lie in the interval $[-2\pi, 2\pi]$ is.......... .

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7. Let $f(x) and g(x)$ be two continuous functions defined from $\overrightarrow{RR}$, such that $f(x_1) > f(x_2)$ and $g(x_1) < g(x_2)$ f or all $x_1 > x_2$. Then what is the solution set of $f\left(g\left(\alpha^2 - 2\alpha\right) > f(g(3\alpha - 4))\right)$

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8. If $f(x) = \frac{t + 3x - x^2}{x - 4}$, where t is a parameter that has minimum and maximum, then the range of values of t is (a) $(0, 4)$ (b) $(0, \infty)$ (c) $(-\infty, 4)$ (d) $(4, \infty)$

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9. Prove that the function $f(x) = \frac{2x - 1}{3x + 4}$ is increasing for all x R.

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10. If $f''(x) + f'(x) + f^2(x) = x^2$ is the differential equation of a curve and let P be the point of maxima, then number of tangents which can be drawn from P to

$x^2 - y^2 = a^2$ is/are

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11. If absolute maximum value of

$f(x) = \frac{1}{|x-4|+1} + \frac{1}{|x+8|+1} is \frac{p}{q}$, (p,q are coprime) the (p-q) is......... .

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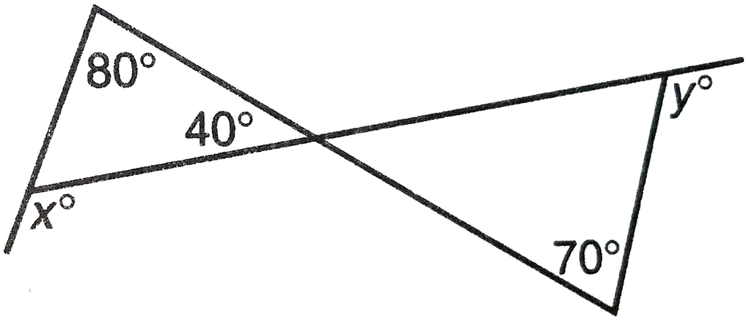
MAXIMA AND MINIMA EXERCISE 6



1.

in the figure above , x+y=

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2. The graph of y=f''(x) for a function f is shown.

Number of points of inflection for y=f(x) is..... .

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Exercise (Questions Asked In Previous 13 Years Exam)

1. The least value of $\alpha \in R$ for which $4ax^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

A. $\frac{1}{64}$

B. $\frac{1}{32}$

C. $\frac{1}{27}$

D. $\frac{1}{25}$

Answer: C

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2. The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is

A. 6

B. 4

C. 2

D. 0

Answer: C

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3. Let $f: R \to (0, \infty)$ and $g: R \to R$ be twice differentiable functions such that f" and g" are continuous functions on R. suppose $f'(2) = g(2) = 0, f(2) \neq 0$ and $g'(2) \neq 0$, If $\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$ then

A. f has a local minimum at x=2

B. f has a local maximum at x=2

C. $f''(2) > f(2)$

D. $f(x) - f''(x) = 0$ for atleast one $x \in R$

Answer: A::D

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4. Let $f: (0, \infty) \vec{R}$ be given by $f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t + \frac{1}{t}\right)} dt}{t}$, then (a) $f(x)$ is monotonically increasing on $[1, \infty)$ (b) $f(x)$ is monotonically decreasing on $(0, 1)$ (c) $f(2^x)$ is an odd function of x on R (d) none of these.

A. f(x) is monotonically increasing on $[1, \infty)$

B. f(x) is monotonically decreasing on $[0, 1]$

C. $f(x) + f\left(\frac{1}{x}\right) = 0, \forall x \in (0, \infty)$

D. $f(2^x)$ is an odd function pf x on R

Answer: C

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5. The fuction $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum respectively at x =

A. -2

B. $\frac{-2}{3}$

C. 2

D. $2/3$

Answer: D

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6. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio $8:15$ is converted into anopen rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the length of the sides of the rectangular sheet are 24 (b) 32 (c) 45 (d) 60

A. 24

B. 32

C. 45

D. 60

Answer: A::C

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7. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ at the points P and Q.Let the tangents to the ellipse at P and Q meet at R. If $\Delta(h)$ Area of triangle ΔPQR, and $\Delta_1=\max_{\frac{1}{2}\leq h\leq 1}\Delta(h)$ and $\Delta_2=\min_{\frac{1}{2}\leq h\leq 1}\Delta(h)$ Then $\frac{8}{\sqrt{5}}\Delta_1-8\Delta_2$

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8. Let f, g and h be real-valued functions defined on the interval $[0, 1]$ by $f(x)=e^{x^2}+e^{-x^2}$, $g(x)=xe^{x^2}+e^{-x^2}$ and $h(x)=x^2e^{x^2}+e^{-x^2}$. if a, b and c denote respectively, the absolute maximum of f, g and h on $[0, 1]$ then

A. $a=b$ and $c\neq b$

B. $a=c$ and $a\neq b$

C. $a \neq bc \neq b$

D. $a = b = c$

Answer: D

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9. e total number of local maxima and local minima of the function `f(x) =

{(2+x)^3, -3

A. 0

B. 1

C. 2

D. 3

Answer: A

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10. If the function $g:(-\infty,\infty)\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is given by $g(u)=2\tan^{-1}(e^u)-\frac{\pi}{2}$. Then, g is

A. even and is strictly increasing in $(0,\infty)$

B. odd and is strictly decreasing in $(-\infty,\infty)$

C. odd is strictly increasing in $(-\infty,\infty)$

D. neither even nor odd but is strictly increasing in $(-\infty,\infty)$

Answer: C

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11. The second degree polynomial f(x), satisfying f(0)=o,

$f(1)=1, f'(x)>0\,\forall x\in(0,1)$

A. $f(x)=\phi$

B. $f(x)=ax+(1-a)x^2,\ \forall a\in(0,\infty)$

C. $f(x)=ax+(1-a)x^2, a\in(0,2)$

D. No such polynomial

Answer: D

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12. If $f(x) = x^3 + bx^2 + cx + d$ and 0<b^2<c, then

A. f(x) is strictly increasing function

B. f(x) has a local maxima

C. f(x) is strictly decreasing function

D. f(x) is bounded

Answer: A

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13. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) > max g(x)$, then the relation between b and c is

A. No real value of b and c

B. $0 < c < b\sqrt{2}$

C. $|c| < |b|\sqrt{2}$

D. $|c| > |b|\sqrt{2}$

Answer: D

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14. The length of the longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing is $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) π

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

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15. If $f(x) = e^{1-x}$ then f(x) is

A. increasing in $[-1/2, 1]$

B. decreasing in R

C. increasing in R

D. decreasing in $[-1/2, 1]$

Answer: A

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16. The maximum value of $(\cos\alpha_1)(\cos\alpha_2)...(\cos\alpha_n)$,

under the restrictions $0 \leq \alpha_1, \alpha_2..., \alpha_n \leq \frac{\pi}{2}$ and

$(\cot\alpha_1)(\cot\alpha_2)......(\cot\alpha_n) = 1$ is

A. $\frac{1}{2^{n/2}}$

B. $\frac{1}{2^n}$

C. $\frac{1}{2n}$

D. 1

Answer: A

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17. If $f(x) = \begin{cases} e^x & ,0 \leq x < 1 \\ 2 - e^{x-1} & ,1 < x \leq 2 \\ x - e & ,2 < x \leq 3 \end{cases}$ and $g(x) = \int_0^x f(t)dt$,

$x \in [1, 3]$, then

A. g(x) has local maxima at $x = 1 + \log_e 2$ and local minima at x=e

B. f(x) has local maxima at x=1 and local minima at x=2

C. g(x) has no local minima

D. f(x) has no local maxima

Answer: A::B

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18. If f(x) is a cubic polynomil which as local maximum at x=-1 . If f(2)=18,f(1)=-1 and f'(x) has minimum at x=0 then

A. the distance between (-1,2) and (a,f(a)), where x=a is the point of local minima, is $2\sqrt{5}$

B. f(x) is increasing for $x \in [1, 2\sqrt{5}]$

C. f(x) has local minima at x=1

D. the value of f(0)=5

Answer: B::C

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19. Consider the function $f:(-\infty, \infty) \rightarrow (-\infty, \infty)$ defined by $f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}; 0 < a < 2$. Which of the following is true ?

A. $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

B. $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

C. $f'(1) f'(-1) = (2-a)^2$

D. $f'(1) f'(-1) = -(2+a)^2$

Answer: A

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20. about to only mathematics

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21. Find a point on the curve $x^2 + 2y^2 = 6$, whose distance from the line $x + y = 7$, is minimum.

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22. Let $IR\overrightarrow{I}R$ be defined as $f(x) = |x| + +x^2 - 1| \cdot$ The total number of points at which f attains either a local maximum or a local minimum is______

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23. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6andp(3) = 2,$ then $p'(0)$ is____

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24. Let f be a function defined on R (the set of all real numbers) such that $f'(x) = 2010(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$, for all $x \in R$. If g is a function defined on R with values in the interval $(0, \infty)$ such that $f(x) = \ln(g(x))$, for all $x \in R$, then the number of point is R at which g has a local maximum is ___

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25. The maximum value of the expression $\dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta}$ is ________.

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26. The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x - 48$ on the set $A = \{x \mid x^2 \mid 20 \leq 9x\}$ is_____.

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27. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a cricle of radius = r units. If the sum of areas of the square and the circle so formed is minimum, then

A. $2x = (\pi + 4)r$

B. $(4 - \pi)x = \pi r$

C. x=2r

D. 2x=r

Answer: C

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28. If x =-1 and x=2 ar extreme points of f(x) =$\alpha \log|x| + \beta x^2 + x$, then

A. $\alpha = -6, \beta = \frac{1}{2}$

B. $\alpha = -6, \beta = -\frac{1}{2}$

C. $\alpha = 2, \beta = -\frac{1}{2}$

D. $\alpha = 2, \beta = \frac{1}{2}$

Answer: C

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29. Let a , b in R be such that the function f given by $f(X) = \ln|x| + bx^2 + asx, x \neq 0$ has extreme values at x=-1 and at x=2

Statement 1: f has local maximum at x =-1 and at x =2

statement 2: a =$\frac{1}{2}$ and b = $\frac{-1}{2}$.

A. Statement I is false, Statement II is true

B. Statement I is true, Statement II is true, Statement II is a correct explanation of Statement I

C. Statement I is true, Statement II is true, Statement II is not a correct explanation of Statement I

D. Statement I is true, Statement II is false.

Answer: C

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