MATHS

BOOKS - IA MARON MATHS (HINGLISH)

IMPROPER INTEGRALS

8 1 Improper Integrals With Infinite Limits

1. Evaluate the following improper integrals with infinite limits on the basis of their

definition :

(a) $\int_2^{\infty} \frac{x dx}{\sqrt{(x^2-3)^3}}$,

(b) $\int_1^{\infty} \frac{dx}{x+x^3}$,

(c) $\int_0^{\infty} \frac{x dx}{\sqrt{(4x^2+1)^3}}$,

(d) $\int_1^{\infty} \frac{dx}{x^2(1+x)}$,

(e) $\int_{-\infty}^{\infty} \frac{dx}{x^2-6x+10}$,

(f) $\int_0^{\infty} e^{-x} \sin x dx$

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2. Prove that the integrals of the form

$$\int_a^{+\infty} e^{-Px}dx \text{and} \int_{-\infty}^{b} e^{Px}dx$$

Converge for any constant $P > 0$ and diverge for $p < 0$.

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3. Test the integral $\int_0^{\infty} \frac{dx}{1+2x^2+3x^4}$ for convergence.

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4. Test the following integrals for convergence

:

(a) $\int_1^{\infty} \frac{1n\left(x^2+1\right)}{x} dx,$

(b) $\int_1^{\infty} \frac{\tan. \frac{1}{x}}{1+x\sqrt{x}} dx,$

(c) $\int_1^{\infty} \frac{2+\cos x}{\sqrt{x}} dx,$

(d) $\int_2^{\infty} \frac{3+\arcsin. \frac{1}{x}}{1+x\sqrt{x}} dx,$

(e) $\int_1^{\infty} \frac{\arctan x}{x} dx.$

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5. Test the integral

$$\int_3^\infty \frac{dx}{\sqrt{x(x-1)(x-2)}}$$

for convergence.

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6. Evaluate the improper integral

$$\int_0^\infty \frac{dx}{(1+x^2)^n},$$ n natural number.

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7. Evaluate the following improper integrals :

(a) $\int_0^{\infty} \frac{1nx}{1+x^2} dx,$

(b) $\int_0^{\infty} e^{-x^2 x^{2m+1}} dx$

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8 2 Improper Integrals Of Unbounded Functions

1. Proceeding from the definition, compute the following improper integrals (or prove their divergence):

(a) $\int_0^{3a} \frac{2xdx}{(x^2-a^2)^{2/3}}$,

(b) $\int_0^{2/\pi} \sin.\ \frac{1}{x} \cdot \frac{dx}{x^2}$,

(c) $\int_0^1 \cos.\ \frac{\pi}{1-x} \cdot \frac{dx}{(1-x)^2}$,

(d) $\int_2^6 \frac{dx}{\sqrt{(4-x)^2}}$,

(e) $\int_{-1}^{-2} \frac{dx}{x\sqrt{x^2-1}}$,

(f) $\int_1^2 \frac{dx}{x1n^p x}$

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2. Evaluate the following improper integrals :

(a) $\int_{-3}^{3} \frac{x^2dx}{\sqrt{9-x^2}}$,

(b) $\int_0^2 \sqrt{\frac{2+x}{2-x}}dx.$

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3. Investigate the following improper integrals for convergence :

(a) $\int_0^1 \frac{e^x dx}{\sqrt{1-x^3}},$

(b) $\int_0^1 \frac{x^2 dx}{\sqrt[3]{(1-x^2)^5}},$

(c) $\int_0^1 \sqrt{\frac{x}{1-x^4}}dx,$

(d) $\int_0^1 \frac{dx}{1-x^3+x^5},$

(e) $\int_0^1 \frac{dx}{x-\sin x}$,

(f) $\int_0^2 \frac{1n\left(\sqrt[4]{x}+1\right)}{e^{\tan x}-1} dx$

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4. Evaluate the following improper integrals (or prove their divergence) :

(a) $\int_0^{1/2} \frac{dx}{x1nx}$,

(b) $\int_1^2 \frac{dx}{x\sqrt{1nx}}$,

(c) $\int_0^1 \frac{3x^2+2}{\sqrt[3]{x^2}} dx$.

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5. Investigate the following integrals for convergence :

(a) $\int_0^1 \frac{dx}{\sqrt{\sin x}}$,

(b) $\int_0^1 \frac{dx}{e^x - \cos x}$,

(c) $\int_0^1 \frac{\cos^2 x dx}{(1-x)^2}$,

(d) $\int_0^1 \frac{\tan x dx}{\sqrt{1-x^2}}$,

(e) $\int_{1/2}^{6/5} \frac{\sin x dx}{\sqrt{|1-x^2|}}$.

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8 3 Geometric And Physical Applicationos Of Improper Integrals

1. Find the area of the figure bounded by the curve $y = \dfrac{1}{1+x^2}$ (the witch of Agnesi) and its asymptote.

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2. Prove that the area of the region bounded by the curve $y = \dfrac{1}{\sqrt{1-x^2}}$ the axis of

abscissas, the axis of ordinates and tha asymptote $x = 1$ is finite and equals $\frac{\pi}{2}$.

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3. Find the volumes of the solids enclosed by the surfaces generated by revolving the lines $y = e^{-x}, x = 0, y = 0 (0 \leq x < +\infty)$:

(a) about the x-axis

(b) about the y-axis.

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4. Compute the area contained between the cissoid $y^2 = \frac{x^3}{2a - x}$ and its asymptote.

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5. Compute the area bounded by the curve $y = e^{-2x}(atx > 0)$ and the axes of coordinates.

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6. Find the volume of the solid generated by revolving, about the x-axis, the infinite branch of the curve $y = 2\left(\frac{1}{x} - \frac{1}{x^2}\right) \text{for} x \geq 1.$

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7. What work has to be performed to move a body of mass m from the Earth.s surface to infinitely ?

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8. Determine the work which has to be done to bring an electric charge $e_2 = 1$ from infinity to a unit distance from a charge e_1.

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8 4 Additional Problems

1. At what values of m does the integral $\int_0^2 \frac{1-\cos x}{x^m} dx$ converge ?

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