MATHS

BOOKS - PATHFINDER MATHS (BENGALI ENGLISH)

APPLICATION OF DERIVATIVE

Question Bank

1. The distance S metres moved by a particle traveling in a straight line in t second is given by S = $45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

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2. If R(x) rupees is the total revenge received from the sale of x tables and $R(x) = 600x - \frac{x^3}{25}$, find the marginal revenue when x =25.

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3. The radius of a spherical balloon is decreasing at the rate of 10 cm per second. At what rate is the surface area of the balloon decreasing when its radius is 15 cm?

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4. At what point of the ellipes $16x^2 + 9y^2 = 400$, does the ordinate decrease at the same rate at which the abscissa increases?

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5. A particle moves along the curve 6y = $x^3 + 2.$ Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

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6. A ladder 16 cm long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/sec. How fast is its height on the wall decreasing when the foot of the ladder is 4 cm away from the wall?

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7. Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 5 cm and the altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top.

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8. Show that $f(x) = \left(\frac{1}{2}\right)^x$ is strictly decreasing on R.

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9. Show that : tanx -x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$.

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10. Show that the following function is strictly increasing : $f(x) = x^2, x > 0.$

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11. Find the intervals on which the following function is strictly increasing and strictly decreasing : $\frac{x}{\log x}$, x>0.

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12. Find the intervals on which the following function is strictly increasing and strictly decreasing : $f(x) = 2x^3 + x^2 - 20.$

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13. Find the intervals on which the following function is strictly increasing and strictly decreasing : $f(x) = 2x^3 + x^2 - 20.$

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14. FInd the slopes of the tangents and normals to the following curve: $x = a\cos^3\theta, y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$.

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15. Show that the tangents to the curve $y = x^2 - 5x + 6$ at the point (2,0) and (3,0) are at right angle.

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16. Find the equations of the tangent and normal to the curve $y = x^2 - 4x - 5$ at x= -2.

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17. Find the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (x= 1,y= 1)`.

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18. For the curve y =$4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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19. FInd the equation of the normal to the curve $3x^2 - y^2 = 8$ which is parallel to the line x+3y=4.

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20. Show that the curve $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

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21. Prove that the curves $x = y^2$ and xy = k cut at right angles* if $8k^2 = 1$.

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22. Find the angle of intersection of the curve $x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0$.

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23. If $y = x^4 - 10$ and if x change from 2 to 1.99, what is the approximate change in y?

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24. If $y = x + \log_5 x + 2^x$, find $\frac{dy}{dx}$

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25. The curve $3y^2 = 2ax^2 + 6b$ passes through the point $P(3. -1)$ and the gradient of curve at P is -1. Find the values of a and b

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26. If $y = e^x \sin x$, find $\frac{dy}{dx}$

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27. If $y = \log_e\left(\tan^{-1}\sqrt{1+x^2}\right)$ find $\frac{dy}{dx}$

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28. If $y = \cos(bx + c)$ find $\frac{dy}{dx}$

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29. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin + \ldots \infty}}}$, find $\frac{dy}{dx}$

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30. If $y\sqrt{1 - x^2} + x\sqrt{1 - y^2} = 1$ when $|x| < 1$ and $|y| < 1$ find $\frac{dy}{dx}$

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31. If $x\sqrt{1 + y} + y\sqrt{1 + x} = 0$, prove that $\frac{dy}{dx} = \frac{-1}{(1 + x)^2}$

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32. If $e^{x}+e^{y}=e^{x+y}$, prove that $\frac{dy}{dx}=-e^{y-x}$

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33. Differentiate $x^{\sin^{-1}x}$ w.r.t 'x'

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34. If $y=(x)^{\cos x}$,find $\frac{dy}{dx}$

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35. If $x^{y}=e^{x-y}$, Prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^{2}}$

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36. If $y=e^{x+e^{x+e^{x+\dots .TO\infty}}}$ prove that $\frac{dy}{dx}=\frac{y}{(1-y)}$

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37. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w,r,t. $\tan^{-1}x$

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38. If $e^y(x+1)=1$, prove that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

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39. If $y=\sin^{-1}$ x, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=0$

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40. If $x=a(\theta+\sin\theta)$ and $y=a(1-\cos\theta)$, find $\frac{d^2y}{dx^2}\text{at}\theta=\frac{\pi}{2}$

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41. A particle moves along the curve $y = \frac{2}{3}x^3$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate.

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42. A man of height 1.6m, walks at a rate of 30m/sec away from a lamp which is 4m in height. How fast the shadow of man is lengthening.

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43. Water is leaking from a conical funnel at the rate of $5cm^3 / \text{sec}$. If the radius of the base of funnel is 10 cm and its height is 20 cm, find the rate at which the water level is dropping when it is 5 cm from the top.

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44. Find the approximate value of $\log_{10} 10.1$ it being given that $\log_{10} e = 0.4343$

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45. The time T of of oscillation of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. Find the percentage error in T, corresponding to an error of 2% in the value of l.

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46. If in a triangle ABC, the side c and the angle C remain constant while the remaining elements are changed slightly, show that $\frac{da}{\cos A} + \frac{db}{\cos B} = 0$

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47. Find the interval in which $f(x) = 2x^3 + 3x^2 - 12x + 1$ is increasing.

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48. Find the interval in which $f(x) = x^3 - 3x^2 - 9x + 30$ is strictly increasing or strictly decreasing.

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49. The function $f(x) = \sin^4 x + \cos^4 x$ increasing if :

A. $0 < x < \pi/8$

B. $\pi/4 < x < 3\pi/8$

C. $3\pi/8 < x < 5\pi/8$

D. $5\pi/8 < x < 3\pi/4$

Answer: B

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50. Let $f(x)=\int_0^x e^t(t-1)(t-2)dt$, Then f decreases in the interval

A. a)$(-\infty, -2)$

B. b)$(-2, -1)$

C. c)(1,2)

D. d)$(2, \infty)$

Answer: C

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51. If $f(x)=x.\, e^{x(1-x)}$, then f(x) is

A. increasing on $\left[-\frac{1}{2}, 1\right]$

B. decreasing on R

C. increasing on R

D. decreasing on $\left[-\frac{1}{2}, 1\right]$

Answer: A

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52. Let $f(x) = 3x + 5$ then show f(x) is strictly increasing and $f^{-1}(x)$ exists and is strictly increasing for $\text{x} \in R$

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53. If $a < 0$, and $f(x) = e^{ax} + e^{-ax}$ Is monotonically decreasing. Find the interval in which x belongs.

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54. If $f''(x) > 0$ and f'(1)=0 such that $g(x) = f\left(\cot^2 x + 2\cot x + 2\right)$, where $0 < x < \pi$ then the interval in which g(x) is decreasing is :

A. $(0, \pi)$

B. $\left(\frac{\pi}{2}, \pi\right)$

C. $\left(\frac{3\pi}{4}, \pi\right)$

D. $\left(0, \frac{3\pi}{4}\right)$

Answer: D

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55. Using calculus find the order relation between x and $\tan^{-1}$ x when $x \in [0, \infty)$

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56. The number of critical points of $f(x) = \frac{|x-1|}{X^2}$ is

A. 1

B. 2

C. 3

D. none of these

Answer: A::B

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57. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 \, dt$ has a local maximum at x =

A. a)0

B. b)1

C. c)2

D. d)3

Answer: C

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58. Use the function $f(x) = x^{1/x}, x > 0$ to determine the bigger of the two numbers e^{π} and π^{e}.

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59. Show that $(x^5 - 5x^4 + 5x^3 - 1)$ has a local maximum when x = 1, a local minimum when x = 3, and neither x = 0

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60. Find the point on the parabola $y^2 = 2x$ which is closest to the point (1,4).

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61. In a right -angled triangle, prove that r+2R=S.

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62. Show that the semivertical angle of a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

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63. Let $f(x) = \{(|x-2| + a^2 - 9a - 9, \text{ifx} < 2, 2x - 3, \text{ifx} \geq 2:\}$ Then find value of 'a' for which f(x) has local minima at x= 2

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64. Find the values of 'm' for which the equation $x^4 - (m-3)x^2 + m$=0 has

Four real roots

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65. The difference between the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$, is

A. a) $\frac{2}{3}$

B. b) $\frac{8}{7}$

C. c) $\frac{9}{4}$

D. d) $\frac{3}{8}$

Answer: C

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66. The tangent and normal to the curve $y = 2\sin x + \sin 2x$ are drown at $P\left(x = \frac{\pi}{3}\right)$, then area of the quadrilateral formed by the tangent, the normal at P and the coordinate axes is

A. $\frac{\pi}{3}$

B. 3π

C. $\frac{\pi\sqrt{3}}{2}$

D. none of these

Answer: C

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67. Show that the sum of the intercepts of the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ on the coordinate axes is constant.

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68. Find the point on the curve $y - e^{xy} + x = 0$. At which we have vertical tangent.

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69. Find the lengths of tangent, subtangent, normal and subnormal to $y^2 = 4ax$ at $(at^2, 2at)$

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70. If the length of subnormal is equal to length of subtangent at a point (3,4) on the curve y = f(x) and the tangent at (3,4) to y=f(x) meets the coordinate axes at A and B, then find the maximum area of the ΔOBB wherer O is orgin

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71. Find the angle of intersection between the following curves, $y^2 = 4x$ and $x^2 = 4y$

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72. Show that $x = y^2$ and xy = k cut orthogonally if $8k^2 = 1$

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73. Show that curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect orthogonally if , $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

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74. Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a,b), whatever be the value of n.

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75. Prove that all points of the curve $y^2 = 4a\left[x + a\frac{\sin x}{a}\right]$ at which the langent is parallel to the axis of x, lie on a parabola.

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76. Verify Rolle's theorem for $f(x) = x^3(x-1)^2$ in the interval $0 \leq x \leq 1$

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77. If $2a + 3b + 6c = 0$ then the equation $ax^2 + bx + c = 0$ has in the interval (0,1)

A. a)at least one root

B. b)at most one root

C. c)no root

D. d)none of these

Answer: A

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78. The value of 'c' of Rolls's throrem is applicable to the function $f(x) = 2x^3 - 5x^2 - 4x + 3$, x $\in [1/2, 3]$ is

A. a)2

B. b)$-\frac{1}{2}$

C. c)-2

D. d)$\frac{2}{3}$

Answer: A

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79. Find 'c' of the Lagrange's mean value theorem for the function $f(x) = 3x^2 + 5x + 7$ in the interval [1,3]

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80. If $A(x_1, y_1)$ an $B(x_2, y_2)$ be two points on the curve $y = ax^2 + bx + c$ then Lagrange's mean value theorem show that there will be at least one point $C(x_3, y_3)$ where the tangent will be parallel to the chord AB, Also show that $x_3 = \frac{x_1 + x_2}{2}$

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81. 0 < a < b < (pi/2) and f(a,b)= (tanb−tana)/b-a

A. $f(a, b) \geq 2$

B. $f(a, b) > 1$

C. $f(a, b) \leq 1$

D. none of these

Answer: B

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82. Let $f, [2, 7] \to [0, \infty)$ be a continuous and differentiable function,

Then the value if $(f(7) - f(2))\dfrac{(f(7))^2 + (f(2))^2 + f(2)xf(7)}{3}$ is (where $c \in (2, 7))$

A. $3f^2(c), f(c)$

B. $5f^2(c), f(c)$

C. $2f^2(c), f(c)$

D. none of these

Answer: C

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83. If

$$y = \frac{1}{1 + x^{n-m} + x^{p-m}} + \frac{1}{1 + x^{m-n} + x^{p-n}} + \frac{1}{1 + x^{m-p} + x^{n-p}},$$

then $\dfrac{dy}{dx}$ is equal to

A. 1

B. 0

C. $m+n+p$

D. $m-n+p$

Answer: B

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84. If $y=|\cos x|+|\sin x|, then\left(\frac{dy}{dx}\right)\text{at}x=\frac{2\pi}{3}$ is

A. $\frac{1}{2}\left(\sqrt{3}+1\right)$

B. $2\left(\sqrt{3}-1\right)$

C. $\frac{1}{2}\left(\sqrt{3}-1\right)$

D. none of these

Answer: C

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85. The differential coefficient of $f(\log_e x)$ with respect to x, where $f(x) = \log_e x$, is :

A. $\dfrac{x}{\log_e x}$

B. $\dfrac{1}{x}\log_e x$

C. $\dfrac{1}{x \log_e x}$

D. none of these

Answer: C

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86. If f(x) = cosx. cos2x. Cos4x. Cos8x.cos16x, then f '(pi/4) is :

A. $\sqrt{2}$

B. $\dfrac{1}{\sqrt{2}}$

C. 1

D. none of these

Answer: A

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87. If $f(x) = \log|2x|, x \neq 0$ then f'(x) is equal to

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. $-\frac{1}{x}$

D. none of these

Answer: B

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88. The derivative of an odd function is always

A. does not exist

B. an odd function

C. an aven function

D. none of these

Answer: C

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89. If $y = \frac{1}{1+x+x^2+x^3}$, then value of $\frac{d^2y}{dx^2}$ at x=0 is

A. a)-1

B. b)1

C. c)0

D. d)none of these

Answer: C

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90. The function $f(x) = (\cos x + i\sin x)(\cos 3x + i\sin 3x)...\{\cos(2n-1)x + i\sin(2n-1)$

, then f'(x) is equal to

A. $n^2 f(x)$

B. $-n^4 f(x)$

C. $-n^2 f(x)$

D. $n^4 f(x)$

Answer: B

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91. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + ...\infty}}}}$ then $\frac{dy}{dx}$ equal to

A. $\dfrac{y^2 - x}{2y^3 - 2xy - 1}$

B. $\dfrac{y^2 + x}{2y^3 + 2xy + 1}$

C. $\frac{x^2 + x}{2y^3 - 2xy - 1}$

D. none of these

Answer: A

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92. If $x^3y^9 = (x + y)^k$ and $x\frac{dy}{dx} = y$ then k =

A. a)10

B. b)11

C. c)12

D. d)none of these

Answer: C

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93. If $y = \log(3x+5)$ and $\frac{d^2y}{dx^2} = \frac{A}{(3x+5)^2}$ then A =

A. 9

B. -9

C. 18

D. none of these

Answer: B

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94. Let f be a function satisfying $f(x+y) = f(x) + f(y)$ and $f(x) = x^3\phi(x)$ for all x and y when ϕ (x) is a continuous function then f''(x) is equal to

A. 0

B. g(0)

C. g'(x)

D. none of these

Answer: A

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95. If $y = \cos^{-1}\cos\left[2\sin^{-1}(\cos x)\right]$ then $\frac{dy}{dx} =$

A. 2

B. -1

C. -2

D. none of these

Answer: C

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96. If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $\frac{dy}{dx} =$

A. $\frac{x}{y}$

B. $-xy$

C. $-\frac{y}{x}$

D. none of these

Answer: C

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97. If $x=\sin^{-1}(3t-4t^3)$, $y=\cos^{-1}(8t^4-8t^2+1)$ then $\frac{dy}{dx}=$

A. $+1$

B. $\frac{2}{3}$

C. $-\frac{3}{4}$

D. $\frac{4}{3}$

Answer: D

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98. If $f(x) = e^x g(2x)$, $g(0) = 4$ and g'(0) = 2 then f'(0) is equal to

A. 4

B. 6

C. 8

D. 12

Answer: C

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99. Let $f(x) = \sin x$, $g(x) = x^2$ and h(x) = log x. If $F(x) = (h(f(g(x))))$, then F'(x) is:

A. a)$2x \cot x^2$

B. b)$2cosec^3 x$

C. c)$-2cosec^2 x$

D. d)none of these

Answer: A

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100. If y = f(x) $\frac{dy}{dx} = f'(x)$ therfore $y = f\{g(x)\}$ then $f'\{g(x)\}$ will be denoted by

A. $\frac{dy}{d\{g(x)\}}$

B. $\frac{dy}{dx}$

C. $\frac{d}{dx}g(x)$

D. none of these

Answer: A

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101. If $f(x) = xf(2a - x)$ then f''(0) equal to

A. 2f'(0)

B. 2f'(a)

C. 2f'(2a)

D. none of these

Answer: D

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102. If $x = e^z$ then $x^2 \frac{d^2y}{dx^2}$ is

A. $\frac{d^2y}{dz^2} - \frac{dy}{dz}$

B. $\frac{d^2y}{dz^2} + \frac{dy}{dz}$

C. $\frac{dy}{dz} + \frac{d^2y}{dz^2}$

D. none of these

Answer: A

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103. If $S = t^3 + 6t^2 - 36t + 100$. Determine acceleration and displacement at the time when velocity vanishes

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104. x and y are the sides of two squares such that $y = x - x^2$ Find the rate of change of the area of the second square with respect to the first square.

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105. If $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$, then in this interval

A. both f(x) and g(x) are increaseing functions

B. both f(x) and g(x) are decreasing functions

C. f(x) is an increasing function

D. g(x) is an increasing function

Answer: C

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106. The function $f(x) = (\sin x + \cos x)$ is an increasing function in

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Answer: D

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107. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1,2], then k lies in the interval

A. $(-\infty, 4)$

B. $(4, \infty)$

C. $(-\infty, 8)$

D. $(8, \infty)$

Answer: A

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108. Let $f(x) = \int e^x (x-1)(x-2)dx$, then f(x) decreases in the interval

A. $(-\infty, -2)$

B. $(-2, -1)$

C. (1,2)

D. $(2, \infty)$

Answer: C

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109. If $f(x) = \begin{cases} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{cases}$

A. f(x) is increasing on $[-1, 2]$

B. f(x) is continuous on $[-1, 3]$

C. f'(2) does not exist

D. all of these

Answer: D

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110. For all θ in $\left[0, \frac{\pi}{2}\right]$ show that $\cos(\sin\theta) > \sin(\cos\theta)$

A. $e^2 < 1 + x$

B. $\log_e(1 + x) < x$

C. $\sin x > x$

D. $\log_e x > x$

Answer: B

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111. Which of the following holds true for $x \in \left[0, \frac{\pi}{2}\right)$

A. $x - \frac{x^2}{3} \leq \sin x \leq x$

B. $x - \frac{x^3}{3} \geq \sin x \geq x$

C. $2\sin x + \tan x \geq 3x$

D. $2\sin x + \tan x \leq 3x$

Answer: A

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112. The maximum value of $\left(\frac{\log x}{x}\right)$ is

A. $\left(\frac{1}{e}\right)$

B. $\frac{2}{e}$

C. e

D. 1

Answer: A

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113. The minimum value of $\left(x^2+\frac{250}{x}\right)$ is

A. 0

B. 25

C. 50

D. 75

Answer: D

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114. The minimum value of $f(x) = 3x^4 - 8x^3 - 48x + 25$ on [0.3] is

A. 16

B. 25

C. -39

D. none of these

Answer: C

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115. The maximum value of $f(x) = (x-2)(x-3)^2$ is

A. $\frac{7}{3}$

B. 3

C. $\frac{4}{27}$

D. 0

Answer: C

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116. The points for which the function $f(x) = \int_1^x \left\{2(t-1)(t-2)^3 + 3(t-1)^2(t-2)^2\right\}$ dt attins maxima and minima is :

A. maximum when $x = \frac{7}{5}$ and minimum when x = 1

B. maximum when x=1 and minimum when x = 0

C. maximum when x = 1 and minimum when x =2

D. maximum when x = 1 and minimum when $x = \frac{7}{5}$

Answer: D

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117. Find the shortest distance between the curve $x^2 + y^2 = 4$ and the point (6,8)

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118. The minimum value of the function $f(x) = 2|x-2| + 5|x-3|$ for all $x \in R$ is

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119. The total number of local maxima and local minima for the function f(x)=(2+x)^3, -3 less than equal to x less than equal 1, (x^(2//3),-1 less than x less than 2

A. 0

B. 1

C. 2

D. 3

Answer: C

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120. Let $f(x) = \begin{cases} |x| & \text{for} \quad 0 < |x| \leq 2 \\ 1 & \text{for} \quad x = 0 \end{cases}$

Then ,at x = 0, f(x) has

A. a local maximum

B. no local maximum

C. a local minimum

D. no extremum

Answer: A

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121. Let f(x) be a function defined as $f(x) = \begin{cases} \sin(x^2 - 3x) & x \leq 0 \\ 6x + 5x^2 & x > 0 \end{cases}$

Then at x=0,f(x)

A. a)has a local maximum

B. b)has a local minimum

C. c)is discontinuous

D. d)none of these

Answer: B

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122. If line $ax + by + c = 0$ is normal to the curve $xy + 5 = 0$ then a and b are of :

A. same sign

B. opposite sign

C. cannot be discussed

D. none of these

Answer: A

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123. If the tangent at (x_0, y_0) to the curve $x^3 + y^3 = a^3$ meets the curve again at (x_1, y_1) then $\frac{x_1}{x_0} + \frac{y_1}{y_0}$ is equal to :

A. -1

B. 2a

C. 1

D. none of these

Answer: D

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124. If $\frac{x}{a} + \frac{y}{b} = 2$ touches the curve $\frac{x^n}{a^n} + \frac{y^n}{b^n} = 2$ at the point (α, β) then:

A. a)$\alpha = a^2, \beta = b^2$

B. b)$\alpha = a, \beta = b$

C. c)$\alpha = -2a, \beta = 4b$

D. d)$\alpha = 3a, \beta = -2b$

Answer: B

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125. The ength of substangent to the curve $y = e^{x/a}$ is :

A. 2a

B. a

C. $a/2$

D. $a/4$

Answer: B

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126. The length of normal to the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$ is:

A. 2a

B. a

C. $\sqrt{2a}$

D. $2\sqrt{2a}$

Answer: C

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127. Between any two real roots of the equation $e^x \sin x - 1 = 0$, the equation $e^x \cos x + 1 = 0$ has :

A. at least one root

B. at most one root

C. exactly one root

D. no root

Answer: A

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128. The value of 'c' of Rolls's throrem is applicable to the function $f(x) = 2x^3 - 5x^2 - 4x + 3, \text{x} \in [1/2, 3]$ is

A. 2

B. $-\frac{1}{3}$

C. -2

D. $\frac{2}{3}$

Answer: A

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129. If $27a+9b+3c+d=0,$ then the equation $4ax^3+3bx^2+2cx+d=0$ has at least one real root lying between

A. a)0 and 1

B. b)1 and 3

C. c)0 and 3

D. d)none of these

Answer: C

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130. If a function is everywhere continuous and differentiable such that $f'(x) \geq 6f$ or $all x \in [2, 4]$ and $f(2) = -4$, then

A. $f(4) < 8$

B. $f(4) \geq 8$

C. $f(4) \geq 2$

D. none of these

Answer: B

Watch Video Solution

131. The value of c in Lagrange's theorem for the function $f(x) = |x|$ in the interval $[-1, 1]$ is

A. 0

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. non-existent in the interval

Answer: D

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132. In a $\triangle ABC$ the sides b and c are given. If there is an error dA in measuring angle A, then the error da in side a is given by

A. $\frac{\triangle}{2a}dA$

B. $\frac{2\triangle}{a}dA$

C. bc sin A dA

D. none of these

Answer: B

Watch Video Solution

133. If a particle moving along a line follows the law $s = \sqrt{1+t}$. then the acceleration is proportional to

A. square of the velocity cube of

B. the displacement cube of the

C. velocity cube of the

D. square of the displacement

Answer: C

Watch Video Solution

134. A variable triangle is incribed in a circle of radius R, If the rate of change of a side is R times the rate of change of the opposite angle, then that angle is

A. $\pi / 6$

B. $\pi / 4$

C. $\pi/3$

D. $\pi/2$

Answer: C

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135. The function $f(x) = x^2(x-2)^2$

A. i) increase on $(0, 1) \cup (2, \infty)$

B. ii) decrease on $(-\infty, 0) \cup (2, \infty)$

C. iii) has a local minimum value 0

D. iv) has a local maximum value 1

Answer: D

Watch Video Solution

136. If $f(x) = x.\, e^{x(1-x)}$, then f(x) is

A. increasing on $[-1/2, 1]$

B. decreasing on R

C. increasing on R

D. decreasing on $[-1/2, 1]$

Answer: A

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137. If 2f(x) + 3f(-x) = 15 - 4x for all real values of x , then show that f(x) = 3 + 4x.

A. a)$b \in (-2, -1) \cup [1, \infty)$

B. b)$b \in [-2, -1) \cup [1, \infty)$

C. c)$b \in (-2, -1] \cup (1, \infty)$

D. d)$b \in (-2, -1) \cup (1, \infty)$

Answer: D

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138. The interval in which $2x^3 + 5$ increases less rapidly than $9x^2 - 12x$, is

A. a)$(- \infty, 1)$

B. b)(1,2)

C. c)$(2, \infty)$

D. d)none of these

Answer: B

Watch Video Solution

139. Let the function $g, (- \infty, \infty) \to \left(- \frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$, Then ,g is

A. even and is strictly increasing in $(0, \infty)$

B. odd and is strictly decreasing in $(-\infty, \infty)$

C. odd and is strictly increasing in $(-\infty, \infty)$

D. neither even nör ödd , but is strictly Increasing in $(-\infty. \infty)$

Answer: C

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140. For the function $f(x) = 2x^2 - In|x|$

A. set of critical points is $\{0, 1/2\}$

B. f(x) is increasing in $(-\infty - 1/2) \cup [0, 1/2]$

C. f(x) is decreasing in $(-1/2, 0) \cup [1/2, \infty)$

D. none of the above

Answer: A

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141. If $Y=\dfrac{(ax-b)}{(x-4)(x-1)}$ has extreme value at $(2,\ -1)$ then the value of $(a+10b)$ must be

A. a)0

B. b)-1

C. c)-2

D. d)1

Answer: D

Watch Video Solution

142. The number of critical points of the function $f(x)=|x|e^{-x}$ must be

A. 0

B. 2

C. 3

D. 4

Answer: B

Watch Video Solution

143. The minimum value of the function $f(x) = 2|x-2| + 5|x-3|$ for all $\mathrm{x} \in R$ is

A. a)3

B. b)2

C. c)5

D. d)7

Answer: B

Watch Video Solution

144. The coordinates of the point on the parabola $y^2 = 8x$ which is at a minimum distance from the circle $x^2 + (y+6)^2 = 1$ are

A. (2,4)

B. $(2, -4)$

C. $(-2, 4)$

D. none of these

Answer: B

Watch Video Solution

145. If $f(x) = |x| + |x-1| + |x-2|$, then

A. a)f(x) has minima at x=1

B. b)f(x) has maxima x=1

C. c)f(x) has neither maxima nor minima at x=1

D. d)none of the above

Answer: A

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146. Let f(x) be a function defined as $f(x) = \begin{cases} \sin(x^2 - 3x) & x \leq 0 \\ 6x + 5x^2 & x > 0 \end{cases}$

Then at x=0,f(x)

A. local maxima at x = 0

B. local minima at x = 0

C. global maxima at x = 0

D. global minima at x = 0

Answer: B

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147. Let $f(x) = \begin{cases} 2 - |x^2 + 5x + 6| & x \neq -2 \\ a^2 + 1 & x = -2 \end{cases}$ then value of 'a' so that f(x) has a maximum at $x = -2$ is

A. $a \in (-1, 1)$

B. $a \in [-1, 1]$

C. $a \in (1, \infty)$

D. $a \in (-\infty, 1)$

Answer: C

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148. Let $f(x) = \begin{cases} |x^2 - 2| & -1 \leq x < \sqrt{3} \\ \frac{x}{\sqrt{3}} & \sqrt{3} \leq x < 2\sqrt{3} \\ 3 - x & 2\sqrt{3} \leq x \leq 4 \end{cases}$

Then the points where f(x)takes maximum and minimum values are

A. 1,4

B. 0,4

C. 2,4

D. none of these

Answer: B

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149. The minimum value of $a\tan^2 x + b\cot^2 x$ equals the maximum value of $a\sin^2\theta + b\cos^2\theta$ where $a > b > 0,$ then a equals to

A. a)a = b

B. b)a = 2b

C. c)a = 3b

D. d)a = 4b

Answer: D

Watch Video Solution

150. All the possible values of the paramter 'a' so that the function , $f(x) = x^3 - 3(7-a)x^2 - 3(9-a^2)x + 2,$ has a negative point of local minimum

A. $a < \frac{29}{7}$

B. $a > 7$

C. $a > 0$

D. No real value of 'a'

Answer: A

Watch Video Solution

151. The equation of the tangent to the curve $y = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ at the origin is

A. x = 0

B. x = y

C. y = 0

D. none of these

Answer: C

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152. The equation of the normal to the curve $y = e^{-2|x|}$ at the point where the curve cuts the line $x = -\frac{1}{2}$, is

A. $2e(ex + 2y) = 4 - e^2$

B. $2e(ex - 2y) = e^2 - 4$

C. $2e(ey - 2x) = e^2 - 4$

D. $2e(ey + 2x) = e^2 - 4$

Answer: A

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153. If the tangent to the curve $2y^3 = ax^2 + x^3$ at the point (a,a) cuts off Intercepts α and β on the coordinate axes, (where $\alpha^2 + \beta^2 = 61)$ then the value of a is

A. a)-30

B. b)10

C. c)20

D. d)none of these

Answer: D

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154. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant a normal is drawn . The normal Intersects the y-axis at the point (0,b). If $\lim_{a-0} b = \frac{1}{2}$, then n equals

A. 2

B. 4

C. 5

D. 6

Answer: A

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155. The point (s) on the curve $y^3 + 3x^2 = 12y$ where line tangent is vertical , is (are)

A. a) $\left(\pm \frac{4}{\sqrt{3}}, -2 \right)$

B. b) $\left(\pm \sqrt{\frac{11}{3}}, 1 \right)$

C. c)(0,0)

D. d) $\left(\pm \frac{4}{\sqrt{3}}, 2 \right)$

Answer: D

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156. The area of the triangle formed by the positive x-axis, and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

A. $3\sqrt{2}$ sq,units

B. $2\sqrt{3}$ sq,units

C. $4\sqrt{3}$ sq,units

D. none of these

Answer: B

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157. The function f satisfying $\frac{f(b) - f(a)}{b - a} \neq f'(x)$ for any x $\in (a, b)$ is

A. $f(x) = x^{1/3}, a = -1, b = 1$

B. $f(x) = x|x| : a = -1, b = 1$

C. $f(x) = \frac{1}{x} : a = 1, b = 4$

D. None of these

Answer: B

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158. Suppose that f is differntiable for all x such that $f'(x) \leq 2$ for all x, If f(1)=2 and f(4) =8 , then f(2) has the least value equal to

A. 3

B. 4

C. 5

D. 6

Answer: B

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159. f(x) is a polynomial of degree 4 with real coefficients such that f(x)=0 is satisfied by x=1,2,3 only then f'(1).f'(2).f'(3) is equal to :`

A. 0

B. 2

C. -1

D. none of these

Answer: A

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160. If f(x) is differentiable and strictly increasing function, then the value of $\lim_{x \to 0} \dfrac{f(x^2) - f(x)}{f(x) - f(0)}$ is

A. 1

B. 0

C. -1

D. 2

Answer: C

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161. Let $f(x) = \max\{\sin x, \cos x\} \forall \text{x} \in R$ then number of critical points of f(x) in $(0, 2\pi)$ is

A. 2

B. 3

C. 4

D. 5

Answer: B

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162. If f(x) and g(x) are differentiable functions for $0 \leq x \leq 1$ such that f(0)=2,g(0)=0,f(1)=6,g(1)=2, then in the interval (0,1)

A. f(x)=0,forall x

B. f(x)=2g'(x), for atleast one x

C. f(x) =2g'(x), for atmost one x

D. none of the above

Answer: B

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163. The number of distinct real root of $x^3 - 3x + 1 = 0$ in (1,2) is

A. 1

B. 2

C. 3

D. none of these

Answer: A

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164. If $|f(x_2) - f(x_1)| \leq (x_2 - x_1)^2 \forall x_1, x_2 \in R,$ then the equation of tangent to the curve y = f(x) at the point (1,2) is

A. y=2

B. $y = -2$

C. $y = 2x + 1$

D. none of these

Answer: D

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165. If $0 < \alpha < \frac{\pi}{6}$ then the value of $(\alpha \cos ec\alpha)$ is:

A. less than $\frac{\pi}{3}$

B. more then $\frac{\pi}{3}$

C. less than $\frac{\pi}{6}$

D. more than $\frac{\pi}{6}$

Answer: A

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166. Let $f(x) = 3x^2 + 4xg'(1) + g''(2)$ and $g(x) = 2x^2 + 3xf'(2) + f''(3)$ for all x $\in R$. Then ,

A. $f'(1) = 22 + 12f'(2)$

B. $g(2) = 44 + 12g'(1)$

C. $f'(3) + g''(2) = 10$

D. all of these

Answer: C

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167. If the value of maximum slope of $f(x) = -x^3 + 8x^2 - 13x - 18$ is λ then the value of $\sqrt[3]{15\lambda}$ must be

A. 1

B. 2

C. 3

D. 5

Answer: D

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168. A tangent to the curve $y = \int_0^x |t| dt$ which is parallel to the line y=x, cuts off an intercept from the y-axis is equal to

A. a)1

B. b) $-\frac{1}{2}, \frac{1}{2}$

C. c) $\frac{1}{2}, 1$

D. d) -1

Answer: B

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169. The angle of intersection of the curves $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 5$ where [] denotes the greatest integer function is

A. $\pi/2$

B. $\pi/4$

C. $\tan^{-1}(2)$

D. $\tan^{-1}\left(\frac{1}{2}\right)$

Answer: C

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170. Let $f(x) = \int_0^x \frac{\cos t}{t} dt (x > 0):$ then at $x = (2n+1)\frac{\pi}{2} n \in 1,$ f(x) has

A. maxima, when$n = -2, -4, -6, ..$

B. maxima, when $n = -1, -3, -5, ..$

C. minima, when $n = 0, 2, 4, ..$

D. none of these

Answer:

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171. If p,q,r are any real number, then

A. $\max(p, q) < \max(p, q, r)$

B. $\min(p, q) = \frac{1}{2}\{p + q - |p - q|\}$

C. max(p,q) < min(p,q,r)

D. none of the above

Answer: B

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172. Consider the two curves $C_1 : y^2 = 4x, C^2 : x^2 - y^2 - 6x + 1 = 0.$

Then.

A. $C_1$and C_2 touch each other only at one point

B. C_1 and C_2 touch each other exaclly at two points

C. C_1 and C_2 intersect (but do not touch) at exactly two points

D. C_1 and C_2 neither intersect nor touch each other

Answer: B

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173. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then (a) $a > 0, b > 0$ (b)$a > 0, b < 0$ (c)$a \langle 0, b \rangle 0$ (d) $a < 0, b > 0$

A. $a > 0, b > 0$

B. $a > 0, b < 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

Answer: B::C

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174. If $f'(x) = x$ and $f(1) = 3$, find f(x)

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175. The interval to which 'b' may belong so that the function $f(x) = \left(1 - \frac{\sqrt{21 - 4b - b^2}}{b+1}\right)x^3 + 5x + \sqrt{6}$ is increasing at every point of its domain is

A. $[-7, -1]$

B. $[-6, -2]$

C. [2,2.5]

D. [2,3]

Answer: A::B::C::D

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176. The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5$ dt has a local minimum at 'x' equal to

A. a)0

B. b)1

C. c)2

D. d)3

Answer: B::D

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177. If a line is tangent to one point and normal at another point on the curve $x = 4t^2 + 3, y = 8t^3 - 1,$ then slope of such a line is

A. -1

B. 1

C. $-\sqrt{2}$

D. $\sqrt{2}$

Answer: C::D

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178. Let $f'(x) > 0$ and $g'(x) < 0$ for all x in R, then

A. $f\{g(x)\} > f\{g(x+1)\}$

B. $f\{g(x) > f\{g(x-1)\}$

C. $g\{f(x)\} > g\{f(x+1)$

D. $g\{f(x)\} > g\{f(x-1)\}$

Answer: A::C

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179. If $f(x) = \begin{cases} 3x^2 + 12x - 1 & -1 \leq x \leq 2 \\ 37 - x & 2 < x \leq 3 \end{cases}$

A. f(x)is increasing on $[-1, 2]$

B. f(x)is continuous on $[-1, 3]$

C. f(x)dose not exist at x=2

D. f(x)has the maximum value at x =2

Answer: A::B::C::D

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180. Let $f(x) = (x-1)^m (x-2)^n$, x $\in R$ then each critical point of f(x) is either local maximum or local minimum where

A. m = 2, n = 3

B. m = 2, n = 4

C. m = 3, n = 4

D. m =, n = 2

Answer: B::D

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181. The difference between the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$, is

A. for at least one x in the interval $[1\infty)$, $f(x+2)-f(x)<2$

B. $\lim_{x \to \infty} f(x)=1$

C. for all x in the intervel $[1, \infty)$, $f(x+2)-f(x)<2$

D. f(x)is strictly decreasing in the interval $[1, \infty)$

Answer: B::C::D

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182. For $f(x)=\int_0^x 2|t|$ dt ,the tangent lines which are parallel to the bisector of the first coordinate angle is

A. a)$y=x-1/4$

B. b)$y=x+1/4$

C. c)$y=x-3/2$

D. d)$y=x+3/2$

Answer: A::B

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183. Let $x^3 + ax^2 + bx + c$ be the given cubic polynomlal and f(x) = 0 be the corresponding cubic equation, where a,b,c, $\in$ R

Now,$f(x) = 3x^2 + 2ax + b$ Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation f(x) = 0 then

If $D = 4a^2 - 12b = (a^2 - 3b) < 0.$ then the roots of the equation f(x) = 0 will be

A. f(x) has all real roots

B. f(x)has one real and two imaginary roots

C. f(x)has repeated roots

D. None of the above

Answer: B

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184. Let $x^3 + ax^2 + bx + c$ be the given cubic polynomlal and

f(x) = 0 be the corresponding cubic equation, where a,b,c, $\in$ R

Now, $f'(x) = 3x^2 + 2ax + b$

Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation

f(x)=0 then

If $D = 4(a^2 - 3b) > 0$ and $f(x_1,).\ f(x_2) > 0.$

where x_1, x_2 are the roots of f'(x),then

A. a) f(x) has all real and disitinct roots

B. b) f(x)would have three real roots but one of the would be repacted

C. c) f(x)would have just one real root

D. d) none of the above

Answer: C

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185. Let f and g are two functions such that f(x) and g (x)are connuous in [a,b] differentlable in (a,b) . then atleast one $c \in (a, b)$ such that

$$f(c) = \frac{f(b) - f(a)}{b - a}$$

If f(a) = f(b), then f'©=0 (RMVT)

If f(a)ne$f(b)$ and $a$ne$b$, $(LMVT) I f g'(x)$ne0, $then$(f(b)-F(a))/(g(b)-g(a))=(f'(c))/(g'(c)$thesetofvaluesof'k'f$ or $whichequation$x^3-3x+k=0` has two disitinct roots in (0.1) is

A. (1,4)

B. $(0, \infty)$

C. (0,1)

D. ϕ

Answer: D

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186. Let f and g are two functions such that f(x) and g (x)are connuous in [a,b] differentlable in (a,b) , then atleast one $c \in (a, b)$ such that $f(c) = \frac{f(b) - f(a)}{b - a}$

If f(a) = f(b), then f'©=0 (RMVT)

If f(a) $\neq$ f(b) and a $\neq$ b, (LMVT)

If g'(x) $\neq$ 0,then $\frac{f(b) - F(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$

Which of the fllowing is true ?

A. $\left|\tan^{-1} x - \tan^{-1} y\right| \geq |x - y|, \forall x, y \in R$

B. $\left|\tan^{-1} x - \tan^{-1} y\right| \geq |x - y| \forall x, y \in R$

C. $|\sin x - \sin y| \geq |x - y|, \forall x, y \in R$

D. none of the above

Answer: A

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187. If $f(x) = \frac{1}{1 + x}$, then find $f[f\{f(x)\}]$.

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188. If f(x)=$\dfrac{4^x}{4^x-2}$ then find the value f(x)+f(1-x).

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189. The number of disitinct real roots of $x^4-4x^3+12x^2+x-1=0$ is

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190. The maximum value of the expression $\dfrac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

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191. Let $f(x)=\begin{cases}|x^3+x^2+3x+\sin x|\left(3+\sin\frac{1}{x}\right) & x\neq 0\\ 0 & x=0\end{cases}$ then number of points (where f(x)attains its minimum value) is

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192. Let p(x)be a polynomial of degree 4 having extremum at x = 1,2 and $\lim_{x\to 0}\left(1+\frac{p(x)}{x^2}\right)=2$ Then the value of p(2) is

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193. Consider the function $f(x)=8x^2-7x+5$ on the interval $[-6,6]$ .the value of c that satisfies the condition of the mean value theorem is

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194. Given, $f(x)=\frac{x-1}{2x^2-7x+5}$ when `xne1 Find the derivative of f(x) at x=1

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195. If $y=\sqrt{\frac{1-x}{1+x}}$ find $\frac{dy}{dx}$ and prove that $\left(1-x^2\right)\frac{dy}{dx}+y=0$

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196. if $y=x^3$ then prove that $x\frac{dy}{dx}-3y=0$

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197. Find the intervals in which the following functions are increasing or decreasing $f(x)=3x^2-4x$

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198. Find the intervals in which the follwing functions are increasing or decreasing $f(x)=5x^2+7x-13$

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199. Find the intervals in which the follwing functions are increasing or decreasing $f(x) = -3x^2 + 12x + 8$

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200. Find $\lim_{x \to 0} \left(\frac{\sin(\pi \sin^2(x))}{x^2}\right)$

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201. $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = ?$

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202. If $f(x) = ax^3 + bx^2 + cx + d$ has local extrema at two points of opposite signs than prove that roods of the quadratic $ax^2 + bx + c = 0$

are real and distinct.

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203. Prove that if $2a_0^2 < 15a$ all roots of $x^5 - a_0x^4 + 3ax^3 + bx^2 + cx + d = 0$ can't be real. It is given that $a_0, a, b, c, d, \in R.$

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204. Let a,b,c be real. If $ax^2 + bx + c = 0$ has two real roots α, β where $\alpha < -1$ and $\beta > 1$,then show that $1 + \frac{c}{a} + \left|\frac{b}{a}\right| < 0.$

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205. Let $a + b = 4$, where a gt 2 and let g(x)be a differentiable function. If $\frac{dg}{dx} > \forall x$prove that $\int_0^a g(x)dx + \int_0^b g(x)dx$ increases as $(b - a)$

increases .

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206. On the curve $x^3 = 12y$ the abscissa changes at a faster rate than the ordinate then y belongs to the interval

A. $(-2, 2)$

B. $\left(-\frac{2}{3}, \frac{2}{3}\right)$

C. $(-1, 1)$

D. $\left(-\frac{1}{12}, \frac{1}{12}\right)$

Answer: B

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207. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50cm^3 / \min$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases is

A. $\frac{1}{36\pi} cm / \min$

B. $\frac{1}{18\pi} cm / \min$

C. $\frac{1}{54\pi} cm / \min$

D. $\frac{5}{6\pi} cm / \min$

Answer: B

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208. The function $f(x) = \tan^{-1} x - x$ decreases in the interval

A. $(1, \infty)$

B. $(-1, \infty)$

C. $(-\infty, \infty)$

D. none of these

Answer: D

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209. The function $f(x) = 2\log(x-2) - x^2 + 4x + 1$ increases in the interval

A. (1,2)

B. (-1,2)

C. $\left(\frac{5}{2}, 3\right)$

D. (2,4)

Answer: C

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210. The function $f(x) = \sin^4 x + \cos^4 x$ increasing if :

A. $0 < x < \frac{\pi}{8}$

B. $\frac{\pi}{4} < x < \frac{3\pi}{8}$

C. $\frac{3\pi}{8} < x < \frac{5\pi}{8}$

D. $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Answer: B

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211. The length of the longest interval in which the function $3\sin x - 4\sin^3 x$ is increasing, is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

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212. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

A. $(\pi/4, \pi/2)$

B. $(-\pi/2, \pi/4)$

C. $(0, \pi/2)$

D. $(-\pi/2, \pi/2)$

Answer: B

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213. The function f defined by $f(x) = 4x^4 - 2x + 1$ is increasing for

A. $x < 1$

B. $x > 0$

C. $x < 1/2$

D. $x > 1/2$

Answer: D

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214. The value of a for which the function $f(x) = a\sin x + \frac{1}{3}\sin 3x$ has an extremum at $x = \frac{\pi}{3}$ is

A. 1

B. -1

C. 0

D. 2

Answer: D

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215. $x > 0, y > 0$ and $x + y = 1$. The minimum value of $x \log x + y \log y$ is

A. 2 log2

B. log2

C. 0

D. $-\log 2$

Answer: D

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216. The minimum value of $2 \log_{10} x - \log_x 0.01$. $x > 1$, is

A. 1

B. $(- 1)$

C. 2

D. none of these

Answer: D

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217. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ is

A. 5

B. (-5)

C. $1/5$

D. none of these

Answer: A

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218. For the function $f(x) = xe^x$, the point

A. $x = -1$ is a maximum

B. x = 0 is a minimum

C. $x = -1$ is a minimum

D. x = 0 is a maximum

Answer: C

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219. The minimum value of $\log_a x + \log_x a, 0 < x < a,$ is

A. 1

B. 2

C. -2

D. none of these

Answer: B

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220. The sides of the rectangle of the greatest area, that can be inscribed in the ellipse $x^2 + 2y^2 = 8$ are given by

A. $4\sqrt{2}, 4$

B. $4, 2\sqrt{2}$

C. $2, \sqrt{2}$

D. $2\sqrt{2}, 2$

Answer: B

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221. Tangents are drawn to the ellipse $x^2 + 2y^2 = 2$, then the locus of the mid-point of the intercept made by the tangents between the coordinate axes is

A. $\frac{1}{2x^2} + \frac{1}{4y^2} = 1$

B. $\frac{1}{4x^2} + \frac{1}{2y^2} = 1$

C. $\frac{x^2}{2} + \frac{y^2}{4} = 1$

D. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Answer: A

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222. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$

A. on the left of x = c

B. on the right of .x = c

C. at no point

D. at all points

Answer: A

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223. The curves $\frac{x^2}{a}+\frac{y^2}{b}=1$and $\frac{x^2}{a_1}+\frac{y^2}{b_1}=1$ will cut orthogonally if (a-b) =

A. a_1+b_1

B. b_1-a_1

C. a_1-b_1

D. none of these

Answer: C

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224. If the tangent to the curve $2y^3=ax^2+x^3$ at the point (a,a) cuts off Intercepts α and β on the coordinate axes, (where $\alpha^2+\beta^2=61)$ then the value of a is

A. 16

B. 28

C. 30

D. 31

Answer: C

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225. The normal to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ at any point θ is such that

A. it passes through the origin

B. it makes an angle $\frac{\pi}{2} + \theta$ with the x-axis

C. it passes through $\left(a\frac{\pi}{2}, -a\right)$

D. it is at a constant distance from the origin

Answer: D

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226. The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at the point x = 0

A. $\frac{2}{\sqrt{3}}$

B. $\frac{\sqrt{2}}{3}$

C. $\frac{\sqrt{3}}{2}$

D. none of these

Answer: D

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227. Let $g(x) = \begin{cases} 2e & \text{if} \quad x \leq 1 \\ \log(x-1) & \text{if} \quad x > 1 \end{cases}$ The equation of the normal to y = g(x)at the point (3,log 2), is

A. $y - 2x = 6 + \log 2$

B. $y + 2x = 6 + \log 2$

C. $y + 2x = 6 - \log 2$

D. $y + 2x = -6 + \log 2$

Answer: B

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228. The intercepts on x-axis made by tangents to the curve, $\int_0^x |t| dt, x \in R$ which are parallel to the line y = 2x, are equal to :

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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229. The Rolle's thorem is applicable in the interval $-1 \leq x \leq 1$ for the function

A. f(x) = x

B. $f(x) = x^2$

C. $f(x) = 2x^3 + 3$

D. f(x)=|x|

Answer: B

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230. A value of 'c' for which the conclusion of mean value theorem holds for the function $f(x) = \log_e x$ on the interval [1,3] is

A. $2 \log_3 e$

B. $\frac{1}{2} \log_e 3$

C. $\log_3 e$

D. $\log_e 3$

Answer: A

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231. A value of 'c' for which conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1,3] is

A. $\log_3 e$

B. $\log_e 3$

C. $2 \log_3 e$

D. $\frac{1}{2} \log_e 3$

Answer: C

Watch Video Solution

232. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$ then in $(-\infty, \infty)$

A. f(x) is striclly increasing function

B. f(x) has a local maxima

C. f(x) is a stricly decreasing function

D. f(x) is unbounded

Answer: A

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233. If f is a twice differentiable function such that $f''(x) = -f(x), f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2$ if h(5)=11, then h(10) equals

A. 8

B. 11

C. 12

D. none of these

Answer: B

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234. Consider a function y = f(x) defined parametrically as $y = 2t + t, x = t^2 \ \forall t \in R$ then the function f(x) is

A. differentiable at x = 0

B. non-differentiable at x = 0

C. nothing can be said about differentiability at x = 0

D. none of these

Answer: A

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235. The range of value of x for which the function $f(x)=\frac{x}{\log x}, x>0$ and x $\neq$ 1, may be decreasing is

A. (0,e)

B. (e, ∞)

C. $(0, e)-\{1\}$

D. none of these

Answer: C

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236. If $F(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, then F'(x) is

A. x^2

B. 2x

C. x

D. none of these

Answer: D

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237. The least value of the function f(x)= $\cos^{-1} x^2$ in the interval $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{-\pi}{3}$

D. none of these

Answer: A

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238. If a function is everywhere continuous and differentiable such that $f'(x) \geq 6f$ or $all x \in [2, 4]$ and $f(2) = -4$, then

A. $f(4) < 8$

B. $f(4) \geq 8$

C. $f(4) \geq 2$

D. none of these

Answer: B

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239. If f'(x) exists for all $x \in R$ and $g(x) = f(x) - (f(x))^2 + (f(x))^3 \, \forall x \in R$, then

A. g(x) is decreasing whenever f(x) is increasing

B. g(x) is increasing whenever f(x) is decreasing

C. g(x) is decreasing whenever f(x) is decreasing

D. none of these

Answer: C

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240. The set of all values of a for which the function

$f(x) = \left(\frac{\sqrt{a+4}}{1-a} - 1\right)x^5 - 3x + \log 5$ decreases for all real x is

A. $(-\infty, \infty)$

B. $\left[-4, \frac{3-\sqrt{21}}{2}\right] \cup (1, \infty)$

C. $\left(-3, \frac{5-\sqrt{21}}{2}\right) \cup (2, \infty)$

D. $[1, \infty)$

Answer: B

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241. The minimum value of the function defined by $f(x) = \max\{x, x+1, 2-x\}$ is

A. 0

B. $\frac{1}{2}$

C. 1

D. $\frac{3}{2}$

Answer: D

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242. If $f(x) > 0$ and $f(x) > 0 \forall x \in R$, then for any two real numbers x_1 and x_2, $(x_1 \neq x_2)$

A. $f\left(\frac{x_1+x_2}{2}\right) > \frac{f(x_1)+f'(x_2)}{2}$

B. $f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$

C. $f'\left(\frac{x_1+x_2}{2}\right) > \frac{f'(x_1)+f'(x_2)}{2}$

D. $f'\left(\frac{x_1+x_2}{2}\right) < \frac{f'(x_1)+f'(x_2)}{2}$

Answer: B

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243. If $0 < \alpha < \beta < \frac{\pi}{2}$ then

A. $\frac{\tan\beta}{\tan\alpha} < \frac{\alpha}{\beta}$

B. $\frac{\tan\beta}{\tan\alpha} > \frac{\alpha}{\beta}$

C. $\frac{\tan\alpha}{\tan\beta} > \frac{\alpha}{\beta}$

D. none of these

Answer: B

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244. A function f is such that f'(4)=f''(4)=0 and f has minimum value 10 at x=4. Then f(x) is equal to

A. $4 + (x - 4)^4$

B. $10 + (x - 4)^4$

C. $(x - 4)^4$

D. none of these

Answer: B

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245. If $f(x) = x^2 + 2bx = 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min $f(x) > \max g(x)$, then the relation between b and c is

A. no relation

B. $0 < c < b/2$

C. $|c| < |b|\sqrt{2}$

D. $|c| > |b|\sqrt{2}$

Answer: D

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246. If $f(x) = x^{\alpha} \log x$ and $f(0) = 0$, then the value of α for which Rolle's theorem can be applied in [0,1] is

A. -2

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D

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247. If $0 < x < \frac{\pi}{2}$,then

A. $\cos x > 1 - \frac{2x}{\pi}$

B. $\cos x < 1 - \frac{2x}{\pi}$

C. $\cos x > \frac{2x}{\pi}$

D. $\cos x < \frac{2x}{\pi}$

Answer: A

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248. The number of values of x where the function $f(x) = \cos x + \cos\left(\sqrt{2}x\right)$ attains its maximum is

A. 0

B. 1

C. 2

D. Infinite

Answer: B

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249. The value of a so that the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a + 1 = 0$ assume the least value, is

A. 2

B. 0

C. 3

D. 1

Answer: D

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250. The equation $e^{x - 1} + x - 2 = 0$ as

A. one real root

B. two real roots

C. three real roots

D. our real roots

Answer: A

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251. For the function $y = f(x) = (x^2 + bx + c)e^x$, which of the following holds ?

A. If $f(x) > 0$ for all real $x \neq f'(x) > 0$

B. If $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

C. If $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

D. If $f(x) > 0$ for all real $x \neq f(x) > 0$

Answer: A::C

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252. The function $y = \frac{x}{1+x^2}$ decreases in the interval

A. $(-1, 1)$

B. $(1, \infty)$

C. $(-\infty, -1)$

D. $(-\infty, \infty)$

Answer: B::C

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253. If $f(x) = \log_e x$ is defined in [1,e], then

A. f(x) is an increasing function in (1,e)

B. f(x) holds Rolle's theorem and $c = \frac{1}{e}$

C. f(x) holds Lagrange's MVT and $f'(c) = \frac{1}{e-1}$

D. area enclosed by y=f(x) and the line passing through (1,0) and (e,1) is

$\frac{3-e}{2}$ sq. units

Answer: A::C::D

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254. The function $f(x)=\int_0^x \sqrt{1-t^4}$dt is such that

A. It is defined on the interval [-1,1]

B. it is an incerasing function

C. it is an odd function

D. the points (0,0) is the point of infiection

Answer: A::B::C::D

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255. Let $g(x)=2f\left(\frac{x}{2}\right)+f(x)$ and $f''(x)<0$ in $0\leq x\leq 1$, then g(x)

A. decreases in $\left[0,\frac{2}{3}\right)$

B. decereases in $\left(\frac{2}{3},1\right]$

C. increases in $\left(0,\frac{2}{3}\right]$

D. increases in $\left(\frac{2}{3},1\right]$

Answer: B::C

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256. The critical points of the function $f(x)=\frac{|x-1|}{x^2}$ are

A. 0

B. 1

C. 2

D. -1

Answer: A::B::C

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257. The extremum values of the function $f(x) - \frac{1}{\sin x + 4} - \frac{1}{\cos x - 4}$ where $x \in R$ is

A. $\frac{4}{8-\sqrt{2}}$

B. $\frac{2\sqrt{2}}{8-\sqrt{2}}$

C. $\frac{2\sqrt{2}}{4\sqrt{2}+1}$

D. $\frac{4\sqrt{2}}{8+\sqrt{2}}$

Answer: A::C

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258. If $f(x) = \sqrt{8x}$, $g(x) = x + 2$, then

A. $y = g(x)$ is the tangent to $y = f(x)$

B. normal at the point of contact cuts x-axis at $(6, 0)$

C. $y = g(x)$ never be a tangent to $y = f(x)$

D. $y = g(x)$ cuts $y = f(x)$ at (2,4)

Answer: A::B::D

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259. Let $f(x) = \begin{cases} x+2 & -1 \leq x \leq 0 \\ 1 & x = 0 \\ \frac{x}{2} & 0 < x \leq 1 \end{cases}$ Then on [-1,1] this function has

A. a minimum

B. a maximum

C. neither a maximum nor a minimum

D. f'(0) does not exist

Answer: C::D

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260. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are

A. $\frac{a}{\sqrt{2}}$

B. $-\frac{a}{\sqrt{2}}$

C. $a\sqrt{2}$

D. $-a\sqrt{2}$

Answer: A::B

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261. The function $f(x) = 4x^2 + 2x + 1$ is increasing for?

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262. The function $f(x)=x^2(x-2)^2$

A. f(x) is decreasing on $(-1, 1)$ and has a local minimum at x=1.

B. f(x) is increasing on $(-1, 1)$ and has a local maximum at x=1.

C. f(x) is increasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at x=1.

D. f(x) is decreasing on $(-1, 1)$ but has neither a local maximum nor a local minimum at x=1.

Answer: A

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263. If $f(x)=|x-2|+|x-4|+|x-6|$.

Find the set of value of x such that f(x) is increases

A. [2,4]

B. [4,6]

C. $[4, \infty)$

D. none of these

Answer: C

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264. If $f(x) = |x-2| + |x-4| + |x-6|$.

Find the set of value of x such that f(x) is decreasing

A. $[4, \infty)$

B. $(-\infty, 4]$

C. $[2, \infty)$

D. (1) and (2) are correct

Answer: D

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265. At birth the body weight of a normal baby should be

A. -1

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 1

Answer: C

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266. At birth the body weight of a normal baby should be

A. ordinate

B. $\sqrt{2}$ ordinate

C. $\sqrt{2(\text{ordinate})}$

D. none of these

Answer: B

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267. If f(x) = is an odd periodic function with period 2, then f(4) equals

A. 2

B. 3

C. 4

D. 5

Answer: C

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268. If f(x) = is an odd periodic function with period 2, then f(4) equals

A. 1

B. 2

C. 3

D. 4

Answer: B

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269. Match List - I with List-II

Let the functions defined in List - I have domain $(-\pi/2, \pi/2)$

	List - I		List - II
(1)	$x + \sin x$	(P)	increasing
(2)	$\sec x$	(Q)	decreasing
(3)	e^{-x}	(R)	neither increasing nor decreasing

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270. Match List - I with List-II

List - I	List - II
(1) The least value of the function $f(x) = 2 \cdot 3^{3x} - 3^{2x} \cdot 4 + 2 \cdot 3^{x}$ in $[-1, 1]$ is	(P) 5
(2) The minimum value of the polynomial $f(x) = (x-1)x(x+1)$ is	(Q) -1
(3) The value of the polynomial $\int_{-1}^{3}(\lvert x-2\rvert - [x])dx$ (where [ ] denotes the greatest integer function) is	(R) 3
(4) If period of the function $f(x) = \sin 36x \tan 42x$ is p, then $\frac{18p}{\pi}$ equals	(S) 0

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271. Match List - I with List-II

List - I		List - II	
(1)	Circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(P)	4
(2)	If an edge of a cube increases by 1%, then percentage increase in volume is	(Q)	0.6π
(3)	If the rate of decrease of $\frac{x^2}{2}-2x+5$ is twice the rate of decrease of x, then x is equal to (rate of decreases is non-zero)	(R)	3
(4)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(S)	$\frac{3\sqrt{3}}{4}$

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272. Find values of x over which the function $f(x) = x^2 + 2$ is increasing

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273. Lel $f(x) = x^3 + 6x^2 + ax + 2$ If the largest possible interval in which f(x) is a decreasing function is $(-3 - 1)$ then the value of a most be

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274. The function $S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right)$ dt has two critical points in the interval [1, 2.4] one of the critical points is a local minimum and the other is a local maximum. Find x where local minimum occurs .

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275. If P(x) be a polynomial of degree 3 satisfyints $P(-1) = 10, P(1) = -6$and P(x) has maximum at x =-1 and P (x)has minima at x=1 If the distance between local maximum and local minimum of the curve is $K\sqrt{65}$ find K.

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276. A line is tangent to the cvrve $f(x) = \frac{41x^3}{3}$ at the point p in the first quardant and has a slope of 2009. The line intersect the y-axis at $\left(0, -\frac{82}{3}K^3\right)$. Find K

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277. Let f(x) be an decreasing function defined on $(0, \infty)$. If $f(3a^2 - 4a + 1) > f(2a^2 + a + 1)$ and the range of a is $\left(o, \frac{K}{3}\right) \cup (K, K + 4)$, find K

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278. The curve $y = ax^3 + bx^2 + cx + d$ cut y axis at (0.5) where gradient is 3 curve touches x axis at $(-2. o)$ then $[a] + [b] + [c] + [d]$=(where [] is GIF)

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279. If tangent at point P(a,b) to the curve $x^3 + y^3 = c^3$ meet the curve agins at $Q(a_1, b_1)$ then $\frac{a_1}{a} + \frac{b_1}{b} =$

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280. Find the number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$

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281. The maximum area of a rectangle that can be inscribed in a circle of radius 2 unit is (in sq unit)

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282. If $x+4y=14$ is a normal to the curve $y^2=ax^3-\beta$at (2,3) then value of $\alpha+\beta$ is

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283. If f(x) and g(x) are differentiable function for$0\leq x\leq 1$such that $f(0)=2, g(0)=0, f(1)=6, g(1)=2,$ then show that these exist c satisfying $0 < c < 1$ and $f'(c)=2g'(c)$

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284. If $ax^2+\frac{b}{x}\geq c$for all positive x where $a>0$ and $b>c$ show that $27ab^2\geq 4c^3$

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285. find all the tangents to the curve $y = \cos(x + y), \; - 2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$

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286. Find the interval in which $f(x) = 4x^3 + 3x^2 + 2x + 1$ is increasing.

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287. A cubic f(x) vanishes at x= -2 and has relative minimum and maximum at x = -1 and $x = \frac{1}{3}$ if $\int_{-1}^{1} f(x)dx = \frac{14}{3}$ find the cubic f(x)

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288. Find the coordinates of the point on the curve $y = \frac{x}{1 + x^2}$ where the tengent to the curve has the greatest slope .

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289. Find the number of points of maxima and minima of the function. $f(x) = \frac{1}{8}\text{In}x - bx + x^2, x > 0$ where $b \geq 0$ is a constant.

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290. Find a point on the curve $x^2 + 2y^2 = 6$ whose distanced from the line $x + y = 7$, is minimum.

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291. Prove that for any two numbers x_1 and x_2. $\frac{2e^{x_1} + e^{x_2}}{3} > e^{\frac{2(x_1) + x_2}{3}}$

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292. If $|f(x_2) - f(x_1)| \leq (x_2 - x_1)^2 \forall x_1, x_2 \in R,$ then the equation of tangent to the curve y = f(x) at the point (1,2) is

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293. If $y = f(x) = \frac{x+2}{x-1}$ then

A. f has a local minimum at x=2

B. f has a local maximum at x=2

C. $f' '(2) > f(2)$

D. $f(x) - f' '(x) = 0$ for at least one $x \in R$

Answer: A::D

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294. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then :

A. 2x=r

B. $2x = (\pi + 4)r$

C. $(4 - \pi)x = \pi r$

D. x=2r

Answer: D

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295. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}, x \in \left(0, \frac{\pi}{2}\right)\right)$. A normal to y=f(x) at $x = \frac{\pi}{6}$ also passes through the point :

A. $\left(\frac{\pi}{4}, 0\right)$

B. (0,0)

C. $\left(0, \frac{2\pi}{3}\right)$

D. $\left(\frac{\pi}{6}, 0\right)$

Answer: C

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296. The line $y = x + \lambda$ is tangent to the ellips $2x^2 + 3y^2 = 1$ Then λ is

A. (-2)

B. 1

C. $\sqrt{\frac{5}{6}}$

D. $\sqrt{\frac{2}{3}}$

Answer: C

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297. The time T of of oscillation of a simple pendulum of length l is given by $T = 2\pi\sqrt{\frac{l}{g}}$. Find the percentage error in T, corresponding to an error of 2% in the value of l.

A. 2%

B. 1%

C. $\frac{1}{2}\%$

D. none of these

Answer: B

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298. The points of the ellips $16x^2 + 9y^2 = 400$ at which the ordinate decreases at the same rate at which the abscissa oncreases is/are given by

A. $\left(3, \frac{16}{3}\right)$ and $\left(-3, \frac{-16}{3}\right)$

B. $\left(3, -\frac{16}{3}\right)$ and $\left(-3, \frac{16}{3}\right)$

C. $\left(\frac{1}{16}, \frac{1}{9}\right)$ and $\left(-\frac{1}{16}, -\frac{1}{9}\right)$

D. $\left(\frac{1}{16}, -\frac{1}{9}\right)$ and $\left(-\frac{1}{16}, \frac{1}{9}\right)$

Answer: A

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299. If f(x) is a function such that f'(x)=$(x-1)^2(4-x)$, then

A. f(0)=0

B. f(x) is increasing in (0,3)

C. x=4 is a critical point of f(x)

D. f(x) is decereasing in (3,5)

Answer: B::C

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300. A cylindrical container is to be made from certain solid material which the following constraints : It has a fixed inner volume of Vmm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the materia! used to make the container is minimum when the inner radius of the container is 10 mm, then the value of $\frac{V}{250\pi}$ is

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301. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$ is

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302. If y = a + bx and M_o is the mode of x, then show that the mode of y must be $a + bM_o$.

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303. Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$.

The common langent to H and S intersects the x-axis at point

M. If (l,m) is the centroid of the triangle $\triangle PMN$, then the

correct expression(s)is(are)

A. $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

B. $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 >$

C. $\frac{dl}{dx_1} = 1 + \frac{1}{3x^2}$ for $x_1 > 1$

D. $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

Answer: A::B::D

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304. The length of the latus rectum of the ellipse $2x^2 + 4y^2 = 16$ is

A. 18

B. $\frac{27}{2}$

C. 27

D. $\frac{27}{4}$

Answer: C

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305. The normal to the curve $x^2 + 2xy - 3y^2 = 0$ at (1,1) :

A. meets the curve again in the second quadrant

B. meets the curve again in the third quadrant

C. meets the curve again in the fourth quadrant

D. does not meet the curve again

Answer: C

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306. For all real values of $a_0 . a_1, a_2, a_3$ satisfying $a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} + a_3 \frac{x^4}{4} = 0,$ the equation $a_0 + a_1 x + a_2 x^2 + a_3 x^3 = 0$ has a real root in the interval

A. [0,1]

B. [-1,0]

C. [1,2]

D. [-2,-1]

Answer: A

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307. A particle starts moving from rest from a fixed point in a fixed direction. The distance s from the fixed point at a time t is given by $s = t^2 + at - b + 17$, where a, b are real numbers. If the particle comes to rest after 5 sec at a distance of s = 25 units from the fixed point, then values of a and b are respeetively

A. $10, \ -33$

B. $-10, \ -33$

C. $-8, 33$

D. $-10, 33$

Answer: B

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308. The least value of $2x^2 + y^2 + 2xy + 2x - 3y + 8$ for realnumbers x and y is

A. $-\frac{1}{2}$

B. 8

C. 3

D. 1

Answer: A

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309. The minimum value of $\cos\theta + \sin\theta + \dfrac{2}{\sin 2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{2}\right)$ is

A. $2+\sqrt{2}$

B. 2

C. $1+\sqrt{2}$

D. $2\sqrt{2}$

Answer: A

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310. Let $y=e^{x^2}$ and $y=e^{x^2}$ sinx be two given curves. Then the angle between the tangents to the curves any point of their intersectons is

A. 0 (zero)

B. π

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer: A

Watch Video Solution

311. If the straight line $(a-1)x - by + 4 = 0$ is normal to the hyperbola $xy = 1$ then which of the following does not hold ?

A. $a > 1, b > 0$

B. $a > 0, b < 0$

C. $a < 1, b < 0$

D. $a < 1, b > 0$

Answer: B::D

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312. If f(x) =x^2 and g(x) = $\sqrt{x}$, then the correct relation will be

A. $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$for some $c \in [0, 1]$

B. $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$for some $c \in [0, 1]$

C. $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$for some $c \in [0, 1]$

D. $(f(c))^2 = (g(c))^2$for some $c \in [0, 1]$

Answer: A::D

Watch Video Solution

313. Let $f : (0, \infty) \to \mathbb{R}$ be given by

$$f(x) = \int_{\frac{1}{x}}^{x} e^{\left(t + \frac{1}{t}\right)} \frac{dt}{t}$$

Then

A. f(x) is monotonically increasing on $[1, \infty)$

B. f(x) is monotonically decreasing on (0,1]

C. $f(x)+f\left(\frac{1}{x}\right)=0$, for all $x \in (0, \infty)$

D. $f(2^x)$ is ann odd function of x on $\mathbb{R}$

Answer: A::C::D

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314. Let $a \in \mathbb{R}$ and R and let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^5 - 5x + a$

Then

A. f(x) has three real roots if $a > 4$

B. f(x) has one real roots if $a > 4$

C. f(x) has three real roots if $a < -4$

D. f(x) has one real roots if $-4 < a < 4$

Answer: B::D

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315. The slope of the tangent to the curve $\left(y-x^5\right)^2=x\left(1+x^2\right)^2$ at the point (1,3) is

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316. If f(x) and g(x) are differentiable functions for $0 \leq x \leq 1$ such that f(0)=2,g(0)=0,f(1)=6,g(1)=2, then in the interval (0,1)

A. $f'(c)=g'(c)$

B. $f'(c)=2g'(c)$

C. $2f'(c)=g'(c)$

D. $2f'(c)=3g'(c)$

Answer: B

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317. If $x = -1$ and x=2 are extreme points of $f(x) = \alpha \log|x| + \beta|x|^2 + x$ then :

A. $\alpha = 2, \beta = -\frac{1}{2}$

B. $\alpha = 2, \beta = \frac{1}{2}$

C. $\alpha = -6, \beta = \frac{1}{2}$

D. $\alpha = -6, \beta = -\frac{1}{2}$

Answer: A

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318. For every real number x,

$f(x) = \frac{x}{1!} + \frac{3}{2!}x^2 + \frac{7}{3!}x^3 + \frac{15}{4!}x^4 + \dots$ < br> Then the equation f(x)=0 has

A. no real solution

B. exactly one real solution

C. exactly two real solution

D. infinite number of real solutions

Answer: B

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319. Let f(x) be a differentiable function in [2,7]. If f(2) = 3 and $f'(x) \leq 5$ all x in (2,7) then the maximum possible value of f(x) at x=7 is

A. 7

B. 15

C. 28

D. 14

Answer: C

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320. Suppose that the equation $f(x) = x^2 + bx + c$ has two distinct real roots α and β. The angle between the tangent to the curve y = f(x) at the point $\left(\frac{\alpha+\beta}{2}, f\left(\frac{\alpha+\beta}{2}\right)\right)$ and the positive direction of the x-axis is

A. 0°

B. 30°

C. 60°

D. 90°

Answer: A

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321. Let $f(x) = \int e^x (x-1)(x-2) dx$, then f(x) decreases in the interval

A. (0,2)

B. (-1,0)

C. (2,3)

D. (-2,-1)

Answer: A

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322. Let R be the set of all real numbers and f:$[1, -1] \to R$ be

defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

A. a) f satisfies the conditions of Rolle's theorem on $[-1, 1]$

B. b) f satisfies the conditions of

Lagrange's Mean Value Theorem on $[-1, 1]$

C. c) f satisfies the conditions of Rolle's theorem on [0,1]

D. d) f satisfies the conditions of

Lagrange's Mean Value Theorem on [0,1]

Answer: D

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323. Suppose that f(x) is a differentiable function such that f'(x) is continuous, f'(0) = 1 and f"(0) does not exist. Let g(x) = xf'(x). Then

A. g'(0) does not exist

B. g'(0)=)

C. g'(0)=1

D. g'(0)=2

Answer: C

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324. Applying Lagrange's Mean Value Theorem for a suitable function f(x) in [0, h], we have $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$. Then for f(x) = cos x, the value of $\lim_{h \to 0^+} \theta$ is

A. 1

B. 0

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: C

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325. The angle of intersection between the curves $y = [|\sin x| + |\cos x|]$ and $x^2 + y^2 = 10$, where [x] denotes the greatest integer $\leq x$, is

A. $\tan^{-1} 3$

B. $\tan^{-1}(-3)$

C. $\tan(-1)\sqrt{3}$

D. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Answer: A::B

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326. The number of points in $(-\infty, \infty)$ for which $x^2 - x\sin x - \cos x = 0$ is

A. 6

B. 4

C. 2

D. 0

Answer: C

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327. If $f(x) = |x| + |x-1| + |x-2|$, then

A. -2

B. $-\frac{2}{3}$

C. 2

D. $\frac{2}{3}$

Answer: A::B

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328. Let $f(x)=x\sin\pi x, x>0$ .Then for all natural number n, f'(x) vanishes at

A. a unique point In the interval $\left(n, n+\frac{1}{2}\right)$

B. a unique point In the interval $\left(n+\frac{1}{2}, n+1\right)$

C. a unique point In the interval $\left(n, n+\frac{1}{2}\right)$

D. two points in the interval $(n, n+1)$

Answer: B::C

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329. The intercepts on x-axis made by tangents to the curve, $\int_0^x |t| dt, x \in R$ which are parallel to the line y = 2x, are equal to :

A. ± 2

B. ± 3

C. ± 4

D. ± 1

Answer: D

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330. The real number k of for which the equation $2x^3 + 3x + k = 0$ has two distinct real roots in [0,1]

A. lies between 2 and 3

B. lies between -1 and 0

C. lies between 1 and 2

D. does not exist

Answer: D

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331. If $y = \sec(\tan^{-1} x)$, then value of $\frac{dy}{dx}$ at x=1 is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. $\sqrt{2}$

Answer: B

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332. If $f(x) = e^x(x-2)^2$ then

A. f is increasing in $(-\infty, 0)$ and $(2, \infty)$ and decereasing in (0,2)

B. f is increasing in $(-\infty, 0)$ and decreasing in (0,2)

C. f is increasing in $(2, \infty)$ and decreasing in $(-\infty, 0)$

D. f is increasing in $(2, \infty)$ and decreasing in $(-\infty, 0)$ and $(2, \infty)$

Answer: A

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333. Let $f(x) = \sin x + 2\cos^2 x, \frac{\pi}{4} \leq x \leq \frac{3\pi}{4}$. then f attains its

A. minimum at $x = \frac{\pi}{4}$

B. maximum at $x = \frac{\pi}{2}$

C. minimum at $x = \frac{\pi}{2}$

D. maximum at $x = \sin^{-1}\left(\frac{1}{4}\right)$

Answer: C

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334. Let exp(x) denote exponential function e^x. If $f(x) = \exp\left(x^{\frac{1}{x}}\right).\ x > 0$ then the minimum value of f in the interval [2,5] is

A. $\exp\left(e^{\frac{1}{e}}\right)$

B. $\exp\left(2^{\frac{1}{2}}\right)$

C. $\exp\left(5^{\frac{1}{5}}\right)$

D. $\exp\left(3^{\frac{1}{3}}\right)$

Answer: C

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335. The minimum value of the function $f(x) = 2|x-1| + |x-2|$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

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336. Let $f(x) = \begin{cases} x^2 \left|\cos \frac{\pi}{x}\right| & x \neq 0 \\ 0 & x = 0 \end{cases}$

A. differentiable both at x = 0 and x = 2

B. differentiable both at x = 0 but not differentiable at x= 2

C. not differentiable atx = 0 but differentiable at x = 2

D. differentiable neither atx = 0 nor at x = 2

Answer: B

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337. The function $f:[0,3] \rightarrow [1,29]$, defined by $f(x)=2x^3-15x^2+36x+1$ is

A. one-one onto

B. one-one but not one-one

C. one-one but not onto

D. neither one-one nor onto

Answer: B

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338. if $f(x)\int_{o}^{x} e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

A. f has a local maxmimum at x=2

B. f is decreasing on (2,3)

C. there exists some $c \in (0, \infty)$ such that $f''(c)=0$

D. f has a localminimum at x=3

Answer: A::B::C::D

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339. Let $f: IR \rightarrow IR$ be defined as $f(x) = |x| + |x^2 - 1|$.The total number of points at which f attains either a local maximum or a local minimum is

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340. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is

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341. xx

A. both P and Q are true

B. P is true and Q is false

C. P is false and Q is true

D. both P and Q are false

Answer: C

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342. Let $f(x) = (1-x)^2 \sin^2 x + x^2$ for all $\xi n R$ and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - Int\right)$f(t)dt for all $\xi n(1, \infty)$

Which of the following is true ?

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on (1, 2) and decreasing on $(2, \infty)$

D. g is increasing on (1, 2) and decreasing on $(2, \infty)$

Answer: B

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343. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{2}{9}$

D. $\frac{9}{2}$

Answer: C

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344. Let a, $b \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and x = 2 Statement 1:f has local maximum ar x=-1 and x=2. Statement 2: $a = \frac{1}{2}$ and $b = \frac{1}{4}$

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true , Statement 2 is true , Statement 2 is a correct explanation for Statemen

C. Statement 1 is true, Statement 2 is true , Statement 2 is not a correct explanation for Statement 1

D. Statement 1 is true, Statement 2 is false.

Answer: B

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345. If f is a real-valued differentiable function such that f(x) $f'(x) < 0$ for all real x, then

A. $|f(x)|$ must be an increasing function

B. $|f(x)|$ must be a decreasing function

C. $|f(x)|$ must be an increasing function

D. $|f(x)|$ must be a decreasing function

Answer: D

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346. Rolle's theorem is applicable in the interval [-2, 2] for the function

A. $f(x) = x^3$

B. $f(x) = 4x^4$

C. $f(x) = 2x^3 + 3$

D. $f(x) = \pi|x|$

Answer: B

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347. Let $y=\left(\frac{3^x-1}{3^x+1}\right)\sin x+\log_e(2+x), x> \ -1$. Then at x=0 $\frac{dy}{dx}$ equals

A. 1

B. 0

C. -1

D. -2

Answer: A

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348. Maximum value of the function $f(x)=\frac{x}{8}+\frac{2}{x}$ on the interval [1,6] is

A. 1

B. $\frac{9}{8}$

C. $\frac{13}{12}$

D. $\frac{17}{8}$

Answer:

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349. For $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, the value of $\frac{d}{dx}\left\{\tan^{-1}\frac{\cos x}{1+\sin x}\right\}$ is equal to

A. $\frac{1}{2}$

B. `-1/2

C. 1

D. $\frac{\sin x}{(1+\sin x)^2}$

Answer: B

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350. If f(x) and g(x) are twice differentiable functions on (0,3) satisfying f"(x) = g"(x), f'(1) = 4, g'(1)=6,f(2)=3, g(2) = 9, then $f(1) - g(1)$ is

A. 4

B. -4

C. 0

D. -2

Answer: B

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