MATHS

BOOKS - DISHA PUBLICATION MATHS (HINGLISH)

APPLICATION OF DERIVATIVES

Jee Main 5 Years At A Glance

1. If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles then the value of b is: (1) 6 (2) $\frac{7}{2}$ (3) 4 (4) $\frac{9}{2}$

A. $\frac{7}{2}$

B. 4

C. $\frac{9}{2}$

D. 6

Answer: C

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2. Let $f(x) = x^2 + \left(\frac{1}{x^2}\right)$ and $g(x) = x - \frac{1}{x}$ $\xi nR - \{-1, 0, 1\}$. If $h(x) = \left(\frac{f(x)}{g(x)}\right)$ then the local minimum value of $h(x)$ is: (1) 3 (2) -3 (3) $-2\sqrt{2}$ (4) $2\sqrt{2}$

A. -3

B. $-2\sqrt{2}$

C. $2\sqrt{2}$

D. 3

Answer: C

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3. Let P be a point on parabola $x^2 = 4y$. If the distance of P from the centre of circle $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of tangent at P on parabola $x^2 = 4y$ is :

A. x + 4y - 2 = 0

B. x + 2y = 0

C. x + y + 1 = 0

D. x - y + 3 = 0

Answer: C

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4. Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then

A. 1

B. 5

C. 4

D. 9

Answer: A

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5. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a\xi s,$ passes through the point : $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{\frac{1}{2,1}}{2}\right)$

A. $\left(\frac{1}{2}, \frac{1}{3}\right)$

B. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

C. $\left(\frac{1}{2}, \frac{1}{2}\right)$

D. $\left(\frac{1}{2}, -\frac{1}{3}\right)$

Answer: C

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6. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. if one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is: $4x + 2y = 7$ (2) $x + 2y = 4$ (3) $2y - x = 2$ (4) $4x - 2y = 1$

A. x + 2y = 4

B. 2y - x = 2

C. 4x - 2y = 1

D. 4x + 2y = 7

Answer: C

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7. Twenty metres of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in $sq\dot{m}$) of the flower-bed is: 25 (2) 30 (3) 12.5 (4) 10

A. 30

B. 12.5

C. 10

D. 25

Answer: D

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8. The tangent at the point $(2, -2)$ to the curve, $x^2y^2 - 2x = 4(1-y)$ does not pass through the point :

A. $\left(4, \frac{1}{3}\right)$

B. (8, 5)

C. (-4, -9)

D. (-2, -7)

Answer: D

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9. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point: (1) (0, 0) (2) $\left(0, \frac{2\pi}{3}\right)$ (3) $\left(\frac{\pi}{6}, 0\right)$ (4) $\left(\frac{\pi}{4}, 0\right)$

A. $\left(\frac{\pi}{6}, 0\right)$

B. $\left(\frac{\pi}{4}, 0\right)$

C. (0, 0)

D. $\left(0, \frac{2\pi}{3}\right)$

Answer: D

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10. A wire of length 2 units is cut into two parts which are bent respectively to form a square of $side = x$ units and a circle of $radius = r$ units. If the sum of the areas of the square and the circle so formed is minimum, then : (1) $2x = (\pi + 4)r$ (2) $(\pi + 4)x = \pi r$ (3) $x = 2r$ (4) $2x = r$

A. x = 2r

B. 2x = r

C. $2x = (\pi + 4)r$

D. $(4 - \pi)x = \pi r$

Answer: A

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11. If the tangent at a point P with parameter t, on the curve $x = 4t^2 + 3, y = 8t^3 - 1\ t \in R$ meets the curve again at a point Q, then the coordinates of Q are

A. $\left(16t^2 + 3, \ - 64t^3 - 1\right)$

B. $\left(4t^2 + 3, \ - 8t^3 - 2\right)$

C. $\left(t^2 + 3, t^3 - 1\right)$

D. $\left(t^2 + 3, \ - t^3 - 1\right)$

Answer: D

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12. The minimum distance of a point on the curve $y = x^2 - 4$ from the origin is :

A. $\frac{\sqrt{15}}{2}$

B. $\sqrt{\frac{19}{2}}$

C. $\sqrt{\frac{15}{2}}$

D. $\frac{\sqrt{19}}{2}$

Answer: A

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13. The normal to the curve $x^2 + 2xy - 3y^2 = 0$, at (1, 1):

A. meets the curve again in the third quadrant.

B. meets the curve again in the fourth quadrant.

C. does not meet the curve again.

D. meets the curve again in the second quadrant.

Answer: B

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14. If the tangent to the conic, $y - 6 = x^2$ at $(2, 10)$ touches the circle, $x^2 + y^2 + 8x - 2y = k$ (for some fixed k) at a point (α, β); then

A. $\left(-\frac{7}{17}, \frac{6}{17}\right)$

B. $\left(-\frac{4}{17}, \frac{1}{17}\right)$

C. $\left(-\frac{6}{17}, \frac{10}{17}\right)$

D. $\left(-\frac{8}{17}, \frac{2}{17}\right)$

Answer: D

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15. The distance, from the origin, of the normal to the curve, $x = 2\cos t + 2t\sin t$, $y = 2\sin t - 2t\cos \text{ t at t } = \frac{\pi}{4}$, is :

A. 2

B. 4

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: A

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16. If x = -1 and x = 2 are extreme points of f(x) = $\alpha \log|x| + \beta x^2 + x$, then

A. $\alpha = 2, \beta = -\frac{1}{2}$

B. $\alpha = 2, \beta = \frac{1}{2}$

C. $\alpha = -6, \beta = \frac{1}{2}$

D. $\alpha = -6, \beta = -\frac{1}{2}$

Answer: A

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Exercise 1

1. The normal to the curve $x = a(1 + \cos\theta), y = a\sin\theta$ at $'\theta'$ always passes through the fixed point

A. (a, a)

B. (0, a)

C. (0, 0)

D. (a, 0)

Answer: D

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2. The chord joining the points (5, 5) and (11, 227) on the curve $y = 3x^2 - 11x - 15$ is parallel to tangent at a point on the curve. Then the abscissa of the point is

A. -4

B. 4

C. -8

D. 8

Answer: D

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3. The line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^{n}+\left(\frac{y}{b}\right)^{n}=2$ at the point (a, b) for

A. n = 1, 2

B. n = 3, 4, -5

C. n = 1, 2, 3

D. any value of n

Answer: D

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4. What is the slope of the normal at the point $\left(\mathrm{at}^2, 2\,\mathrm{at}\right)$ of the parabola $y^2 = 4ax$?

A. $\frac{1}{t}$

B. t

C. $-t$

D. $-\frac{1}{t}$

Answer: C

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5. If the line, $ax + by + c = 0$ is a normal to the curve $xy = 2$

A. $a > 0, b > 0$

B. $a > 0, b < 0$

C. $a < 0, b < 0$

D. Data is insufficient

Answer: B

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6. The area of the triangle formed by normal at the point $(1, 0)$ on the curve $x = e^{\sin y}$ with axes is

A. $1/4$

B. $1/2$

C. $3/4$

D. 1

Answer: B

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7. If at each point of the curve $y = x^3 - ax^2 + x + 1$, the tangent is inclined at an acute angle with the positive direction of the x-axis, then $a > 0$ (b) $a < -\sqrt{3}$ $-\sqrt{3} \leq a \leq \sqrt{3}$ (d) $noneofthese$

A. $a > 0$

B. $a \leq \sqrt{3}$

C. $-\sqrt{3} \leq a \leq \sqrt{3}$

D. None of these

Answer: D

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8. The slope of the tangent to the curve $y = 3x^2 - 5x + 6$ at (1, 4) is

A. -2

B. 1

C. 0

D. -1

Answer: B

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9. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, -1), is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{-6}{7}$

D. -6

Answer: B

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10. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point

:

A. (1, 1)

B. (0, 1)

C. (1, 0)

D. no point

Answer: C

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11. What is the x coordinate of the point on the curve f(x)= $\sqrt{x}(7x-6)$ where the tangent is parallel to x axis ?

A. $-\frac{1}{3}$

B. $\frac{2}{7}$

C. $\frac{6}{7}$

D. $\frac{1}{2}$

Answer: B

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12. The slope of the tangent to the curves $x = 3t^2 + 1, y = t^2 - 1$ at t=1 is

A. $\frac{1}{2}$

B. 0

C. -2

D. ∞

Answer: B

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13. Find the distance between the point (1, 1) and the tangent to the curve $y = e^{2x} + x^2$ drawn from the point x = 0.

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{3}{\sqrt{5}}$

D. None of these

Answer: B

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14. If OT is the perpendicular drawn from the origin to the tangent at any point t to the curve $x = a\cos^3 t, y = a\sin^3 t$, then OT is equal to :

A. $a \sin 2t$

B. $\frac{a}{2} \sin 2t$

C. $2a \sin 2t$

D. 2a

Answer: B

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15. Determine p such that the length of the such-tangent and sub-normal is equal for the curve $y = e^{px} + px$ at the point $(0, 1)$.

A. $\pm \frac{1}{2}$

B. ± 1

C. ± 2

D. 0

Answer: A

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16. The angle at which the curve $y = ke^{kx}$ intersects Y-axis is

A. $\tan^{-1}(k^2)$

B. $\cot^{-1}(k^2)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$

D. $\sec^{-1}\sqrt{1+k^4}$

Answer: B

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17. The angle of intersection of the curves $y = x^2, 6y = 7 - x^3$ at (1, 1), is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. π

Answer: A

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18. If the curves $\frac{x^2}{a^2} + \frac{y^2}{12} = 1$ and $y^3 = 8x$ intersect at right angles, then the value of a^2 is equal to

A. 16

B. 12

C. 8

D. 4

Answer: D

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19. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{6}$

Answer: C

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20. Show the condition that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$ should intersect orthogonally is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a'} - \frac{a}{b'}$.

A. $\frac{a - a_1}{aa_1} = \frac{b - b_1}{bb_1}$

B. $\frac{a + a_1}{aa_1} = \frac{b + b_1}{bb_1}$

C. $\frac{a - a_1}{a + a_1} = \frac{b - b_1}{b + b_1}$

D. None of these

Answer: A

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21. The normal to the curve $x^2 = 4y$passing (1,2) is(A) $x + y = 3$

(B) $xy = 3$ (C) $x + y = 1$ (D) $xy = 1$

A. The length of sub-tangent to the curve $x^2y^2 = 16a^4$ at the point (-2a, 2a) is 2a.

B. x + y = 3 is a normal to the curve $x^2 = 4y$

C. Curves $y = -4x^2$ and $y = e^{-x/2}$ are orthogonal.

D. If $a \in (-1, 0)$, then tangent at each point of the curve $y = \frac{2}{3}x^3 - 2ax^2 + 2x + 5$ makes an acute angle with the positive direction of x-axis.

Answer: C

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22. The shortest distance between the lines $y - x = 1$ and the curve $x = y^2$ is

A. $\dfrac{3\sqrt{2}}{8}$

B. $\dfrac{2\sqrt{3}}{8}$

C. $\dfrac{3\sqrt{2}}{5}$

D. $\dfrac{\sqrt{3}}{4}$

Answer: A

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23. If curve $x^2 = 9a(9 - y)$ and $x^2 = a(y + 1)$ intersect orthogonally then value of 'a' is

A. 3

B. 4

C. 5

D. 7

Answer: B

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24. The function $f(x) = \cot^{-1} x + x$ increases in the interval (a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $(-\infty, \infty)$ (d) $(0, \infty)$

A. $(1, \infty)$

B. $(-1, \infty)$

C. $(-\infty, \infty)$

D. $(0, \infty)$

Answer: C

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25. If f(x) = cos x, then

A. f(x) is strictly decreasing in $(0, \pi)$

B. f(x) is strictly increasing in $(0, 2\pi)$

C. f(x) is neither increasing nor decreasing in $(\pi, 2\pi)$

D. All the above are correct

Answer: A

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26. The function $f(x) = \frac{a \sin x + b \cos x}{c \sin x + d \cos x}$ is decreasing, if

A. $ad - bc < 0$

B. $ad - bc > 0$

C. $ab - cd < 0$

D. $ab - cd > 0$

Answer: A

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27. Show that the equation $x^5 - 3x - 1 = 0$ has a unique root in $[1, 2]$.

A. at least one root

B. at most one roots

C. no roots

D. a unique root

Answer: D

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28. Find the interval in which the function $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is increasing or decreasing.

A. $(-\infty, \infty)$

B. $(-\infty, 0)$

C. $(0, \infty)$

D. $(1, \infty)$

Answer: B

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29. If $f(x)=x\sqrt{ax-x^2}$ for $a>0$, then $f(x)$ is

A. increasing in (0, 3a), decreasing in (3a, 4a)

B. increasing in (a, 4a), decreasing in $(5a, \infty)$

C. increasing in (0, 4a)

D. None of these

Answer: A

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30. The function f(x)=$[x(x-2)]^2$ is increasing in the set

A. $(-\infty, 0) \cup (2, \infty)$

B. $(-\infty, 1)$

C. $(0, 1) \cup (2, \infty)$

D. (1, 2)

Answer: C

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31. The function $f(x) = \tan x - x$

A. always increases

B. always decreases

C. never decreases

D. some times increases and some times decreases

Answer: A

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32. The function $f(x) = x^{1/x}$ is increasing in the interval

A. (e, ∞)

B. $(-\infty, e)$

C. (-e, e)

D. None of these

Answer: B

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33. If $f(x) = kx^3 - 9x^2 + 9x + 3$ monotonically increasing in R, then $k < 3$ (b) $k \leq 2$ $k \geq 3$ (d) none of these

A. $k < 3$

B. $k \leq 3$

C. $k < 3$

D. $k \geq 3$

Answer: D

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34. The function $f(x) = \frac{\log(\pi + x)}{\log(e + x)}$s is

A. increasing in $[0, \infty)$

B. decreasing in $[0, \infty)$

C. decreasing in $\left[0, \frac{\pi}{e}\right]$ & increasing in $\left[\frac{\pi}{e}, \infty\right)$

D. increasing in $\left[0, \frac{\pi}{e}\right] \& decrea \sin g \in$ [(pi)/(e), oo)`

Answer: B

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35. If the function $f(x) = 2x^3 - 9ax^2 + 12x^2x + 1, where a > 0,$ attains its maximum and minimum at $p and q$, respectively, such that $p^2 = q,$ then a equal to 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

A. $\frac{1}{2}$

B. 3

C. 1

D. 2

Answer: D

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36. The minimum value of the function, $f(x) = x^{3/2} + x^{-3/2} - 4\left(x + \frac{1}{x}\right)$. For all permissible real values of x is

A. -10

B. -6

C. -7

D. -8

Answer: A

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37. The minmumu value of the fucntion

$f(x) = \frac{a^2}{x} + \frac{b^2}{a - x}, a > 0, b > 0$, in (0,a) is

A. a + b

B. $\frac{1}{a+b}$

C. $\frac{1}{a}(a+b)^2$

D. $\frac{1}{a^2}(a+b)$

Answer: C

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38. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$. Find the value of a .

A. 110

B. 10

C. 55

D. None of these

Answer: D

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39. Find the maximum and minimum values of f , if any, of the function given by $f(x) = |x|, x \in R$.

A. point of minimum value of f is x = 1

B. f has no point of maximum value in R

C. Both (a) and (b) are true

D. Both (a) and (b) are not true

Answer: B

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40. The function $f(x) = 1 + x(\sin x)[\cos x], 0 < x \leq \frac{\pi}{2}$ (where [.] is G.I.F.)

A. is continuous on $\left(0, \frac{\pi}{2}\right)$

B. is strictly increasing in $\left(0, \frac{\pi}{2}\right)$

C. is strictly decreasing in $\left(0, \frac{\pi}{2}\right)$

D. has global maximum value 2

Answer: A

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41. Find two positive numbers x and y such that $x + y = 60 \, and \, x^3 y$ is maximum.

A. 25 and 35

B. 30 and 30

C. 40 and 20

D. 45 and 15

Answer: D

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42. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

A. 2h = r

B. h = 4r

C. h = 2r

D. h = r

Answer: D

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43. All possible value of $f(x)=(x+1)^{\frac{1}{3}}-(x-1)^{\frac{1}{3}}$ on [0,1] is 1 (b) 2 (c) 3 (d) $\frac{1}{3}$

A. 2

B. 3

C. -1

D. 9

Answer: A

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44. Let f(x) be a function defined as below :

$$f(x) = \sin\left(x^2 - 3x\right), x \le 0$$

$$= 6x + 5x^2, x > 0$$

then at x=0 ,f(x)

A. has a local maximum

B. has a local minimum

C. is discontinuous

D. None of these

Answer: B

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45. Tangents are drawn to $x^2 + y^2 = 16$ from the point $P(0, h)$. These tangents meet the $x - a\xi s$ at $AandB$. If the area of triangle PAB is minimum, then $h = 12\sqrt{2}$ (b) $h = 6\sqrt{2}$ $h = 8\sqrt{2}$ (d) $h = 4\sqrt{2}$

A. $h = 12\sqrt{2}$

B. $h = 6\sqrt{2}$

C. $h = 8\sqrt{2}$

D. $h = 4\sqrt{2}$

Answer: D

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46. A cylindrical gas container is closed at the top and open at the bottom. If the iron plate of the top is 5/4 times as thick as the plate forming the cylindrical sides, the ratio of the radius to the height of the cylinder using minimum material for the same capacity is 3:4 (b) 5:6 (c) 4:5 (d) none of these

A. $\frac{2}{3}$

B. $\frac{1}{2}$

C. $\frac{4}{5}$

D. $\frac{1}{3}$

Answer: C

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47. Let $f(x) = \frac{a}{x} + x^2$. If it has a maximum at $x = -3$, then find the value of a.

A. $a < -27$

B. $a > -27$

C. $a > 27$

D. $a < 27$

Answer: A

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48. The radius of a right circular cylinder increases at the rate of 0.1 cm/min, and the height decreases at the rate of 0.2 cm/min.

The rate of change of the volume of the cylinder, in $cm^2 / m \in$, when the radius is $2cm$ and the height is 3cm is $-2p$ (b) $-\frac{8\pi}{5}$ $-\frac{3\pi}{5}$ (d) $\frac{2\pi}{5}$

A. -2π

B. $-\frac{8\pi}{5}$

C. $-\frac{3\pi}{5}$

D. $\frac{2\pi}{5}$

Answer: D

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49. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is (2,6) (b) $(2, -6)$ $\left(\frac{9}{8}, -\frac{9}{2}\right)$ (d) $\left(\frac{9}{8}, \frac{9}{2}\right)$

A. $\left(\frac{9}{8}, \frac{9}{2}\right)$

B. (2, -4)

C. $\left(\frac{-9}{8}, \frac{9}{2}\right)$

D. (2, 4)

Answer: A

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50. A ball is dropped from a platform 19.6 m high. Its position function is :

A. $x = -4.9t^2 + 19.6(0 \le t \le 1)$

B. $x = -4.9t^2 + 19.6(0 \le t \le 2)$

C. $x = -9.8t^2 + 19.6(0 \le t \le 2)$

D. $x = -4.9t^2 - 19.6(0 \le t \le 2)$

Answer: B

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51. The diagonal of square is changing at the rate of $0.5cms^{-1}$. Then the rate of change of area, when the area is $400cm^2$, is equal to

A. $20\sqrt{2}cm^2/\sec$

B. $10\sqrt{2}cm^2/\sec$

C. $\frac{1}{10\sqrt{2}}cm^2/\sec$

D. $\frac{10}{\sqrt{2}}cm^2/\sec$

Answer: B

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52. A stone is dropped into a quiet lake and waves move in circles at a speed of 3.5 cm per second. At the instant when the radius of the circular wave is 7.5 cm, how fast is the enclosed area increasing ? (Take $\pi = 22/7$)

A. 32.5 $\pi cm^2 / \sec$

B. 31.5 $\pi cm^2 / \sec$

C. 52.5 $\pi cm^2 / \sec$

D. None of these

Answer: C

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53. If a circular plate is heated uniformly, its area expands 3c times as fast as its radius, then the value of c when the radius is 6 units, is

A. 4π

B. 2π

C. 6π

D. 3π

Answer: A

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54. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50cm^3 / \min$. When the thickness of ice is 5 cm, then the rate at which the thickness of ice decreases, is:

A. $\frac{1}{36\pi}$ cm/min

B. $\frac{1}{18\pi}$ cm/min

C. $\frac{1}{54\pi}$ cm/min

D. $\frac{5}{6\pi}$ cm/min

Answer: B

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55. A ladder is resting with the wall at an angle of 30°. A man is ascending the ladder at the rate of 3 ft/sec. His rate of approaching the wall is

A. 3 ft/sec

B. $\frac{3}{2}$ ft/sec

C. $\frac{3}{4}$ ft/sec

D. $\frac{3}{\sqrt{2}}$ ft/sec

Answer: B

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56. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2units/sec, then at x=3 the slope of the curve is changing at ___

A. -78 units/s

B. -72 units/s

C. -36 units/s

D. -18 units/s

Answer: B

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57. An edge of a variable cube is increasing at the rate of 10 cm/s. How fast the volume of the cube is increasing when the edge is 5 cm long?

A. 750 $cm^3/\sec$

B. 75 $cm^3/\sec$

C. 300 $cm^3/\sec$

D. 150 $cm^3/\sec$

Answer: A

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58. The altitude of a cone is 20 cm and its semi-vertical angle is 30°. If the semi-vertical angle is increasing at the rate of 2° per second, then the radius of the base is increasing at the rate of

A. 30 cm/sec

B. $\frac{160}{3}$ cm/sec

C. 10 cm/sec

D. 160 cm/sec

Answer: B

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59. The aproximate value of square root of 25.2 is

A. 5.08

B. 5.02

C. 5.01

D. 5.03

Answer: B

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60. The possible percentage error in computing the parallel resistance R of three resistances R_1, R_2, R_3 from the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$, if R_1, R_2, R_3 are each 1.2% in error, is

A. 1.2%

B. 2.4%

C. 0.6%

D. 1.8%

Answer: A

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61. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

A. 34.995

B. -30.995

C. 24.875

D. None of these

Answer: D

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62. The approximate value of $\left\{(3.92)^2 + 3(2.1)^4\right\}^{1/6}$ is

A. 2.040

B. 3.567

C. 1.562

D. 2.577

Answer: A

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63. Using differentials, find the approximate value of $(0.007)^{1/3}$

A. $\frac{23}{120}$

B. $\frac{27}{120}$

C. $\frac{19}{120}$

D. $\frac{17}{120}$

Answer: A

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64. If there is an error of $\pm 0.04 cm$ in themeasurement of the diameter of sphere then the percentage error in its volume, when radius is $10 cm$

A. ± 1.2

B. ± 1.0

C. ± 0.6

D. ± 0.8

Answer: C

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65. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximating error in calculating its volume.

A. 2.46 πcm^3

B. 8.62 πcm^3

C. 9.72 πcm^3

D. 7.6 πcm^3

Answer: C

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Exercise 2

1. Find the distance of the point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line $y = 2x - 1$

A. $\frac{2}{\sqrt{5}}$

B. $\sqrt{5}$

C. $\frac{1}{\sqrt{5}}$

D. $5\sqrt{5}$

Answer: C

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2. If the tangents at any point on the curve $x^4 + y^4 = a^4$ cuts off intercept p and q on the axes, the value of $p^{-\frac{4}{3}} + q^{-\frac{4}{3}}$ is

A. $a^{-4/3}$

B. $a^{-1/2}$

C. $a^{1/2}$

D. None

Answer: A

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3. $f(x) = \frac{x}{\sin x}$ and $g(x) = \frac{x}{\tan x}$, where $0 < x \leq 1$ then in the interval

A. both f(x) and g(x) are increasing functions

B. both f(x) and g(x) are decreasing functions

C. f(x) is an increasing function

D. g(x) is an increasing function

Answer: C

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4. If the tangent at any point $(4m^2, 8m^2)$ of $x^3 - y^2 = 0$ is a normal to the curve $x^3 - y^2 = 0$, then find the value of m.

A. $\pm\sqrt{\frac{2}{9}}$

B. $\pm\frac{1}{3}$

C. $\pm\sqrt{\frac{9}{2}}$

D. $\pm\frac{2}{3}$

Answer: A

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5. The point P of the curve $y^2 = 2x^3$ such that the tangent at P is perpendicular to the line $4x - 3y + 2 = 0$ is given by

A. (2, 4)

B. $(1, \sqrt{2})$

C. $\left(\frac{1}{2}, \frac{1}{2}\right)$

D. $\left(\frac{1}{8}, -\frac{1}{16}\right)$

Answer: D

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6. Find the number of solutions of the equation $3\tan x + x^3 = 2 \in \left(0, \frac{\pi}{4}\right)$

A. 1

B. 2

C. 3

D. infinite

Answer: A

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7. For the curve $by^2 = (x+a)^3$ the square of subtangent is proportional to

A. 8a/27

B. 27/8b

C. 8b/27

D. $8/27$

Answer: C

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8. Find the point of intersection of the tangents drawn to the curve $x^2y = 1 - y$ at the points where it is intersected by the curve $xy = 1 - y$.

A. (0, -1)

B. (1, 1)

C. (0, 1)

D. None of these

Answer: C

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9. The function $f(x) = tan^{-1}(\sin x + \cos x)$ is an increasing function in (1) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (2) $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ (3) $\left(0, \frac{\pi}{2}\right)$ (4) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

A. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

B. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

C. $\left(0, \frac{\pi}{2}\right)$

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: B

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10. The function $f(x) = 2\log(x-2) - x^2 + 4x + 1$ increases on the interval (a) $(1,\ 2)$ (b) $(2,\ 3)$ (c) $(1,\ 3)$ (d) $(2,\ 4)$

A. (1, 2)

B. (2, 3)

C. (1/2, 3)

D. (2, 4)

Answer: B

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11. Find the intervals in which the function f given by $f(x)=\dfrac{4\sin x-2x-x\cos x}{2+\cos x} is$ increasing decreasing, $x\in(0,2\pi)$

A. $(0,\pi)\cup(2\pi,4\pi)$

B. $\left(0,\dfrac{\pi}{2}\right)\cup\left(\dfrac{3\pi}{2},2\pi\right)$

C. $\left(0,\dfrac{\pi}{4}\right)\cup\left(\dfrac{\pi}{2},\pi\right)$

D. None of these

Answer: B

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12. Let $f(x)=x^3+ax^2+bx+5\sin^2 x$ be an increasing function in the set of real numbers R Then a and b satisfy the condition

A. $a^2 - 3b - 15 > 0$

B. $a^2 - 3b + 15 > 0$

C. $a^2 + 3b - 15 < 0$

D. $a > 0$ and $b > 0$

Answer: C

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13. The function $f(x) = 3\cos^4 x + 10\cos^3 x + 6\cos^2 x - 3, (0 \leq x \leq \pi)$ is -

A. Increasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

B. Increasing in $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{2\pi}{3}, \pi\right)$

C. Decreasing in $\left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$

D. All of the above

Answer: A

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14. $f(x) = 2x^2 - \log|x| (x \neq 0)$ is monotonic increasing in the interval -

A. $(1/2, \infty)$

B. $(-\infty, -1/2) \cup (1/2, \infty)$

C. $(-\infty, -1/2) \cup (0, 1/2)$

D. $(-1/2, 0) \cup (1/2, \infty)$

Answer: D

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15. The minimum value of $e^{(2x^2-2x+1)\sin^2 x}$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

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16. The number of solutions of the equation $x^3+2x^2+5x+2\cos x=0$ in $(0, 2\pi]$ is

A. one

B. two

C. three

D. zero

Answer: D

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17. Let $g(x) = 2f\left(\frac{x}{2}\right) + f(1 - x)$ and $f^{''}(x) < 0$ in $0 \leq x \leq 1$ then g(x)

A. (1/2, 2)

B. (4/3, 2)

C. (0, 2)

D. (0, 4/3)

Answer: D

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18. If $f(x) = x + \sin x, g(x) = e^{-x}, u = \sqrt{c+1} - \sqrt{c}\ v = \sqrt{c}$
$- \sqrt{c-1}, (c > 1),$ then `fog(u)gof(v)(d)fog(u)

A. $fog(u) < fog(v)$

B. $gof(u) < gof(v)$

C. $gof(u) > gof(v)$

D. $fog(u) < fog(v)$

Answer: C

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19. $f(x) = (\sin^2 x)e^{-2\sin^2 x} \cdot \ \max\ f(x) - \ \min\ f(x)$=

A. $\dfrac{1}{e^2}$

B. $\frac{1}{2e} - \frac{1}{e^2}$

C. 1

D. None of these

Answer: D

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20. Given $P(x) = x^4 + ax^3 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0. If P(1) < P(1)$, then in the interval $[1, 1]$. (1) P(1) is the minimum and P(1) is the maximum of P (2) P(1) is not minimum but P(1) is the maximum of P (3) P(1) is the minimum but P(1) is not the maximum of P (4) neither P(1) is the minimum nor P(1) is the maximum of P

A. P(-1) is not minimum but P(1) is the maximum of P

B. P(-1) is the minimum but P(1) is not the maximum of P

C. Neither P(-1) is the minimum nor P(1) is the maximum of P

D. P(-1) is the minimum and P(1) is the maximum of P

Answer: A

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21. LL' is the latus sectum of the parabola $y^2 = 4axandPP'$ is a double ordinate drawn between the vertex and the latus rectum. Show that the area of the trapezium $PP'LL'$ is maximum when the distance PP' from the vertex is $a/9$.

A. 1

B. 4

C. 9

D. 36

Answer: A

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22. Let $y = x^2 e^{-x}$ then the interval in which y increases with respect to x is

A. (-1, 1)

B. (-2, 0)

C. (2, 1)

D. (0, 2)

Answer: D

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23. The function $\log(1+x) - \frac{2x}{x+2}$ is increasing in the interval:

A. $(0, \infty)$

B. $(-\infty, 0)$

C. $(-\infty, \infty)$

D. None of these

Answer: A

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24. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $(-\pi/3, \pi/3)$.

A. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

B. $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

C. $\left(-\frac{\pi}{3}, \frac{\pi}{2}\right)$

D. $\left(\frac{\pi}{2}, \pi\right)$

Answer: A

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25. Which one of the following statements is correct in respect of the curve $4y - x^2 - 8 = 0$?

A. The curve is increasing in (-4, 4)

B. The curve is increasing in (-4, 0)

C. The curve is increasing in (0,4)

D. The curve is decreasing in (-4, 4)

Answer: C

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26. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ [where $\theta \in \left(0, \frac{\pi}{2}\right)$] Then the value of θ such that sum of intercepts on axes made by this tangent is minimum is $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

A. $\pi/3$

B. $\pi/6$

C. $\pi/8$

D. $\pi/4$

Answer: B

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27. Let domain and range of $f(x)$ and $g(x)$ are respectively $(0, \infty)$. If $f(x)$ be an increasing function $g(x)$ be an decreasing function. Also, $h(x) = f(g(x)), h(0) = 0$ and $p(x) = h\left(x^2 - 2x^2 + 2x\right) - \in (4)$ then for every $x \in (0, 2]$

A. $f\{g(x)\} \geq f\{g(0)\}$

B. $g\{f(x)\} \leq g\{f(0)\}$

C. $f\{g(2)\} = 7$

D. None of these

Answer: B

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28. The volume V and depth x of water in a vessel are connected by the relation $V = 5x - \frac{x^2}{6}$ and the volume of water is increasing at the rate of $5cm^3/\sec$ when x=2cm. The rate at which the depth of water is increasing , is

A. $\frac{5}{18}$ cm/sec

B. $\frac{1}{4}$ cm/sec

C. $\frac{5}{16}$ cm/sec

D. None of these

Answer: D

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29. Find the maximum value and the minimum value and the minimum value of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval

$[0, 3]$.

A. 50 and -19

B. 40 and -18

C. 25 and -39

D. 40 and 0

Answer: C

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30. Find the points of inflection for $f(x) = \sin x$ $f(x) = 3x^4 - 4x^3$ $f(x) = x^{\frac{1}{3}}$

A. x = 1 and x = 2

B. x = 3 and x = -1

C. x = 0 and $x = \dfrac{2}{3}$

D. $x = \frac{4}{5}$ and $x = -1$

Answer: C

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