MATHS

BOOKS - DISHA PUBLICATION MATHS (HINGLISH)

CONTINUITY AND DIFFERENTIABILITY

Jee Main 5 Years At A Glance

1. Let $S = \left\{t \in R: f(x) = |x - \pi|\left(e^{|x|} - 1\right)\sin|x|\right.$ is not differentiable at t} Then the set S is equal to: (1) ϕ (2) {0} (3) $\{\pi\}$ (4) $\{0, \pi\}$

A. {0}

B. $\{\pi\}$

C. $\{0, \pi\}$

D. $\emptyset(anemptyset)$

Answer: D

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2. If the function f defined as $f(x) = \frac{1}{x} - \frac{k-1}{e^{2x}-1}, x \neq 0,$ is continuous at $x = 0,$ then the ordered pair $(k, f(0))$ is equal to :

A. (3,1)

B. (3,2)

C. (1/3,2)

D. (2,1)

Answer: A

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3. If $x = \sqrt{2^{\cos ec^{-1}t}}$ and $y = \sqrt{2^{\sec^{-1}t}}(|t| \geq 1)$then (dy)/(dx) is equal to.

A. y/x

B. $-\frac{y}{x}$

C. $-\frac{x}{y}$

D. x/y

Answer: B

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4. the derivative of $\frac{\tan^{-1}(6x\sqrt{x})}{1-9x^3}$ is $\sqrt{x}g(x)$ then $g(x)$ is:

A. $\frac{3}{1+9x^3}$

B. $\frac{9}{1+9x^3}$

C. $\frac{3x\sqrt{x}}{1-9x^3}$

D. $\frac{3x}{1-9x^3}$

Answer: B

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5. Let f be a polynomial function such that $f(3x) = f'(x); f''(x),$ for all $x \in R.$ Then :

A. f(b)+f'(b)=28

B. f"(b)-f'(b)=0

C. f"(b)-f'(b)=4

D. f(b)-f'(b)+f"(b)=10

Answer: B

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6. If $y = \left[x + \sqrt{x^2 - 1}\right]^{15} + \left[x - \sqrt{x^2 - 1}\right]^{15},$ then $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is eqaul to

A. 12 y

B. $224y^2$

C. $225y^2$

D. 225 y

Answer: D

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7. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g'(0)= -cos (log2)

B. g is differentiable at x = 0 and g'(0) = -sin(log2)

C. g is not differentiable at x = 0

D. g'(0) = cos(log2)

Answer: D

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8. Let $a, b \in R, (a \in 0)$. If the funtion f defined as

$$f(x) = \begin{cases} \frac{2x^2}{a} & 0 \leq x < 1 \\ a & 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^3} & \sqrt{2} < x < \infty \end{cases}$$ is a continous in $[0, \infty)$. Then, (a,b)=

A. $(-\sqrt{2}, 1 - \sqrt{3})$

B. $(\sqrt{2}, 1 + \sqrt{3})$

C. $(\sqrt{2}, 1 - \sqrt{3})$

D. $(-\sqrt{2}, 1 + \sqrt{3})$

Answer: C

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9. If the function

$$f(x) = \begin{cases} -x & x < 1 \\ a + \cos^{-1}(x + b) & 1 \leq x \leq 2 \end{cases}$$ is differentiable at x=1, then $\frac{a}{b}$ is equal to

A. $\frac{\pi + 2}{2}$

B. $\frac{\pi - 2}{2}$

C. $\frac{-\pi - 2}{2}$

D. $-1 - \cos^{-1}(2)$

Answer: A

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10. If the function.

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx + 2 & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of k+m is :

A. (10)/(3)

B. 4

C. 2

D. (16)/(5)

Answer: C

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11. If Rolle's theoram holds for the function f(x)$2x^3 + bx^2 + cx$, $x\varepsilon[-1, 1]$, at the point $\mathrm{x} = \frac{1}{2}$,then 2b + c equals :

A. -3

B. -1

C. 2

D. 1

Answer: B

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12. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

A. f'(c) g'(c)

B. f'(c) 2g'(c)

C. 2f'(c) g'(c)

D. 2f'(c) 3g'(c)

Answer: B

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13. If f(x) is continuous and $f\left(\frac{9}{2}\right) = \frac{2}{9}$, then : $\lim_{x \to 0} f\left(\frac{1 - \cos 3x}{x^2}\right)$=

A. $\frac{9}{2}$

B. $\frac{2}{9}$

C. 0

D. $\frac{8}{9}$

Answer: B

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14. If the Rolle's theorem holds for the function f(x) = $2x^3 + ax^2 + bx$in the interval [-1,1]for the point c=$\frac{1}{2}$,then the value of 2a+b is :

A. 1

B. -1

C. 2

D. -2

Answer: B

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Exercise 1 Concept Builder Topicwise

1. Let $\frac{(e^x - 1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1 + \frac{x}{4}\right)}$ for $x \neq 0$ and $f(0) = 12$. If f is continuous at x = 0, then the value of a is equal to

A. 1

B. -1

C. 2

D. 3

Answer: D

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2. If the function $f(X) = \begin{cases} (\cos x)^{1/x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x=0, then the value of k, is

A. 1

B. -1

C. 0

D. e

Answer: A

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3. The value of a for which the function $f(x) = f(x) = \left\{ \frac{(4^x - 1)\hat{3}}{\sin(xa)\log\{(1 + x^2 3)\}}, x \neq 012(\log 4)^3, x = 0 \right.$ may be continuous at $x = 0$ is 1 (b) 2 (c) 3 (d) none of these

A. 1

B. 2

C. 3

D. Nne of these

Answer: D

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4. If $R \to R$ is defined by

$$f(x) = \begin{cases} \frac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$

then the value of a so that f is continuous at x = 0 is

A. 2

B. 1

C. -1

D. 0

Answer: D

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5. Let $f(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \geq 0 \end{cases}$ and $\lambda \in R$, then at x = 0

A. f is discontinuous

B. f is continuous only,if λ=0

C. f is continuous only,whatever λ may be

D. None of these

Answer: A

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6. Determine the value of a, b, c for which the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} c \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{\frac{3}{2}}} \end{cases},$$

$fx < 0f \text{ or } x = 0iscont \in uousatx = 0f \text{ or } x > 0$

A. $a = \frac{-3}{2}, c = \frac{1}{2}, b = 0$

B. $a = \frac{3}{2}, c = \frac{1}{2}, b \neq 0$

C. $a = \frac{-3}{2}, c = \frac{1}{2}, b \neq 0$

D. None of these

Answer: C

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7. If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x}, & x \neq \frac{\pi}{4} \\ a \quad , & x = \frac{\pi}{4} \end{cases}$

is continuous at $x = \frac{\pi}{4}$, then a =

A. 4

B. 2

C. 1

D. $\frac{1}{4}$

Answer: D

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8. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$.

The value of $f(\pi)$ so that f(x) is continuous at $x = \pi$ is

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -1

D. 1

Answer: C

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9. Determine a & b so that f is continuous at $x = \frac{\pi}{2}$

whre $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & \text{if} \quad x < \frac{\pi}{2} \\ a & \text{if} \quad x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & \text{if} \quad x > \frac{\pi}{2} \end{cases}$

A. (1,4)

B. $\left(\frac{1}{2}, 2\right)$

C. $\left(\frac{1}{2}, 4\right)$

D. None of these

Answer: C

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10. Let $f(x)$ be a function defined as $f(x) = \left\{ \frac{x^2 - 1}{x^2 - 2|x - 1| - 1}, x \neq 1 \frac{1}{2}, x = 1 \right.$ Discuss the continuity of the function at $x = 1$.

A. The function is continuous for all values of x

B. The function is continuous only for x $>$ 1

C. The function is continuous at x = 1

D. The function is not continuous at x = 1

Answer: D

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11. If $f(x) = \left[\frac{8^x - 4^x - 2^x + 1^2}{x^2}, \ x > 0 e^x \sin x + 4x + k \ln 4, \ x \leq 0 \right.$ is continuous at $x = 0,$ then find the value of k.

A. 4log2

B. 2log2

C. log2

D. None of these

Answer: C

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12. The number of points at which the function $f(x) = \frac{1}{\log}|x|$ is discontinuous is (1) 0 (2) 1 (3) 2 (4) 3

A. 1

B. 2

C. 3

D. 4

Answer: C

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13. If $f(x) = \begin{cases} -x^2, & when x \leq 0 \\ 5x - 4, & \text{when } 0 < x \leq 1 \\ 4x^2 - 3x, & \text{when } 1 < x < 2 \\ 3x + 4, & when x \geq 2 \end{cases}$,then

A. f(x) is continuous at x = 0

B. f(x) is continuous at x = 2

C. f(x) is discontinuous at x = 1

D. None of these

Answer: B

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14. The points of discontinuity of the function $\lim_{n \to \infty} \left(\frac{(2 \sin x)^{2n}}{3^n - (2 \cos x)^{2n}} \right)$

A. R

B. $\left\{n\pi \pm \frac{\pi}{3}, n\varepsilon I\right\}$

C. $\left\{n\pi \pm \frac{\pi}{6}, n\varepsilon I\right\}$

D. None of these

Answer: C

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15. Let $f(x) = \left[x^3 - 3\right]$, where [.] is the greatest integer function, then the number of points in the interval (1,2) where function is discontinuous is (A) 4 (B) 5 (C) 6 (D) 7

A. 5

B. 4

C. 6

D. 3

Answer: C

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16. If $f(x) = \begin{cases} x-1, & x<0 \\ \frac{1}{4}, & x=0 \\ x^2, & x>0 \end{cases}$, then

A. $\lim_{x \to 0+} f(x) = 1$

B. $\lim_{x \to 0-} f(x) = 1$

C. f (X) is discontinuous at x = 0

D. None of these

Answer: C

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17. The number of points at which the function $f(x) = \frac{1}{x-[x]}$ ([.] denotes, the greatest integer function) is not continuous is

A. 1

B. 2

C. 3

D. None of these

Answer: D

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18. If $f(x) = \{(\sin x; x \text{ rational}) \ (\cos x; x \text{ is irrational})$ then the function is

A. discontinuous at $x = n\pi + \pi/4$

B. continuous at $x = n\pi + \frac{\pi}{4}$

C. discontinuous at all x

D. None of these

Answer: B

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19. If $f(x) = \begin{cases} 1 & , \text{ when} 0 < x \leq \frac{3\pi}{4} \\ 2(\sin)\frac{2}{9}x & , \text{ when} \frac{3\pi}{4} < x < \pi \end{cases}$, then

A. f(x) is continuous at x = 0

B. f(x) is continuous at $x = \pi$

C. f(x) is continuous at $x = \dfrac{3\pi}{4}$

D. f(x) is discontinuous at $x = \dfrac{3\pi}{4}$

Answer: C

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20. If f(x) = sgn (x) and g (x) = $\left(1 - x^2\right)$,then the number of points of discontinuity of function f (g (x)) is -

A. exact two

B. exactly three

C. finite and more than 3

D. infinitely many

Answer: A

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21. If $f(x) = \begin{cases} x, x \leq 1 \\ x^2 + bx + c, \quad x > 1 \end{cases}$ then find the values of b and c if f(x) is diferentiable at x=1.

A. b=c=1

B. b=-1,c=1

C. b=0,c=1

D. b=-1,c=0

Answer: B

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22. Let f(x) = |sin x| .Then

A. f is everywhere differentiable

B. f is everywhere continuous but no differentiable at x = nπ, $n\varepsilon$Z

C. f is everywhere continuous but not differentiable at x= $(2n+1)\frac{\pi}{2}, n\varepsilon Z$

D. None of these

Answer: B

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23. f(x)=maximum {2sinx,1-cosx} is not differentiable when x is equal to

A. 1

B. -1

C. 0

D. $\pi - \cos^{-1}\left(\frac{3}{5}\right)$

Answer: D

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24. Let $f: R \to R$ be such that f (1) = 3 and f'(1) = 6. Then, $\lim_{x \to 0}\left[\frac{f(1+x)}{f(1)}\right]^{1/x}$ equals

A. 1

B. $e^{1/2}$

C. e^2

D. e^3

Answer: C

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25. If f (x) = $ae^{|x|} + b|x|^2$; $a, b \varepsilon R$ and f(x) is differentiable at x=0.Then ,a and b are

A. a= 0,$b \varepsilon R$

B. a=1,b=2

C. b=0,$a \varepsilon R$

D. a=4 , b=5

Answer: A

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26. Let $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x^2}\right) & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$, then $\lim_{x \to \infty} f(x)$ is equal to

A. f is differentiable at x = 0 an x= 1

B. f is differentiable at x = 0 but not at x = 1

C. f is differentiable at x =1 but not at x = 0

D. f is neither differentiable at x = 0 nor at x =1

Answer: B

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27. If $f(x) = \begin{cases} \sqrt{x}\left(1 + x\sin\frac{1}{x}\right), x > 0 \\ -\sqrt{-x}\left(1 + x\sin\frac{1}{x}\right), x < 0 \\ 0, \qquad\qquad\qquad\qquad , x = 0 \end{cases}$,

thenf(x) is

A. continuous as well as differentiable at x = 0

B. continuous but not differentiable at x = 0

C. differentiable but not continuous at x = 0

D. neither continuous nor differentiable at x = 0

Answer: B

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28. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k$, k is an integer, is

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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29. Let $f: R \to R$ be a function defined by $f(x) = \max . \{x, x^3\}$. The set of all points where $f(x)$ is NOT differenctiable is

(a) $\{-1, 1\}$

(b) $\{-1, 0\}$

(c) $\{0, 1\}$

(d) $\{-1, 0, 1\}$

A. {-1,1}

B. {-1,0}

C. {0,1}

D. {-1,0,1}

Answer: D

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30. If $f(x) = \begin{cases} \frac{[x]-1}{x-1}, & x \neq 1 \\ 0 \quad , & x = 1 \end{cases}$ then f(x) is

A. continuous as well as differentiable at x = 1

B. differentiable but not continuous at x = 1

C. ontinuous but not differentiable at x = 1

D. neither continuous nor differentiable at x = 1

Answer: D

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31. If f(x) = $\begin{cases} e^x + ax & \text{for } x < 0 \\ b(x-1)^2 & \text{for } x \geq 0 \end{cases}$, is differentiable at x =0,then (a,b) is

A. (-3,-1)

B. (-3,1)

C. (3,1)

D. (3,-1)

Answer: B

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32. If $f(x) = \lim_{n \to \infty} \frac{\tan \pi x^2 + (x+1)^n \sin x}{x^2 + (x+1)^n}$ then

A. f is continuous at x = 0

B. f is differentiable at x =0

C. f is continuous but not differentiable at x = 0

D. None of these

Answer: D

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33. If $f(x) = \sqrt[3]{\frac{x^4}{|x|}}$,x $\neq$ 0 and f(0)=0 is:

A. continuous for all x but not differentiable for any x

B. continuous and differentiable for all x

C. continuous for all x and differentiable for all $x \neq 0$

D. continuous and differentiable for all $x \neq 0$

Answer: C

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34. Let $f: R \to R$ be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, $f(0) = 0$ and $f'(0) = 3$,then

A. f(x) is a quadratic function

B. f(x) is continuous but not differentiable

C. f(x) is differentiable in R

D. f(x) is bounded in R

Answer: C

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35. If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}(\log a)^n$, then at $x = 0$, $f(x)$

A. has no limit

B. is discontinuous

C. is continuous but not differentiable

D. is differentiable

Answer: D

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36. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then f(x) is

A. discontinuous everywhere

B. continuous as well as differentiabe for all x

C. continuous for all x but not differentiable at x = 0

D. neither differentiable nor continuous at x = 0

Answer: C

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37. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, andg'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2$ $\frac{2}{3}\sin x^3 \sec x^2 \tan x$ (d) none of these

A. $\frac{1}{2}x\cos x^3 \cos ec^2$

B. $\frac{3}{2}x\cos x^3 \cos ec^2$

C. $\frac{1}{2}x\sec x^3 \sin x^2$

D. $\frac{3}{2}x\sec x^3 \cos ecx^2$

Answer: B

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38. If $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{a}, then \frac{dy}{dx}$ wherever it is defined is $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$ (b) $\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}} - \frac{(a+b)}{2\sqrt{(a-x)(x-b)}}$ (d) $\frac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

A. $\frac{x+(a+b)}{\sqrt{(a-x)(x-b)}}$

B. $\dfrac{2x-a-b}{2\sqrt{(a-x)(x-b)}}$

C. $\dfrac{(a+b)}{2\sqrt{(a-x)(x-b)}}$

D. $\dfrac{2x+(a+b)}{2\sqrt{(a-x)(x-b)}}$

Answer: B

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39. If $x=\dfrac{1-t^2}{1+t^2}$ and $y=\dfrac{2t}{1+t^2}$, then $\dfrac{dy}{dx}$ is equal to

A. $-\dfrac{y}{x}$

B. $\dfrac{y}{x}$

C. $-\dfrac{x}{y}$

D. $\dfrac{x}{y}$

Answer: C

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40. $\frac{d}{dx}\left[\sin^2 \cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$ is

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 1

Answer: B

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41. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x \in (-\infty, -1)$

A. 1

B. -1

C. $-1/2$

D. x

Answer: A

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42. If $f(x) = 2\sin^{-1}\sqrt{1-x} + \sin^{-1}\left(2\sqrt{x(1-x)}\right)$ where $x \in \left(0, \frac{1}{2}\right)$, then $f'(x)$ has the value equal to (i) $\frac{2}{x\sqrt{1-x}}$ (ii) 0 (iii) $-\frac{2}{x\sqrt{1-x}}$ (iv) π

A. $\frac{2}{\sqrt{x(1-x)}}$

B. zero

C. $-\frac{2}{\sqrt{x(1-x)}}$

D. π

Answer: B

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43. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is

A. 1

B. -1

C. 2

D. None of these

Answer: A

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44. If $x = \sin t \cos 2t$ and $y = \cos t \sin 2t$, then at $t = \frac{\pi}{4}$, the value of $\frac{dy}{dx}$ is equal to:

A. -2

B. 2

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: C

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45. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$ then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$

A. $\frac{y^2+4}{n^2(x^2+4)}$

B. $\frac{y^2+4}{n(x^2+4)}$

C. $\frac{n^2(y^2+4)}{x^2+4}$

D. $\frac{n(y^2+4)}{x^2+4}$

Answer: C

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46. Let $y = x^3 - 8x + 7 and x = f(t)$. If $\frac{dy}{dt} = 2$ and $x = 3$ at $t = 0$, then $\frac{dx}{dt}$ at $t = 0$ is given by 1 (b) $\frac{19}{2}$ (c) $\frac{2}{19}$ (d) none of these

A. $\frac{2}{19}$

B. $\frac{3}{5}$

C. $\frac{-1}{17}$

D. $\frac{5}{16}$

Answer: A

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47. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$?

A. $(1+\log x)^2$

B. (1+logx)logx

C. $\frac{\log x}{((1+\log x))^2}$

D. $(\log x)^2.2$

Answer: C

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48. If $y = \sin x + e^x$, then $\frac{d^2x}{dy^2} =$

A. $\frac{\sin x - e^x}{(\cos x + e^x)^2}$

B. $\frac{\sin x - e^x}{(\cos x + e^x)^3}$

C. $\frac{\sin x - e^x}{(\cos x - e^x)^2}$

D. $(-\sin x + e^x)^{-1}$

Answer: B

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49. If f(x) $= |\log_e |x||$, then f'(x) equals

A. $\frac{1}{|x|}, x\pi 0$

B. 1/x for |x| gt 1 and -1/x for |x|lt1

C. $-\frac{1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

D. 1/x for x gt 0 and -1/x for x lt 0

Answer: B

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50. If $f(x) = x^4 \tan(x^3) - x1n(1 + x^2)$, then the value of $\frac{d^4(f(x))}{dx^4}$ at $x = 0$ is 0 (b) 6 (c) 12 (d) 24

A. 0

B. 6

C. 12

D. 24

Answer: A

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51. let $y = t^{10} + 1,$ and $x = t^8 + 1,$ then $\frac{d^2y}{dx^2}$ is

A. 5/2t

B. $20t^8$

C. $\frac{5}{16t^6}$

D. None of these

Answer: C

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52. If $x = \exp\left\{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)\right\}$ then dy/dx equals

A. $2x[1 + \tan(\log x)] + x\sec^2(\log x)$

B. $x[1 + \tan(\log x)] + x\sec^2(\log x)$

C. $2x[1 + \tan(\log x)] + x^2\sec^2(\log x)$

D. $2x[1 + \tan(\log x)] + \sec^2(\log x)$

Answer: A

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53. If $y = \log^n x$, where $\log^n$ means log log log...

(repeated n time), then x log $x \log^2 x \log^3 x \ldots \log^{n-1} x \log^n$xdy/dx is equal

to

A. log x

B. $\log^n x$

C. 1/logx

D. 1

Answer: B

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54. If $x = e^{y + e^{y + \ldots \text{to}00}}, x > 0, \text{then} \frac{dy}{dx}$

A. (1+x)/x

B. 1/x

C. (1-x)/x

D. x/(1+x)

Answer: C

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55. $\frac{d}{dx}\left[\log\left\{e^x\left(\frac{x-2}{x+2}\right)^{3/4}\right\}\right]$ equals $\frac{x^2-1}{x^2-4}$ (b) 1 (c) $\frac{x^2+1}{x^2-4}$ (d) $e^x\frac{x^2-1}{x^2-4}$

A. $\frac{x^2-1}{x^2-4}$

B. 1

C. $\frac{x^2+1}{x^2-4}$

D. $e^x\frac{x^2+1}{x^2-4}$

Answer: A

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56. If : $y = \frac{1}{x}$, then: $\frac{dy}{\sqrt{1+y^4}} + \frac{dx}{\sqrt{1+x^4}} =$

A. 0

B. 1

C. x/y

D. y/x

Answer: A

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57. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{y \cos x}{2y - 1}$

B. $\frac{\cos x}{2y + 1}$

C. $\frac{\cos x}{2y-1}$

D. None of these

Answer: C

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58. The equation $x \log x = 3 - x$ has, in the interval (1,3) :

A. exactly one root

B. atmost one root

C. atleast one root

D. no root

Answer: C

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59. In $[0, 1]$ Lagranges Mean Value theorem in NOT applicable to

$f(x) = \left\{ \frac{1}{2} - \times < \frac{1}{2}\left(\frac{1}{2} - x\right)^2 x \geq \frac{1}{2} \right.$ b. $f(x) = \left\{ \frac{s \in x}{x}, x \neq 01, x = 0 \right.$ c. $f(x) = x|x|$ d. $f(x) = |x|$

A. $f(x) = \begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$

B. $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

C. f(x)=x|x|

D. f(x)=|x|

Answer: A

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60. Rolle's theorem hold for the function $f(x) = x^3 + bx^2 + cx, 1 \leq x \leq 2$ at the point 4/3, the values of b and c are

A. b=8,c=-5

B. b=-5,c=8

C. b=5,c=-8

D. b=-5,c=-8

Answer: B

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61. The value of x in the interval [4,9] at which the function f(x) = $\sqrt{x}$ satisfies the mean value theoram is

A. $\frac{13}{4}$

B. $\frac{17}{4}$

C. $\frac{21}{4}$

D. $\frac{25}{4}$

Answer: D

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62. The value of c in (0,2) satisfying the Mean Value theorem for the function $f(x) = x(x-1)^2, x\varepsilon[0, 2]$ is equal to

A. $\frac{3}{4}$

B. $\frac{4}{3}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: B

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63. If $f(x) > x; \forall x \in R.$ Then the equation $f(f(x)) - x = 0,$ has

A. exactly once in(a,b)

B. atmost once in (a,b)

C. atleast once in (a,b)

D. None of these

Answer: B

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64. If $f(x) = x^a \log x$ and $f(0) = 0$ then the value of α for which Rolle's theorem can be applied in [0,1] is

A. -2

B. -1

C. 0

D. $1/2$

Answer: D

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65. If f(x) satisfies the requirements of Lagrange's mean value theorem on [0, 2] and if f(0)= 0 and $f'(x) \leq \frac{1}{2}$

A. |f(x)|le2

B. f(x)le1

C. f(x)=2x

D. f(x)=3 for atleast one x in [0,2]

Answer: B

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Exercise 2 Concept Applicator

1. In order that the function $f(x) = (x+1)^{\cot x}$ is continuous at x = 0, the value of f(0) must be defined as

A. e

B. C^2

C. e^3

D. None of these

Answer: A

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2. The function $f: R - \{0\} \to R$ given by $f(x) = \frac{1}{x} - \frac{2}{e^2x - 1}$ can be made continuous at x=0 by defining f(0) as

A. 0

B. 1

C. 2

D. -1

Answer: B

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3. Let $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 3-x & 5-3x^2 & 3x^3-1 \\ 2x^2-1 & 3x^5-1 & 7x^8-1 \end{vmatrix}$ then the equation of $f(x) = 0$ has

A. f(x) = 0 has at least two real roots

B. f'(x) = 0 has at least one real root

C. f (x) is many-one function

D. All of the above

Answer: D

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4. Find the value of a for which the following function is continuous at a given point.

$$f(x) = \begin{cases} \frac{\sin[x]}{[x+1]}, & f \text{ or } x > 0 \\ \frac{\frac{\cos \pi}{2}[x]}{[x]}, & f \text{ or } x < 0 \end{cases} at\ x = 0,\ a,\ at\ x =$$

where $[x]$ denotes the greatest less than or equal to x.

A. equals to 0

B. equal to 1

C. equal to -1

D. indeterminate

Answer: A

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5. The value of a for which the function $f(x) = f(x) = \left\{ \frac{(4^x - 1)\hat{3}}{\sin(xa)\log\{(1 + x^2 3)\}}, x \neq 012(\log 4)^3, x = 0 \right.$ may be continuous at $x = 0$ is 1 (b) 2 (c) 3 (d) none of these

A. 1

B. 2

C. 3

D. None of these

Answer: D

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6. If f(x) $\begin{array}{ll} = \frac{8^x - 4^x - 2^x + 1}{x^2}, & \ldots \quad x > 0 \\ = e^x \sin x + \pi x + \lambda. \text{In } 4, & \ldots \quad x \leq 0 \end{array}$ is continuous at x = 0 , then : λ =

A. $4 \log_e 2$

B. $2 \log_e 2$

C. $\log_e 2$

D. None of these

Answer: C

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7. The function defined by $f(x) = \begin{cases} \left(\frac{1}{x^2+e^2-x}\right)^{-1} & x \neq 2 \\ k & , \quad x = 2 \end{cases}$,is continuous from right at the point x = 2 ,then k is equal to

A. 0

B. $1/4$

C. $-1/4$

D. None of these

Answer: B

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8. In the Mean Value theorem $\frac{f(b)-f(a)}{b-a} = f'(c)$ if $a = 0, b = \frac{1}{2}$ and f(x)=x(x-1)(x-2) the value of c is

A. $1 - \frac{\sqrt{15}}{6}$

B. $1 + \sqrt{15}$

C. $1 - \frac{\sqrt{21}}{6}$

D. $1 + \sqrt{21}$

Answer: C

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9. Leg $f(x+y) = f(x) + f(y) f \text{ or } all x, y \in R,$ If $f(x) iscont \in uousatx = 0, showthatf(x)$ is continuous at all x.

A. (-∞,∞)

B. (0,∞)

C. (-∞,0)

D. (2,∞)

Answer: A

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10. Which of the following is true about

$$f(x) = \begin{cases} \frac{(x-2)}{|x-2|}\left(\frac{x^2-1}{x^2+1}\right) & x \neq 2 \\ \frac{3}{5} & x = 2 \end{cases}$$

A. f(x) is continuous at x=2

B. f(x) has removable discontinuity at x=2

C. f(x) has non-removable discontinuity at x = 2

D. Discontinuity at x = 2 cab be removed by redefining function at x = 2

Answer: C

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11. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2+1}$ Then the value of $f(0)$ is

A. 1

B. 2

C. -1

D. -2

Answer: A

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12. if $y = \log x \cdot e^{(\tan x + x^2)}$, then $\frac{dy}{dx}$ is equal to

A. $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x + x)\log x\right]$

B. $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x - x)\log x\right]$

C. $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x + 2x)\log x\right]$

D. $e^{(\tan x + x^2)}\left[\frac{1}{x} + (\sec^2 x - 2x)\log x\right]$

Answer: C

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13. Find the values of $aandb$ if $f(x) = \{a + \sin^{-1}(x+b), x \geq 1x, x < 1$ is differentiable at $x = 1$.

A. a=1,b=-1

B. a=-1,b=1

C. a=2,b=0

D. a=0,b=0

Answer: A

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14. Which of the following functions is not differentiable at x=1?

A. $f(x) = (x^2 - 1)|(x-1)(x-2)|$

B. f(x)=sin(|x-1|)-|x-1|

C. f(x)=tan(|x-1|)+|x-1|

D. None of these

Answer: C

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15. If

$$f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots \text{to } \infty$$

,then at x = 0,f(x)

(a) has no limit

(b) is discontinuous

(c) is continuous but not differentiable

(d) is differentiable

A. 1

B. 2

C. 3

D. 4

Answer: D

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16. if $f(x) = (x^2 - 4)|(x^3 - 6x^2 + 11x - 6)| + \frac{x}{1 + |x|}$ then set of points at which the function if non differentiable is

A. {-2,2,1,3}

B. {-2,0,3}

C. {-2,2,0}

D. {1,3}

Answer: D

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17. Number of point where function f(x) defined as $f: [0, 2\pi] \to R, f(x) = \begin{cases} 3 - \left|\cos x - \frac{1}{\sqrt{2}}\right|, & |\sin x| < \frac{1}{\sqrt{2}} \\ 2 + \left|\cos x + \frac{1}{\sqrt{2}}\right|, & |\sin x| \geq \frac{1}{\sqrt{2}} \end{cases}$ is non differentiable is

A. 2

B. 3

C. 4

D. 5

Answer: C

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18. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

A. $a\varepsilon[8, 64]$

B. $a\varepsilon(0, 8]$

C. $a\varepsilon[64, \infty]$

D. None of these

Answer: C

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19. If g is the inverse function of $fandf'(x) = \sin x, theng'(x)$ is $\cos ec\{g(x)\}$ (b) $\sin\{g(x)\} - \frac{1}{\sin\{g(x)\}}$ (d) none of these

A. cosec{g(x)}

B. sin{g(x)}

C. $-\frac{1}{\sin\{g(x)\}}$

D. cos{g(x)}

Answer: A

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20. Which of the following function is not differentiable at $x = 0$?

$f(x) = \min\{x, \sin x\}$ $f(x) = \{0, x \geq 0x^2, x < 0$ (c)

$f(x) = x^2 sgn(x)$

A. Only (i)

B. Only (ii)

C. Only (iii)

D. All of the above

Answer: D

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21. If $y = \left[(\tan x)^{\tan x}\right]^{\tan x}$,then at $x = \frac{\pi}{4}$, the value of $\frac{dy}{dx} =$

A. 1

B. 3

C. 4

D. 0

Answer: D

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22. Given $F(x) = f(x)\phi(x)$ and $f'(x)\phi'(x) = c$ then prove that $F''\frac{x}{F}(x) = \frac{f''}{f} + \frac{\phi''}{\phi} + 2c$

A. $\frac{d^2y}{dx^2}$

B. $\frac{1}{y}\frac{d^2y}{dx^2}$

C. $y\frac{d^2y}{dx^2}$

D. None of these

Answer: B

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23. Let $f: R\overrightarrow{R}$ be a function defined by $f(x) = M \in \{x+1, |x|+1\}$. Then which of the following is true? (1) $f(x) \geq 1f$ or $allx \in R(2)f(x)$ is not differentiable at $x = 1$ (3) f(x) is differentiable everywhere (4) f(x) is not differentiable at $x = 0$

A. f(x) is differentiable everywhere

B. f(x) is not differentiable at x = 0

C. f(x) $\geq$ 1 for all x ε R

D. f(x) is not differentiable at x = 1

Answer: A

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24. If $(x-a)^2+(y-b)^2=c^2,$ for some $c>0,$ $provethat\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}isaconstant \in dependen \rightarrow f$ a and b.

A. is a constant dependent on a

B. is a constant dependent on b

C. is a constant independent of a and b

D. 0

Answer: C

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25. Let f''(x) be continuous at x = 0 and f''(0) = 4 then value of $\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$

A. 12

B. 10

C. 6

D. 4

Answer: A

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26. If f(x) and g(x) ar edifferentiable function for $0 \leq x \leq 1$ such that $f(0) = 2, g(0), f(1) = 6, g(1) = 2,$ then in the interval (0,1)

A. f'(x) = 0,for all x

B. f'(x) = 2g'(x),for atleast one x

C. f'(x) = 2g'(x),for atmost one x

D. None of these

Answer: B

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27. Let $f[2, 7] \to [0, \infty)$ be a continuous and differentiable function. Then, the value of

$$(f(7) - f(2))\frac{(f(7))^2 + (f(2))^2 + f(2).\, f(7)}{3}$$ is

(where $c \in (2, 7)$)

A. $5f^2(c)f'(c)$

B. $5f'(c)$

C. $f(c)f'(c)$

D. None of these

Answer: A

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28. The equation $e^{x-8}+2x-17=0$ has :-

A. two real roots

B. one real root

C. eight real roots

D. four real roots

Answer: B

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29. If $f(x)=\frac{\tan[x]\pi}{[1+|\log(\sin^2 x+1)|]}$,where [.] denotes the greatest integer function and |.| stands for the modulus of the function,thenf(x) is

A. discontinuous $\forall \quad x \quad \varepsilon \quad I$

B. continuous $\forall$ x

C. non differentiable $\forall \quad x \quad \varepsilon \quad I$

D. a periodic function with fundamental period I.

Answer: B

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30. If f''(x) =- f(x) and g(x) = f'(x) and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that F(5) =5, then F(10) is

A. 5

B. 10

C. 0

D. 15

Answer: A

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