MATHS

BOOKS - IA MARON MATHS (HINGLISH)

INTRODUCTION OF MATHEMATICAL ANALYSIS

Real Numbers

1. Prove that the sum of or the difference between, a rational number α and an irrational number β is an irrational number.

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2. (a) Find all rational values of x at which $y = \sqrt{x^2 + x + 3}$ is a rational number.

(b) Prove that $\sqrt{2}$ is an irrational number.

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3. Prove that for every positive number s satisfying the condition $s^2 > 2$ one can always find a smaller rational number $s - k(k > 0)$, forwhich $(s - k)^2 > 2$.

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4. Solve the following inequalities.

(a) $|2x - 3| < 1$, (b) $(x - 2)^2 \geq 4$,

(c) $x^2 \geq 4$,

(c) $x^2 + 2x - 8 \leq 0$,

(d) $\left|x^2 - 7x + 12\right| > x^2 - 7x + 12$.

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5. Find out the whether the following equations have any solutions:

(a) $|x| = x + 5$, (b) $|x| = x - 5$?

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6. Determine the values of x satisfying the following equalities:

(a) $\left|\frac{x-1}{x+1}\right| = \frac{x-1}{x+1}$ (b) $\left|x^2-5x+6\right| = -\left(x^2 \pm 5x+6\right)$

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7. Solve the inequalities:

(a) $|\sin x| = \sin x + 1$

(b) $\left|x^2-5x\right| > \left|x^2\right| - |5x|$

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8. Find the roots of the following equations:

(a) $|\sin x| = \sin x + 1$

(b) $x^2 - 2|x| - 3 = 0$

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Function Domain Of Definition

1. Given the function $f(x) = (x+1)(x-1)$

Find $f(2x), 2f(x), f(x^2), [f(x)]^2$.

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2. Give the function $f(x) = (x+1)(x^3-1)$. Find $f(-1), f(a+1), f(a)+1$.

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3. Given the function $f(x) = x^3 - 1$. Find $\frac{f(b)-f(a)}{b-a} (b \neq a)$ and $f\left(\frac{a+b}{2}\right)$

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4. Given the function $f(x) = \begin{cases} 3^{-x} - 1 & -1 \le x < 0 \\ \tan(x/2) & 0 \le x < \pi \\ x/(x^2 - 2) & \pi \le x \le 6 \end{cases}$

Find f(-1), $f(\pi/2)$, $f\left(\frac{2\pi}{3}\right)$, $f(4)$, $f(6)$.

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5. The function f(x) is defined over the whole number scale by the following law:

$$f(x) = \begin{cases} 2x^3 + 1 & \text{if } x \le 2 \\ 1/(x-2) & \text{if } 2 < x \le 3 \\ 2x - 5 & \text{if } x > 3 \end{cases}$$

Find $f(\sqrt{2})$, $f(\sqrt{8})$, $f(\sqrt{\log 2})$, $f(1024)$.

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6. Calculate $f(x) = \frac{49}{x^2} + x^2$ at the points for which $\frac{7}{x} + x = 3$

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7. Find a function of the form $f(x) = ax^2 + bx + C$. If it is known that

$f(0) = 5,\ f(-1) = 10,\ f(1) = 6.$

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8. Find a function of the form $f(x) = a + bc^x (c > 0)$

if $f(0) = 15,\ f(2) = 30,\ f(4) = 90$

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9. Given the function $f(x) = \dfrac{5x^2 + 1}{2 - x}$

Find $f(3x),\ f(x^3),\ 3f(x),\ [f(x)]^3.$

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10. Let $f(x) = \begin{cases} 3^x & at \quad -1 < x < 0 \\ 4 & at \quad 0 \leq x < 1 \\ 3x - 1 & at \quad 1 \leq x \leq 3 \end{cases}$

Find f(2), f(0), f(0.5), f(-0.5), f(3).

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11. Prove that if for an exponential function $y = a^x (a > 0, a \neq 1)$ the values of the argument $x = x_n (n = 1, 2, \ldots.)$ form an arithmetic progression,then the corresponding values of the function $y_n = a^x (n = 1, 2, 3..)$ form a geometric progression.

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12. $f(x) = x^2 + 6, \omega(x) = 5x$. Solve the equation $f(x) = |\omega(x)|$.

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13. Find f(x) if $f(x+1) = x^2 - 3x + 2$

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14. Evaluate the functions $f(x) = x^2 + \frac{1}{x^2}$ and $\phi(x) = x^4 + \frac{1}{x^4}$ for the points at which $\frac{1}{x} + x = 5$

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15. $f(x) = x + 1, \omega(x)x = x - 2$, solve the equation

$|f(x) + \phi(x)| = |f(x)| + |\omega(x)|$.

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16. A rectangle with altitude x is inscribed in a triangle ABC with the bae b and altitude h. Express the perimeter P and area S of the rectangle as fuction of x.

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17. Find the domains of definaton of the following functions:

(a) $f(x) = \sqrt{x-1} + \sqrt{6-x}$

(b) $f(x) = \sqrt{x^2 - x - 2} + \frac{1}{\sqrt{3 + 2x - x^2}}$

(c) $f(x) = \frac{x}{\sqrt{(x^2 - x - 2)}}$

(d) $f(x) = \sqrt{\sin x - 1}$

(e) $f(x) = \sqrt{\log \frac{5x - x^2}{4}},$

(f) $f(x) = \log_x 5,$

(g) $f(x) = \log \frac{x^2 - 5x + 6}{x^2 + 4x + 6}$

(h) $f(x) = arc \sin \frac{x-3}{2} - \log(4-x)$

(i) $f(x) = \frac{1}{\log(1-x)} + \sqrt{x+2},$

(j) $f(x) = \log \cos x,$

(k) $f(x) = arc \cos \frac{3}{4 + 2 \sin x}$

(l) $y = \frac{1}{\sqrt{|x| - x}}$

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18. Find the domains of definaton of the following functions:

(a) $f(x) = \sqrt{arc \sin(\log_2 x)},$

(b) $f(x) = \log_2 \log_3 \log_4 x,$

(c) $f(x) = \frac{1}{x} + 2^{arc\sin x} + \frac{1}{\sqrt{x-2}}$

(d) $f(x) = \log|4 - x^2|,$

(e) $f(x) = \sqrt{\cos(\sin x) + arc\sin\frac{1+x^2}{2x}}$

Find the ranges of the following functions:

(f) $y = \frac{1}{2 - \cos 3x}$

(g) $y = \frac{x}{1+x^2}$

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19. Find the domains of definition of the following functions:

(a) $y = \frac{2x-3}{\sqrt{x^2+2x+3}}$ (b) $y = \log\sin(x-3) + \sqrt{16 - x^2},$

(c) $y = \sqrt{3-x} + arc\cos\frac{x-2}{3}$ (d) $y = \frac{x}{\log(1+x)}$

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20. The function f(x) is defined on the interval (0,1). What are the domains of definition of the following functions.

(a) $f(3x^2)$, $(b)\, f(x-5)$, $(c)\, f(\tan x)$?

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21. The function f(x) is defined on the interval [0,1]. What are the domains of definition of functions.

(a) $f(\sin x)$, $(b)\, f(2x+3)$?

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Investigation Of Functions

1. (a) Find the minimum value of the function $y = 3x^2 + 5x - 1$

(b) Find the reactangle with the maximum area from among all rectangles of a given perimeter.

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2. Show that

(a) the function $f(x) = x^3 + 3x + 5$ increase in the entire domain of the definition,

(b) the function $g(x) = \dfrac{x}{(1+x)^2}$ decrease in the interval $(1, \infty)$

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3. Find the intervals of increase and decrease for the following functions:

(a) $f(x) = \sin x + \cos x$,

(b) $\tan\left(x + \dfrac{\pi}{3}\right)$

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4. Find the minimum and maximum values of the function $f(x) = a\cos x + b\sin x\left(a^2 + b^2 > 0\right)$

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5. Find the minimum value of the function $f(x) = 3^{\left(x^2-2\right)^3} + 8$

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6. Test the function $f(x) = \tan x + \cot \quad \text{x, where} \quad 0 < x < \frac{\pi}{2},$ for increase and decrease.

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7. Which of the given functions is (are) even, odd, and which of them is (are) neither even, nor odd?

$f(x) = \log x + \sqrt{1+x^2}$, (b) $f(x) = \log \frac{1-x}{1+x}$
(c) $f(x) = 2x^3 - x + 1$, (d) $f(x) = x \frac{a^x+1}{a^x-1}$

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8. Which of the following functions is (are) even and which is (are) odd?

(a) $f(x) = 4 - 2x^4 + \sin^2 x$ (b) $f(x) = \sqrt{1+x+x^2} - \sqrt{1-x+x^2}$

(c) $f(x) = \frac{1+a^{kx}}{1-a^{kx}}$ (d) $f(x) = \sin x + \cos x$,

(e) f(x)=const.

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9. Indicate the amplitude |A|, frequency ω initial phase ω' and period T of the following harmonics:

(a) $f(x) = 5\sin 4x$, (b) $f(x) = 4\sin(3x + \pi/4)$

(c) $f(x) = 3\sin(x/2) + 4\cos(x/2)$

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10. Find the period for each of the following functions:

(a) $f(x) = \tan 2x$

(b) $f(x) = \cot\left(\frac{x}{2}\right)$,

() $f(x) = \sin 2\pi x$.

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11. Find the period for each of the following functions

(a) $f(x) = \sin^4 x + \cos^4 x$

(b) $f(x) = |\cos x|$

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12. Find the greatest value of the function $f(x) = \dfrac{2}{\sqrt{2x^2 - 4x + 3}}$

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13. Which of the following functions are even, and which are odd :

(a) $f(x) = \sqrt[3]{(1-x)^2} + \sqrt[3]{(1+x)^2}$,

(b) $f(x) = x^2 - |x|$,

(c) $f(x) = x \sin^2 x - x^3$,

(d) $f(x) = (1 + 2^x)^2 / 2^x$?

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14. Find the period for each of the following functions:

(a) $f(x) = arc\tan(\tan x)$

(b) $f(x) = 2\cos\frac{x-\pi}{3}$

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15. Prove that the functions

(a) $f(x) = x + \sin x, (b) f(x) = \cos\sqrt{x}$ are non-periodic.

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Inverse Functions

1. Find the inverse to the function y=3x+5.

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2. Show that the function $y = \frac{k}{x}(k \neq 0)$ is inverse to itself.

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3. Find the inverse of the function $y = \log_a\left(x + \sqrt{x^2 + 1}\right), (a > 0, a \neq 1)$

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4. Show that the functions $f(x) = x^2 - x + 1, x \geq \frac{1}{2}$ and $\phi(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$ and mutually inverse and solve the equation $x^2 - x + 1 = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$.

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5. Find the inverse of the given functions:

(a) $y = \sin(3x - 1) at - \left(\frac{\pi}{6} + \frac{1}{3}\right) \leq x \leq \left(\frac{\pi}{6} + \frac{1}{3}\right)$

(b) $y = arc\sin\left(\frac{x}{3}\right) at - 3 \leq x \leq 3,$

(c) $y = 5^{\log x}$

$(d) y = 2^{x(x-1)}$

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6. Show that the inverse of the function $f(x) = \frac{1-x}{1+x}$, where $x \neq -1$, is itself

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Graphical Representation Of Functions

1. Sketch the graph of the following functions:

(a) $y = \cos x + |\cos x|,$

(b) $y = |x+2|x,$

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2. Sketch the graph of the function $y = 2|x-2| - |x+1| + x$.

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3. Sketch the graph of the function $Y = \sin x$.

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4. Graph the function $y = 3\cos x - \sqrt{3}\sin x$ by transforming the cosine curve.

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Number Sequences Limit Of A Sequence

1. Given the general term of the sequence (x_n), $x_n = \frac{\sin(n\pi/2)}{n}$. Write the first five term of this sequence.

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2. Find the first several term of the sequence if the gerneral term is given by one of the following formulas:

(a) $x_n = \sin\left(\frac{n\pi}{3}\right)$,

(b) $x_n = 2^{-n} \cos n\pi$

(c) $x_n = (1 + 1/n)^n$.

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3. Prove that $\lim_{n \to \infty} x_n = 1, \quad \text{if} \quad x_n = \frac{3^n + 1}{3^n}$.

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4. Prove that $\lim x_n = 2, \quad \text{if} \quad x_n = (2n + 3)/(n + 1)$. Find the number of the term beginning with which the inequality $|(2n + 3)/(n + 1) - 2| \leq \varepsilon$, where e=0.1, 0.01, 0.001, is fulfilled.

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5. Prove that the sequence $\frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{7}{8}, \frac{1}{8}......$ with the general term $x_n = \begin{cases} 1 - \frac{1}{2^{(n+1)/2}} & \text{if n is odd} \\ \frac{1}{2^{\pi/2}} & \text{if n is odd} \end{cases}$ has no limit.

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6. Test the following sequences for limits:

(a) $x_n = \frac{1}{(2\pi)}, (b) x_n = \begin{cases} 1 & \text{for an even n} \\ 1/n & \text{for an odd } \pi \end{cases},$

(c) $x_n = \frac{1}{n} \cos \frac{n\pi}{2}, \quad (d) x_n = n\{1 - (-1)^n]$

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7. Prove that the sequence with the general term $x_n = \frac{1}{n^k}(k > 0)$ is an infinitely small sequence.

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8. Prove that the sequence with the general term

(a) $x_1 = \dfrac{1-(-1)^n}{n}$

(b) $x_n = \dfrac{1}{n}\sin\left[(2n-1)\dfrac{\pi}{2}\right]$

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9. Prove that $\lim_{n \to \infty} \sqrt[n]{a} = 1(a > 0.$

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Evaluation Of Limits Of Sequences

1. Find $\lim_{n \to \infty} x_n$ if

(a) $x_n = \dfrac{3n^2+5n+4}{2+n^2}$ (b) $x_n = \dfrac{5n^3+2n^2-3n+7}{4n^3-2n+11}$

(c) $x_n = \dfrac{4n^2-4n+3}{2n^3+3n+4}$ (d) $x_n = \dfrac{1^2+2^2+\ldots.+n^2}{5n^3+n+1}$

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2. Find $\lim_{n \to \infty} x_n$, if

(a) $x_n = \left(\frac{3n^2+n-2}{4n^2+2n+7}\right)^3$, (b) $x_n = \left(\frac{2n^3+2n^2+1}{4n^3+7n^2+3n+4}\right)^4$,

(c) $x_n = \sqrt[n]{5n}$, (d) $x_n = \sqrt[n]{n^8}$,

(e) $x_n = \sqrt[n]{n^5}$, (f) $x_n = \sqrt[n]{6n+3}$.

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3. Find $\lim_{n \to \infty} \left(\frac{2n^3}{2n^2+3} + \frac{1-5n^2}{5n+1}\right)$

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4. Find $\lim_{n \to \infty} x_n$ if

(a) $x_{n=\sqrt{2n+3}-\sqrt{n-1}}$

(b) $x_n = \sqrt{n^2+n+1} - \sqrt{n^2-n+1}$,

(c) $x_n = n^2\left(n - \sqrt{n^2+1}\right)$

(d) $x_n = \sqrt[3]{n^2-n^3} + \pi$

(e) $x_n = \frac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[5]{n^3+n}-\sqrt{n}}$

(f) $x_n = \sqrt[n+1]{\hat{}}(2) - \sqrt[3]{n-1}^2$,

(g) $x_n = \dfrac{1-2+3-4+5-6+\ldots-2n}{\sqrt{n^2+!}+\sqrt{4n^2-1}}$

(g) $x_n = \dfrac{1}{1.2} + \dfrac{1}{2.3} + \dfrac{1}{3.4} + \ldots. + \dfrac{1}{n(n+10)}$

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5. Find $\lim_{n \to \infty} x_n$ if

(a) $x_n = \dfrac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$

(b) $x_n = \dfrac{\sqrt{n^2+4n}}{\sqrt[3]{n^3-3n^2}}$

(c) $x_n = \sqrt[3]{1-n^3}+n$,

(d) $x_n = \dfrac{1}{2n}\cos n^3 - \dfrac{3n}{6n+1}$

(e) $x_n = \dfrac{2n}{2n^2-1}\cos\dfrac{n+1}{2n-1} - \dfrac{n}{1-2n}\dfrac{n(-1)^n}{1-2n}\left(n^2+1\right)$

(f) $x_n = \dfrac{1+\frac{1}{2}+\frac{1}{4}+\ldots.+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\ldots\ldots+\frac{1}{3^n}}$

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Testing Sequences For Convergence

1. Taking advantage of the theorem on the existence of a limit of monotonic bounded prove that the following sequence are convergent.

$x_n = \frac{n^2-1}{n^2}$, (b) $x_n = 2 + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!}$

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2. Prove that the following sequences converage and find their limits:

(a)

$x_1 = \sqrt{2}, x_2 = \sqrt{2+\sqrt{2}}, x_3 = \sqrt{2+\sqrt{2+\sqrt{2}}}, \ldots., x_n = \sqrt{(2+\sqrt{2+}$

(b) $x_n = \frac{2^n}{(n+2)!}$

(c) $x_n = (E(ny))$

the sequence of successive decimal appromations 1:1:4,, 1.41, 1.414, of the irratioanl number $\sqrt{2}$

(e) $x_n = \frac{n!}{n^n}$.

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3. Uusing the theorem on passing to the limit in inequalities prove

$$\lim_{n \to \infty} \sqrt[n]{a} = 1(a > 0)$$

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4. Prove the existence of the limit of the sequence $y_n = a^{\frac{1}{2^n}}(a > 1)$ and calculate it.

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5. Taking advantage of the theorem on the limit of a monotomic sequence prove the existence of a finite limit of the sequence.

$$x_n = 1 + \frac{1}{2^n} + \frac{1}{3^2} + \dots. + \frac{1}{n^2}$$

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6. Taking advantage of the theorem on passing to the limit in inequalities, prove that $\lim_{n \to \infty} x_n = 1$ if $x_n = 2n\left(\sqrt{n^2+1} - n\right)$

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7. Prove that the sequence $x_1 = \sqrt{x}, x_2 = \sqrt{a + \sqrt{a}},$ $x_3 = \sqrt{a + \sqrt{+ \sqrt{a}}}, x_n = \sqrt{a + \sqrt{a + \dots . + \sqrt{a}}}$ has the limits $b = \left(\sqrt{4a+1} + 1\right)/2$

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8. Prove that the sequence with the general term $x_n = \frac{1}{3+1} + \frac{1}{3^2+2} + \dots + \frac{1}{3^n+n}$ has a finite limit.

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The Limit Of A Function

1. Proceeding from the definiton of the limit of a function after Cauchy (ie, in the term of $e-\delta le-M''$ etc). Prove that

(a) $\lim_{x\to 1}(3x-8)=-5$, (b) $\lim_{x\to\infty}\frac{5x+1}{3x+9}=\frac{5}{3}$

(c) $\lim_{x\to 1}\frac{1}{(1-x)^2}=+\infty$, (d) $\lim_{x\to\infty}\log_a x=\infty(a>1)$,

(e) $\lim_{x\to\infty} arc\tan x=\frac{\pi}{2}$, (f) $\sin_{x\to\frac{\pi}{6}} x=\frac{1}{2}$

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Calculation Of Limits Of Functions

1. Find the limits :

(a) $\lim_{x\to 1}\frac{4x^5+9x+7}{3x^5+x^2+1}$ (b) $\frac{x^3+3x^2-9x-2_{x\to 2}}{x^3-x-6}$

(c) $\lim_{x\to -1}\frac{x+1}{\sqrt{6x^2+3+3x}}$ (d) $\lim_{x\to 1}\frac{x^p-1}{x^q-1}$ (p and q integers)

(e) $\lim_{x\to 0}\sqrt{\frac{9+5x+4x^2-3}{x}}$ (f) $\lim_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}$

(g) $\lim_{x\to 0}\frac{\sqrt{x+7}-3\sqrt{2x-3}}{\sqrt[3]{x+6}-2\sqrt[3]{3x-5}}$ (h) $\log_a\frac{x-3}{\sqrt{x+6}-3}_{x\to 3}$

(i) $\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3-3x+2}$ (j) $\frac{\sqrt{x+8}-\sqrt{8x+1}_{x\to 1}}{\sqrt{5-x}-\sqrt{7x-3}}$

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2. Find the limits :

(a) $\lim_{x \to \infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)$

(b) $\lim_{x \to +\infty} \left(\sqrt{9x^2 + 1} - 3x \right)$

(c) $\lim_{x \to 1} \frac{2\sqrt{x + 3\sqrt[3]{x + 5\sqrt[5]{x}}}}{\sqrt{3x - 2 + \sqrt[3]{2x - 3}}},$

(d) $\lim_{x \to \infty} \left(\sqrt{2x^2 - 3 - 5x} \right)$

(e) $\lim_{x \to +\infty} x \left(\sqrt{x^2 + 1 - x}, \right.$

(f) $\lim_{x \to +\infty} \frac{\sqrt{2x^2 + 3}}{4x + 2}$ and $\lim_{x \to \infty} \frac{\sqrt{2x^2 + 3}}{4x + 2}$

(g) $\lim_{x \to \infty} 5^{2x / (x + 3)}$

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3. Find the limits:

(a) $\lim_{x \to 1} \frac{2x - 2}{\sqrt[3]{26 + x} - 3}$:

(b) $\frac{\underset{x \to -1}{x + 1}}{\sqrt[4]{x + 17} - 2}$

(c) $\lim_{x \to -1} \frac{1 + \sqrt[3]{x}}{1 + \sqrt[5]{x}}$

(d) $\lim_{x \to 0} \frac{\sqrt[k]{1+x}-1}{x}$ (k positive integer)

(e) $\lim_{x \to \pi/6} \frac{\sin(x-\pi/6)}{\sqrt{3}-2\cos x}$

(f) $\lim_{x \to \pi/2} \frac{\cos x}{\sqrt[3]{1-\sin x}^2}$

(g) $\lim_{x \to \pi/6} \frac{2\sin^2 x+\sin x-1}{2\sin^2 x-3\sin x+1}$

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4. Find the limits:

(a) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$ (b) $\lim_{x \to 0} \frac{\tan x-\sin x}{x^3}$

(c) $\lim_{x \to 1} \frac{\cos(\pi x/2)}{1-x}$

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5. Find the limits:

(a) $\lim_{x \to \infty} (1+1/x)^{7x}$, (b) $\lim_{x \to 0} (1+x)^{1/3x}$

(c) $\lim_{x \to \infty} \left(\frac{x}{1+x}\right)^x$, (d) $\lim_{x \to \infty} (1+k/x)^{mx}$

(e) $\lim_{x \to 0} \frac{\ln(1+x)}{3^x-1}$ (f) $\lim_{x \to 0} \frac{e^{4x}-1}{\tan x}$

(g) $\lim_{x \to 0} \frac{\ln(a+x) - \ln a}{x}$ (h) $\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x}$

(i) $\lim_{x \to e} \frac{\ln x - 1}{x - e}$

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6. Find $\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$

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7. Find the limits :

$\lim_{x \to \infty} \left(\frac{1+x}{2+x}\right)^{(1-\sqrt{x})/(1-x)}$

(b) $\lim_{x \to \infty} \left(\frac{x^2 + 2x - 1}{2x^2 - 3x - 2}\right)^{(2x+1)/(x-1)}$

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8. Find the limits :

(a) $\lim_{x \to \infty} \left(\frac{2x^2 + 3}{2x^2 + 5}\right)^{8x^2+3}$

(b) $\lim_{x \to 0} \left(\frac{1 + \tan x}{1 + \tan x} \right)^{1/\sin x}$

(c) $\lim_{x \to 1} (1 + \sin \pi x)^{\cot \pi x}$

(d) $\lim_{x \to a} \left(\frac{\sin x}{\sin a} \right)^{1/(x-a)} \quad (a \neq k\pi, \quad \text{with k integer})$

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9. Find the limits

(a) $\lim_{x \to 0} \frac{\cos x + 4 \tan x}{2 - x - 2x^4}$ (b) $\lim_{x \to -2} \frac{2x^2 + 5x - 7}{3x^2 - x - 2}$

(c) $\lim_{x \to 1} \frac{\sqrt{5 - x} - 2}{\sqrt{2 - x} - 1}$ (d) $\lim_{x \to \infty} \frac{2x^2 - 5x + 4}{5x^2 - 2x - 3}$

(e) $\lim_{x \to \infty} \left(\sqrt{x^2 + 1} - \sqrt{x^2 - 1} \right)$, (f) $\lim_{x \to a} \frac{1 - 2x}{\sqrt[3]{1 + 8x^3} + 2^{-x}}$

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10. Find the limits

(a) $\lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{\sin 5x}$ (b) $\lim_{x \to 1} \frac{\sin(1 - x)}{\sqrt{x} - 1}$

(c) $\lim_{\alpha \to \pi} \frac{\sin \alpha}{1 - \alpha^2/\pi^2}$, (d) $\lim_{x \to \pi/4} \tan 2x \tan(\pi/4 - x)$,

(e) $\lim_{x \to \pi/3} \frac{\tan^3 x - 3 \tan x}{\cos(x + \pi/6)}$

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11. Find the limits

(a) $\lim_{x \to \infty} \left(1+\frac{4}{x}\right)^{x+3}$ (b) $\lim_{x \to \infty} \frac{e^{-x}-1}{x}$,

(c) $\lim_{x \to 0} \frac{a^{2x}-1}{x}$ (d) $\lim_{x \to 0} \left(1+3\tan^2 x\right)^{\cot 2}x$

(e) $\lim_{x \to \pi/4} (\sin 2x)^{\tan 2x}$ (h) $\lim_{x \to \pi/2} \left(\frac{2x-1}{2x+1}\right)^{x}$

(g) $\lim_{x \to \pi/2} (\tan x)^{\tan 2x}$ (h) $\lim_{x \to \pi/2} (\sin x)^{\tan x}$

(i) $\lim_{x \to \infty} \left(\frac{3x^2+2x+1}{x^2+x+2}\right)^{(6x+1)(3x+2)}$ (j)

$\lim_{x \to \infty} \left(\frac{1+3x}{2+3x}\right)^{(1-\sqrt{x})/(1-x)}$

(k) $\lim_{x \to 0} \frac{e^{ax}-e^{\beta x}}{x}$

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12. Find the limits:

(a) $\lim_{x \to 0} \frac{arc\cos(1-x)}{\sqrt{x}}$ (b) $\lim_{x \to \pi/4} \frac{\ln\tan x}{1-\cot x}$

(c) $\lim_{x \to 0} \frac{1}{\sin x}\ln(1+a\sin x)$

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Infinitesimal And Infinite Functions

1. Determine the order of smallness of the quantity β with respect of the infinitesimal α.

(a) $\beta = \cos\alpha - \cos 2\alpha$, (b) $\beta = \tan\alpha - \sin\alpha$

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2. Assuming $x \to \infty$, compare the following infinitely large quantities.

(a) $f(x) = 3x^2 + 2x + 5 \text{ and } \omega(x) = 2x^3 + 2x - 1$

(b) $f(x) = 2x^2 + 3x \text{ and } \omega(x) = (x+2)^2$,

(c) $f(x) = \sqrt[3]{x+a} \text{ and } \omega(x) = \sqrt[3]{x}$

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3. Prove that the infinitesimals $\alpha = x$ and $\beta = x\cos(1/x)(asx \to 0)$ are not comparable, ie, their ratio has no limit.

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Equivalent Infinitesimals

1. For $x \to 0$ determine the order of smallness, relative to the infinitesimal $\beta(x) = x$, of the following infinitesimals

(a) $\sqrt{\sin^2 x + x^4}$ (b) $\frac{x^2(1+x)}{1+\sqrt[3]{x}}$

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2. For $x \to 2$ determine the order of smallness, relative to the infinitesimal $\beta(x) = x - 2$, of the following infinitesimals

(a) $3(x-2)^2 + 2\left(x^2 - 4\right)$, (b) $\sqrt[3]{\sin \pi x}$

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3. Making use of the method of replacing an infinitesimal with an equivalent one, find the following limits:

$\lim_{x \to 0} \frac{\sin 3x}{\ln(1+5x)}$ (b) $\lim_{x \to 0} \frac{\ln(1+\sin 4x)}{e^{\sin 5x}-1}$

(c) $\frac{e^{\sin 3x}-1}{\ln(1+\tan 2x)}$ $x \to 0$ (d) $\lim_{x \to 0} \frac{arc\tan 3x}{arc\sin 2x}$

(e) $\lim_{x \to 0} \frac{\ln(2-\cos 2x)}{\ln^2(\sin 3x+1)}$ (f) $\lim_{x \to 0} \frac{\sqrt{1+\sin 3x}-1}{\ln(1+\tan 2x)}$

(g) $\lim_{x \to 0} \frac{\ln(1+2x-3x^2+4x^3)}{\ln(1-x+2x^2-7x^3)}$ (h) $\lim_{x \to 0} \frac{\sqrt{1+x^2}-1}{1-\cos x}$

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4. Find an approximate value of the root $\sqrt[3]{1042}$.

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One Sided Limits

1. Find the one-sided limits of the functions :

$$f(x) = \begin{cases} -2x+3 & \text{if } x \le 1 \\ 3x-5 & \text{if } x > 1 \end{cases} as x \to 1:$$

(b) $f(x) = \dfrac{x^2-1}{|x-1|} as x \to 1$

(c) $f(x) = \dfrac{\sqrt{1-\cos 2x}}{x} \quad \text{as} \quad x \to 0$

(d) $f(x) = 3 + \dfrac{1}{1+7^{1/(1-x)} as x \to 1}$

(e) $f(x) = \cos\left(\dfrac{\pi}{2}\right) as x \to 0$

(f) $f(x) = 5/(x-2)^3 as x \to 2$

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2. Prove that as $x \to 1$, the functions $f(x) = \begin{cases} x+1 & 0 \le x < 1 \\ 3x+2 & 1 < x \le 3 \end{cases}$

has a limit to the left equal to 2 and a limit to the right equal to 5.

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3. Find the one-sided limits of the following functions as $x \to 0$

(a) $f(x) = \dfrac{1}{2-2^{1/x}}$

(b) $f(x) = e^{1/x}$.

(c) $f(x) = \frac{|\sin x|}{x}$

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Continuity Of A Function

1. Test the following functions for continuity

(a) $f(x) = \begin{cases} \frac{\sin x}{x} & f \text{ or } x \neq 0 \\ 1 & f \text{ or } x = 0 \end{cases}$

(b) $f(x) = \sin(1/x)$:

(c) $f(x) = \begin{cases} x\sin(1/x) & f \text{ or } x \neq 0 \\ 0 & f \text{ or } x = 0 \end{cases}$

(d) $f(x) = \begin{matrix} 4.3^x & f \text{ or } x < 0 \\ 2a + x & f \text{ or } x \geq 0 \end{matrix}$

(e) $f(x) = arc\tan(1/x)$, $(f) f(x) = (x^3 + 1)/(x + 1)$

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2. Redfine the following functions at the point x=0 so as to make them continuous:

(a) $f(x)=\frac{\tan x}{x}$ (b) $f(x)=\frac{5x^2-3x}{2x}$

(f) $f(x)=\frac{\sqrt{1+x}-1}{x}$ (d) $f(x)=\frac{\sin^2 x}{1-\cos x}$

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Arithmetical Operations On Continous Function Continuity Of A Composite Functions

1. Test the following composite functions for continuity

(a) $y=\cos x^n$, where n is a natrual number.

(b) $y=\cos\log x$

(c) $y=\sqrt{1/2-\cos^2 x}$

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2. For each of the following functions find the points of discontinuity and determine their character :

(a) $y=\frac{1}{u^2}+u-2, \quad \text{where u}=\frac{1}{x-1}.$

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The Properties Of A Function Continuous A Closed Interval Continuity Of An Inverse Function

1. Give a function on the interval [-2,+2]

$f(x)=\begin{cases}x^2+2 & \text{if} & -2\leq x<0\\ -x^2+2 & \text{if} & 0\leq x\leq 2\end{cases}$ Is there a point on this closed interval at which f(x)=0?

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2. Prove that the function $f(x)=\begin{cases}x+1 & at & -1\leq x\leq 0\\ -x & at & 0<x\leq 1\end{cases}$ is discontinuous at the point x=0 and still has the maximum and the minimum value on [-1,1].

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Additional Problems

1. Prove the inequalities :

(a) $n! < \left(\frac{n+1}{2}\right)^n$ for a natural $n > 1$:

(b) $\frac{1}{3}\frac{.3}{4}.\frac{5}{6}......\frac{2n-1}{2n} < \frac{1}{\sqrt{2n+1}}$

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2. Prove the inequalities :

(a) $202^{303} > 303^{202}$ (b) $202!Lt100^{2000}$

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3. Solve the inequalities :

(a) $|x| - 2 | \ \leq 1,$ (b) $||2-3x|-1| > 2,$

(c) $(x-2)\sqrt{x^2+1} > x^2+2$

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4. Can a sum difference, product or quotient of irrational numbers be a rational number?

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5. Do the equations

(a) |sin x|=sin x+3, (b) |tan x|=tan x+3 have any roots?

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6. Prove the identity $\left(\frac{x+|x|}{2}\right)^2+\left(\frac{x-|x|}{2}\right)^2=x^2$

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7. Prove that Bernoulli inqeuality $(1+x_1)[1+x_2)\ldots.(1+x_n)\geq 1+x_1+x_2+\ldots\ldots+x_n.$ Where $x_1, x_2, \ldots.x_n$ are numbers of like sign, and $1+x_1>0(i=1,2,\ldots.n)$

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8. Find the domains of defination of the following functions:

(a) $f(x)=\sqrt{x^3-x^2}$,

(b $f(x)=\sqrt{\sin\sqrt{x}}$

(c) $fx\Big)=\sqrt{-\sin^2\pi x}$,

(d) $f(x)=\dfrac{1}{\sqrt{|x|-x|}}$ and $g(x)=\dfrac{1}{\sqrt{x-|x|}}$

(e) $f(x)=arc\sin(|x|x-3)$,

(f) $f(x)=arc\cos\dfrac{1}{\sin x}$

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9. Are the following functions identical?

(a) $f(x)=\dfrac{x}{x}$ and $\phi(x)=1$

(b) $f(x)=x$ and $\phi(x)=(\sqrt{x})^2$,

(c) $f(x)=1$ and $\phi(x)=\sin^2x+\cos^2x$

(d) $f(x)=\log(x-1)+\log(x-2)$ and $\phi(x)=\log(x-1)(x-2)$

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10. In what interval are the following functions identical?

(a) $f(x) = x$ and $\phi(x) = 10^{\log x}$.

(b) $f(x) = \sqrt{x}\sqrt{x-1}$ and $\phi(x) = \sqrt{x(x-1)}$.

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11. An isosceles triangle of a given perimeter 2p=12 revolves about its base. Write the function V(x), where V is the volume of the solid of revolution thus obtained and x is the length of the lateral side of the triangle.

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12. Investigating the domain of definition of functions.

(a) Solve the inequality $\sqrt{x+2} + \sqrt{x-5} \geq \sqrt{5-x}$

(b) Prove that the equality

$\log_{2-x}(x-3) \geq -5$ has no solution.

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13. Prove that the product of two even or two odd functions is an even function, whereas the product of an even and an odd function is an odd function.

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14. Prove that if the domain of definition of the function f(x) is symmetrical with respect to x=0, then f(x) + f(-x) is an even function and f(x)-f(-x) is an odd one.

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15. Prove that any function f(x) defined in a symmetrical interval (-l, l) can be presented as a sum of an even and an odd function. Rewrite the following functions in the form of a sum of an even and an odd function :

$(a) f(x) = \frac{x+2}{1+x^2}$ $(b) y = a^x.$

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16. Extend the function $f(x) = x^2 + x$ defined on the interval (0,3) onto the interval(-3, 3) in an even and an odd way.

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17. Prove theat the Drichlet function $\lambda(x)$ is a periodic one but has not period.

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18. Prove that if the function $f(x) = \sin x + \cos ax$ is a periodic, than a is a rational number.

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19. Prove that the sum of two functions increasing on a certain open interval is a function monotonically increasing on this interval. Will the difference of increasing functions be a monotomic function?s

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20. Give an examole of non-monotonic functions that has an inverse.

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21. Determine the inverse function and its domain of definition, if (a)

$$y = \tan hx, \qquad (b) y = \begin{cases} x & \text{if } -\infty < x < 1 \\ x^2 & \text{if } 1 \le x \le 4 \\ 2^x & \text{if } 4 < x < \infty \end{cases}.$$

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22. Show that the equation $x^2 + 2x + 1 + \sqrt{x}$ has no real roots.

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23. Prove that if the graph of the function y=f(x), defined throughout the number scale, is symmetrical about two vertical axes x =a and $x = b(a < b)$, then this function is a periodic one.

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24. Let the sequence x_n converge and the sequence y_n diverge. What can be said about convergence of the sequences

(a) $x_n + y_n$, (b) $x_n y_n$?

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25. Let the sequences x_n and y_n diverge. Can one assert that the sequences $x_n + y_n$, $x_n y_n$ diverge too?

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26. Let α_n be an interior angle of a regular n-gon (n=3,4,...). Write the first several terms of the sequence α_n. Prove that lime $a_n = \pi$.

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27. Prove that from $\underset{n \to \infty}{x} {}_n = a$ a it follows that $\lim_{n \to \infty} |x_n| = |a|$. Is the converse true ?

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28. Prove that if the sequence $\{a_n / b_n\}\{b_n > 0\}$ is monotonic, then the sequence. $\left\{\frac{a_1 + a_2 + \dots . + a_n}{b_1 + b_2 + \dots . + b_n}\right\}$ will also be monotonic.

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29. Prove the existence of limits of the following sequences and find them.

(a) $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}......,$

(b) $x_n = c^n / \sqrt[k]{n!}(c > 0, k > 0,$

(c) $x_n = \frac{\alpha_n}{n}$, where α_n is the nth of the number π.

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30. Prove that if the function f(x): (1) is defined and monotonic on the interval [a, b], (2) traverses all intermediate values between fla) and fb). then it is continuous on the interval (a, b).

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31. Let the function y =f(x)be continuous on the interval [a, b], its range being the same interval $a \le y \le b$. Prove that on this closed interval

there exists at least one point x such that f(x) =x. Explain this geometrically.

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32. Prove that if the function f(x) is continuous on the interval (a, b) and $x_1, x_2, \ldots, x_n,$ are any values from this open interval, then we can find among them a number ε such that $f(\varepsilon) = \frac{1}{n}[f(x_1) + f(x_2 + \ldots . + f(x_n)]$

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33. Prove that the equation $x2^x = 1$ has at least one positive root which is less than unity.

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34. Prove that the inverse of the discontinuous function $y = (1 + x^2)$ sing x is a continuous function.

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