MATHS

BOOKS - DISHA PUBLICATION MATHS (HINGLISH)

PRINCIPLE OF MATHEMATICAL INDUCTION

Jee Main 5 Years At A Glance

1. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true ? (A) S(1) is correct (B) S(k)=S(k+1) (C) $S(k) \neq S(k+1)$ (D) Principal of mathematical induction can be used to prove the formula

A. Principle of mathematical induction can be used to prove the formula

B. S(K) $\Rightarrow$ S(K+1)

C. S(K) $\not\Rightarrow$ S(K+1)

D. S(1) is correct

Answer: B

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2. If $a_n = \sqrt{7 + \sqrt{7 + \sqrt{7} + \ldots\ldots}}$ having n radical signs then by methods of mathematical induction which is true

A. $a_n > 7 \forall n \geq 1$

B. $a_n < 7 \forall n \geq 1$

C. $a_n < 4 \forall n \geq 1$

D. $a_n < 3 \forall n \geq 1$

Answer: B

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Exercise 1 Concept Builder

1. Let T(k) be the statement 1+3+5+...+(2k-1)=$k^2 + 10$ Which of the following is correct ?

A. T(1) is true

B. T(k) is true $\Rightarrow$ T (k+1) is true

C. T(n) is true for all $n \in N$

D. All above are correct

Answer: B

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2. A student was asked to prove a statement by induction. He proved (i) P(5) is true and (ii) truth of P(n) => truth of P(n+1), n $\in$ N . On the basis of this, he could conclude that P(n) is true

A. for all $n \in N$

B. for all $n > 5$

C. for all $n \geq 5$

D. for all $n < 5$

Answer: C

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3. For a positive integer n , let $a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{2^n - 1}$:

Then

A. $a(100) \leq 100$

B. $a(100) > 100$

C. $a(200) \leq 100$

D. $a(200) < 100$

Answer: A

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4. The statement P(n) 1 x 1! + 2 x 2! + 3 x 3! + ...+ n x n! = (n+1)! -1 is True for all n $>$ 1

A. True for all n $>$ 1

B. Not true for any n

C. True for all $n \in N$

D. None of these

Answer: C

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5. Let $P(n): 2^n < (1 \times 2 \times 3 \times \quad \times n)$. Then the smallest positive integer for which $P\ (n)$ is true is 1 b. 2 c. 3 d. 4

A. 1

B. 2

C. 3

D. 4

Answer: D

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6. Let $P(n): n^2 + n + 1$ is an even integer. If $P(k)$ is assumed $true \Rightarrow P(k+1)$ is true. Therefore P(n) is true:

A. for $n > 1$

B. for all $n \in N$

C. for $n > 2$

D. None of these

Answer: D

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7. Principle of mathematical induction is used

A. to prove any statement

B. to prove results which are true for all real numbers

C. to prove that statements which are formulated in terms of n, where n is positive integer

D. None of these

Answer: C

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8. For each $n \in N$, the correct statement is

A. $2^n < n$

B. $n^2 < 2n$

C. $n^4 < 10^n$

D. $2^{3n} > 7n + 1$

Answer: C

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9. If n is a natural number then $\left(\frac{n+1}{2}\right)^n \geq n!$ is true when

A. $n > 1$

B. $n \geq 1$

C. $n > 2$

D. $n \geq 2$

Answer: B

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10. For natural number n , $2^{n}(n-1)! < n^{n}$, if

A. $n < 2$

B. $n > 2$

C. $n \geq 2$

D. $n > 3$

Answer: B

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11. If $P(n): 2n < n!, n \in N$ then P(n) is true for all $\leq$ ....

A. all n

B. all n $>$ 2

C. all $n > 3$

D. None of these

Answer: C

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12. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then P(n) is true for

A. $n \geq 1$

B. $n > 0$

C. $n < 0$

D. $n \geq 2$

Answer: D

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13. For every positive integral value of $n, 3^n > n^3$ when

A. $n > 2$

B. $n \geq 3$

C. $n \geq 4$

D. $n < 4$

Answer: C

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14. If $x > -1$, then the statement $(1+x)^n > 1 + nx$ is true for

A. all $n \in N$

B. all n $>$ 2

C. all $n > 1$ provided $x \neq 0$

D. None of these

Answer: C

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15. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by

A. 2

B. 4

C. 8

D. 12

Answer: A

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16. For every natural number n, $n(n^2-1)$ is divisible by

A. 4

B. 6

C. 10

D. None of these

Answer: B

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17. Prove the following by the principle of mathematical induction:

$2.\ 7^n + 3.\ 5^n - 5$ is divisible 25 for all $n \in N$.

A. $24, \ \forall n \in N$

B. $21, \ \forall n \in N$

C. $35, \ \forall n \in N$

D. $50, \ \forall n \in N$

Answer: A

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18. The remainder when 5^{4n} is divided by 13, is

A. 1

B. 8

C. 9

D. 10

Answer: A

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19. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 17

Answer: C

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20. If $P(n)$ is the statement $n^3 + n$ is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

A. Natural number greater than 1

B. Irrational number

C. Complex number

D. Odd number

Answer: A

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21. For all $n \in N, 3.5^{2n+1} + 2^{3n+1}$ is divisble by-

A. 19

B. 17

C. 23

D. 25

Answer: B

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22. Prove the following by using the principle of mathematical induction for all $n \in N$:$10^{2n-1}+1$is divisible by 11.

A. 11

B. 12

C. 13

D. 9

Answer: A

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23. The greatest positive integer , which divides (n+1)(n+2)(n+3)...(n+r) for all $n \in W$, is

A. r

B. r!

C. n+r

D. (r+1)!

Answer: B

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24. Prove the following by using the Principle of mathematical induction $\forall n \in N$

3^{2n} when divided by 8 leaves the remainder 1.

A. 2

B. 3

C. 7

D. 1

Answer: D

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25. For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

A. 7

B. 3

C. 4

D. 5

Answer: C

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Exercise 2 Concept Applicator

1. For all $n \geq 1$, prove that

$$\frac{1}{1.\,2} + \frac{1}{2.\,3} + \frac{1}{3.\,4} + \vdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

A. $\dfrac{n}{n+1}$

B. $\dfrac{1}{n+1}$

C. $\dfrac{1}{n(n+1)}$

D. None of these

Answer: A

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2. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)1+\overset{\vdots}{\frac{(2n+1)}{n^2}}=(n+1)^2$$

A. $(n+1)^2$

B. $(n-1)^2$

C. $n(n+1)$

D. None of these

Answer: A

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3. If $n \in N$, then the number $\left(2+\sqrt{3}\right)^n + \left(2-\sqrt{3}\right)^n$ is

A. an integer for all values of n

B. an integer if n is even

C. an integer if n is odd

D. always an irrational number

Answer: A

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4. Prove the following by using the principle of mathematical induction for all $n \in N$:

$$1.3 + 2.3^2 + 3.3^3 + \vdots + n.3^n = \frac{(2n-1)3^{n+1}+3}{4}$$

A. $\dfrac{(2n+1)3^{n+1}+3}{4}$

B. $\dfrac{(2n-1)3^{n+1}+3}{4}$

C. $\dfrac{(2n+1)3^{n}+3}{4}$

D. $\dfrac{(2n-1)3^{n}+1}{4}$

Answer: B

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5. Show by the Principle of Mathematical induction that the sum S_n, of the nterms of the series

$1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + 7^2 + \ldots.$ is given by

$$S_n = \begin{cases} \frac{n(n+1)^2}{2}, \text{if n is even , then } \frac{n^2(n+1)}{2} \text{, if n is odd} \end{cases}$$

A. $S_n = \frac{n(n+1)^2}{2}$, If n is even

B. $S_n = \frac{n^2(n+1)}{2}$, if n is odd

C. Both (a) and (b) are true

D. Both (a) and (b) are false

Answer: C

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6. If $P(n0: 49^n + 16^n + \lambda$ is divisible by 64 for nN is true, then the least negative integral value of λ is -3 b. -2 c. -1 d. -4

A. -1

B. 1

C. 2

D. -2

Answer: A

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7. Prove the rule of exponents $(ab)^n = a^n b^n$ by using principle of mathematical induction for every natural number.

A. $a^n b^n$

B. $a^n b$

C. ab^n

D. 1

Answer: A

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8. $11^{n+2} + 12^{2n+1}$ is divisible by 133.

A. 113

B. 123

C. 133

D. None of these

Answer: C

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9. Prove the following by using the principle of mathematical induction for all $n \in N$:$10^{2n-1} + 1$is divisible by 11.

A. 11

B. 12

C. 13

D. 9

Answer: A

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10. $41^n - 14^n$ is a multiple of 27

A. 26

B. 27

C. 25

D. None of these

Answer: B

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11. Using the principle of mathematical induction prove that $\frac{1}{1.\ 2.\ 3}+\frac{1}{2.\ 3.\ 4}+\frac{1}{3.\ 4.\ 5}+\ +\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$ for all $n \in N$

A. $\frac{n(n+1)}{4(n+2)(n+3)}$

B. $\frac{n(n+3)}{4(n+1)(n+2)}$

C. $\frac{n(n+2)}{4(n+1)(n+3)}$

D. None of these

Answer: B

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12. $x^{2n-1}+y^{2n-1}$ is divisible by $x+y$

A. x+y

B. x-y

C. $x^2 + y^2$

D. $x^2 + xy$

Answer: A

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13. When 2^{301} is divided by 5, the least positive remainder is

A. 4

B. 8

C. 2

D. 6

Answer: C

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14. If n is a positive integer, then $2.4^{2n+1} + 3^{3n+1}$ is divisible by :

A. 2

B. 7

C. 11

D. 27

Answer: C

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15. $5^{2n+2} - 24n + 25$ is divisible by 576

A. 574

B. 575

C. 674

D. 576

Answer: D

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16. Prove the following by the principle of mathematical induction: $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$ for all $n \in N$.

A. x

B. x+1

C. $x^2 + x + 1$

D. $x^2 - x + 1$

Answer: C

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17. If $P(n0: 49^n + 16^n + \lambda$ is divisible by 64 for nN is true, then the least negative integral value of λ is -3 b. -2 c. -1 d. -4

A. -2

B. -1

C. -3

D. -4

Answer: B

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18. If n is any odd number greater than 1, then $n\left(n^2-1\right)$ is divisible by 24 always (b) divisible by 48 always (c) divisible by 96 always (d) None of these

A. 24

B. 16

C. 32

D. 8

Answer: A

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19.

$$\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}++\frac{1}{2n-1}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}++\frac{1}{2n-1}$$

A. all $n \in N$

B. for even values of n

C. for odd values of n

D. not true for any n

Answer: A

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20. Show using mathematical induciton that $n! < \left(\frac{n+1}{2}\right)^n$. Where $n \in N$ and $n > 1$.

A. $n! > \left(\frac{n+1}{2}\right)^n$

B. $n! \geq \left(\frac{n+1}{2}\right)^n$

C. $n! < \left(\frac{n+1}{2}\right)^n$

D. None of these

Answer: C

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