# MATHS

# BOOKS - IA MARON MATHS (HINGLISH)

# THE DEFINITE INTEGRAL

## 6 1 Statement Of The Problem The Lower And Upper Integral Sums

**1.** For the integral

$$\int_0^{\pi} \sin x dx$$

find the upper and lower integral sums corresponding to the division of the closed interval $[0, \pi]$ into 3 and 6 equal subintervals.

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**2.** At what $\delta > 0$ does the relation

$$\left|\int_0 \pi \sin x dx - \sum_{i=0}^{n-1} \sin \zeta_k \Delta x_k\right| < 0.001$$

follow from the inequality max $\Delta x_i < \delta$

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**3.** Show that the Dirichlet function [see Problem 1 . 14 . 4 (b)] is not integrable in the interval [0,1].

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**4.** Find the distance covered by a body in a free fall within the time interval t = a sec to t = b sec

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**5.** Proceeding from the definition, compute the integral $\int_0^1 x dx$

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**6.** compute the integral :

$$\int_a^b x^m dx (m \neq -1, 0 < a < b)$$

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**7.** compute the integral :

$$\int_1^2 \frac{dx}{x}$$

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**8.** Evaluate the integral

$$I = \int_0^5 \sqrt{25 - x^2} dx$$

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**9.** Evaluate the integral :

$$I=\int_1^5 (4x-1)dx$$

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**10.** Prove that

$$I=\int_0^x \sqrt{a^2-x^2}dx$$

$$=\frac{1}{2}x\sqrt{a^2-x^2}+\frac{a^2}{2}\text{arc}\sin\ \frac{x}{a}(0<x\le a)$$

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**11.** show that

(a) $\int_0^{2\pi}\sin^3 x dx=0$ ,

(b) $\int_{-1}^1 e^{-x^2}dx=2\int_0^1 e^{-x^2}dx$

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**12.** Given the function $f(x) = x^3$ on the interval [-2,3] , find the lower $(s_n)$ and the upper $(S_n)$ integral sums for the given interval by subdividing it into n equal parts.

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## 6 2 Evaluating Definite Integrals By The Newton Leibnitz Formula

**1.** Evaluate: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

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**2.** Compute the integrals :

(a) $\int_0^{\pi/2} \sin 2x dx$ ,

(b) $\int_{\pi/6}^{\pi/2} \frac{\cos x}{\sin^3 x} dx$

(c) $\int_0^1 \frac{dx}{\sqrt{16 - x^2}}$

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**3.** Given fuction, $\begin{cases} x^2 & \text{for} 0 \leq x < 1 \\ \sqrt{x} & \text{for} 1 \leq x \leq 2 \end{cases}$

Evaluate $\int_0^2 f(x) dx.$

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**4.** Evaluate the integral

$I = \int_0^2 |1 - x| dx$

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**5.** $\int_0^\pi \frac{\sqrt{1 + \cos 2x}}{2} dx$

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**6.** Evaluate: $\int_0^{100\pi} \sqrt{(1 - \cos 2x)} dx$.

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**7.** Evaluate the integrals : $I = \int_{-2}^{-1} \frac{dx}{(11 + 5x)^3}$,

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**8.** Evaluate the integrals : $I = \int_{-3}^{-2} \frac{dx}{x^2 - 1}$,

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**9.** Evaluate the integrals : $I = \int_{-\pi}^{\pi} \sin^2\left(\frac{x}{2}\right) dx$,

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**10.** Evaluate the integrals : $I = \int_0^{\pi/4} \frac{x^2}{x^2+1} dx,$

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**11.** Evaluate the integrals : $I = \int_e^{e^2} \frac{dx}{xInx}$ ,

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**12.** Evaluate the integrals : $I = \int_{1/\pi}^{2/\pi} \frac{\sin\frac{1}{x}}{x^2} dx,$

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**13.** Evaluate the integrals : $I = \int_0^1 \frac{e^x}{1+e^{2x}} dx$ ,

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**14.** Evaluate the integrals : $I = \int_0^1 \frac{x^3 dx}{1 + x^3}$,

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**15.** Evaluate the integrals : $I = \int_0^3 \frac{x dx}{\sqrt{x+1} + \sqrt{5x+1}}$,

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**16.** The value of the integral $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}$ dx is

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**17.** Evaluate the integrals : $I = \int_1^{\sqrt{3}} \frac{dx}{(1 + x^2)^{\frac{3}{2}}}$

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# 6 3 Estimating An Integral The Definite Integral As A Function Of Its Limits

**1.** Estimate the integral from above

$$I = \int_0^1 \frac{\sin x}{1+x^3} dx$$

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**2.** Proceeding from geometric reasoning, prove that :

(a) if the function f(x) increases and has a concave graph in the interval [a,b], then

$$(b, a) f(a) < \int_a^b f(x) dx < (b-a)\frac{f(a)+f(b)}{2}$$

(b) if the function f(x) increases and has a convex graph in the interval [a,b], then

$$(b-a)\frac{f(a)+f(b)}{2} < \int_a^b f(x) dx < (b-a) f(b)$$

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**3.** Estimate the integral $\int_0^1 \sqrt{1+x^4}$ dx using

(a) the mean-value theorem for a definite integral,

(b) the result of the preceding problem,

(c) the inequality $\sqrt{1+x^4} < 1 + \frac{x^4}{2}$

(d) the Schwarz-Bunyakovasky inequality (see Problem 6.3.6)

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**4.** Find the derivative with respect to x of the following functions :

(a) $F(x) = \int_{x^2}^{x^3} \text{In t dt} \quad (x > 0)$

(b) $f(x) = \int_{1/x}^{\sqrt{x}} \cos(t^2) dt (x > 0)$

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**5.** Find the derivative with respect to x of the following functions :

(a) $F(x) = \int_0^{2x} \frac{\sin t}{t}$ dt ,

(b) $f(x) = \int_x^0 \sqrt{1+t^4}dt$

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**6.** Find the points of extremum of the function $F(x) = \int_0^x \frac{\sin t}{t}$ dt in the domain $x > 0$

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**7.** Find the derivative of y, with respect to x, of the function represented parametrically :

$x = \int_1^{t^3} 3\sqrt{x}$ In dx, y $= \int_{\sqrt{t}}^3 z^2 In$ zdz

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**8.** Find the limits :

(a) $\lim_{x \to 0} \frac{\int_0^{x^3} \sin\sqrt{x}dx}{x^3}$

(b) $\lim_{x \to +\infty} \frac{\int_0^x (\arctan x)^2 dx}{\sqrt{x^2+1}}$

(c) $\lim_{x \to +\infty} \frac{\left(\int_0^x e^{x^2} dx\right)^2}{\int_0^\pi e^{2x^2} dx}$

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**9.** Find the derivative $\frac{dy}{dx}$ of the following implicit functions :

(a) $\int_0^y e^{-t^2} dt + \int_0^{x^3} \sin^2 t dt = 0$

(b) $\int_0^y e^t dt + \int_0^x \sin t dt = 0$

(c) $\int_{\pi/2}^x \sqrt{3 - 2\sin^2 z} dz + \int_0^y \cos t dt = 0$

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**10.** Find : (a) the points of extrumum and the points of inflection on the graph of the function.

$I = \int_0^x (t-1)(t-2)^2 dt$

(b) curvature of the line defined by the parametric equations :

$$\begin{cases} x = a\sqrt{x}\int_0^t \cos\frac{\pi t^2}{2}dt \\ y = a\sqrt{\pi}\int_0^t \sin\frac{\pi t^2}{2}dt \end{cases}$$

(the Cornu spiral)

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**11.** Prove that the function L(x), defined in the interval $(0, \infty)$ by the integral

$$L(x) = \int_1^x \frac{dt}{t}$$

is an inverse of the function $e^x$

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**12.** Given the graph of the function y = f(x) (Fig. 62), find the shape of the graph of the antiderivative $I = \int_0^x f(t)dt$

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**13.** Find the polynomial P(x) of the least degree that has a maximum equal to 6 at x = 1, and a minimum equal to 2 at x = 3

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**14.** Find the polynomial P(x) of the least degree whose graph has three points of inflection : (-1,-1),(1,1) and a point with abscissa 0 at which the curve is inclined to the axis of abscissas at an angle of $60^\circ$ .

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**15.** Find the derivatives of the following functions :

(a) $F(x) = \int_1^x \text{In t dt} \quad (x > 0)$

(b) $F(x) = \int_{2/x}^{x^2} \frac{dt}{t}$

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**16.** Find the derivative $\frac{dy}{dx}$ of functions represented parametrically :

(a) $x = \int_2^1 \frac{Inz}{z} dz, y = \int_5^{Int} e^z dz$

(b) $x = \int_{c^2}^{\sin t} \text{arc} \sin \text{z dz}, \quad y = \int_n^{\sqrt{t}} \frac{\sin z}{z} dz$

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**17.** Find the points of extremum of the following functions :

(a) $F(x) = \int_1^x e^{-\frac{t^2}{2}} (1 - t^2) dt$

(b) $F(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$

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## 6 4 Changing The Variable In A Definite Integral

**1.** Compute in integral $\int_{-\sqrt{3}}^{\sqrt{2}} \sqrt{4 - x^2} dx$

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**2.** The value of the integral $\int_{2}^{4} \frac{\sqrt{x^2-4}}{x^4} dx$ is

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**3.** Compute the integrals :

(a) $\int_0^0 x^2\sqrt{a^2-x^2}dx$

(b) $\int_1^{\sqrt{x}} \frac{dx}{\sqrt{(1+x^2)^3}}$

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**4.** Compute the integrals :

(a) $\int_0^{\pi/2} \frac{\cos x dx}{6-5\sin x+\sin^2 x}$

(b) $\int_0^{\pi/2} \frac{dx}{2+\cos x}$

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**5.** Compute the integral

$$\int_0^{\pi/4} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}(a > 0, b > 0)$$

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**6.** Compute the integrals

(a) $\int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx$

(b) $\int_1^{0^2} \frac{dx}{x\sqrt{1+Inx}}$

(c) $\int_3^2 \frac{3\sqrt{(x-2)^2}}{3+3\sqrt{(x-2)^2}} dx$

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**7.** The value of the integral $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$, is

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**8.** Evaluate the integral

$$I = \int_0^1 \frac{In(1+x)}{1+x} dx$$

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**9.** Prove that for any given integral with finite limits a and b one can always choose the linear substitution x = pt + q (p,q constants) so as to transform this integral into a new one with limits 0 and 1

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**10.** Evaluate $\int_{-4}^{-5} e^{(x+5^2)} dx + 3\int_{1/3}^{2/3} e^{9\left(x-\frac{2}{3}\right)^2} dx$

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**11.** Prove that the integral

$$\int_0^{\pi} \frac{\sin 2kx}{\sin x} dx$$

equals zero if k an integer.

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**12.** Compute the integral

$$\int_{1/2}^{\sqrt{3/2}} \frac{dx}{x\sqrt{1-x^2}}$$

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**13.** Prove that the function L(x) defined on the interval $(0, \infty)$ by the integral $L(x) = \int_1^x \frac{dt}{t}$ possesses the following properties.

$$L(x_1 x_2) = L(x_1) + L(x_2)$$

$$L\left(\frac{x_1}{x_2}\right) = L(x_1) - L(x_2)$$

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**14.** Transform the integral $\int_0^3 (x-2)^2 dx$ by the substitution $(x-2) = t$

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**15.** Compute the integrals

$$I = \int_0^1 \frac{dx}{1+\sqrt{x}}$$

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**16.** Compute the integrals

$$I = \int_0^5 \frac{dx}{2x+\sqrt{3x+1}}$$

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**17.** Compute the integrals

$$I = \int_{\pi/4}^{\pi/3} \frac{dx}{1 - \sin x}$$

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**18.** Compute the integrals

$$I = \int_0^1 \sqrt{2x - x^2} dx$$

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**19.** Compute the integrals

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$$

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**20.** Compute the integrals

$$I = \int_0^a x^2 \sqrt{\frac{a-x}{a+x}} dx, a > 0$$

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**21.** Compute the integrals

$$I = \int_0^{2a} \sqrt{2ax - x^2} dx$$

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**22.** Compute the integrals

$$I = \int_{-1}^{1} \frac{dx}{1+x^2}$$

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**23.** Applying a suitable change of the variable, find the following definite integrals :

(a) $\int_0^2 \frac{dx}{(\sqrt{x}-1)+\sqrt{(x+1)^3}}$

(b) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$

(c) $\int_1^2 \frac{dx}{x(1+x^4)}$

(d) $\int_{\sqrt{(3a^2+b^2)/2}}^{\sqrt{(a^2+b^2)/2}} \frac{xdx}{\sqrt{(x^2-a^2)(b^2-x^2)}}$

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**24.** Consider the integral $\int_{-2}^{2} \frac{dx}{4+x^2}$ . It is easy to conclude that it is equal to $\frac{\pi}{4}$ .Indeed

$$\int_{-2}^{2} \frac{dx}{4+x^2} = \frac{1}{2}\arctan\ \frac{x}{2}\ |_{-2}^{2} = \frac{1}{2}\left[\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$$

On the other hand, making the substitution x $= \frac{1}{t}$ we have

$$dx = -\frac{dt}{t^2}\left|\begin{matrix} x & t \\ -2 & -1/2 \\ 2 & 1/2 \end{matrix}\right|$$

$$\int_{-2}^{2} \frac{dx}{4+x^2} = -\int_{-1/2}^{1/2} \frac{dt}{t^2\left(4+\frac{1}{t^2}\right)} = -\int_{-1/2}^{+1/2} \frac{dt}{4t^2+1}$$

$$= \frac{1}{2} arc\tan 2t \mid_{\frac{1}{2}}^{\frac{1}{2}} = -\frac{\pi}{4}$$

This result is obviously wrong, since the integrand $\frac{1}{4+x^2} > 0$ , and consequently, the definite integral of this function cannot be equal to a negative number $-\frac{\pi}{4}$ . Find the mistake .

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**25.** Consider the integral $I = \int_0^{2\pi} \frac{dx}{5 - 2\cos x}$ . Making the substitution tan $\frac{x}{2}$ = t we have

$$\int_0^{2\pi} \frac{dx}{5 - 2\cos x} = \int_0^0 \frac{2dt}{(1+t^2)\left(5 - 2\frac{1-t^2}{1+t^2}\right)} = 0$$

The result is obviously wrong, since the integrand is positive, and , consequently, the integral of this function cannot be equal to zero . Find the mistake.

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**26.** Make sure that a formal change of the variable $t = x^{\frac{2}{5}}$ leads to the wrong result in the integral $\int_{-2}^{2} 5\sqrt{x^2}$ dx. find the mistake and explain it

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**27.** Is is possible to make the substitution x = sec t in the integral $I = \int_0^1 \sqrt{x^2+1}dx$ ?

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**28.** Given the integral $\int_0^1 \sqrt{1-x^2}$ dx. Made the substitution x = sin t . It is possible to take the number $\pi$ and $\pi/2$ as the limits for t ?

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**29.** Prove the equality

$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(-x)]dx$ for any continuous function f(x)

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**30.** Transform the definite integral $\int_{0}^{2\pi} f(x) \cos x dx$ by the substitution sin x = t

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## 6 5 Simplification Of Integrals Based On The Properties Of Symmetry Of Integrands

**1.** What is the value the integral $\int_{-1}^{1} |x| dx$ ?

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**2.** Compute the integral

$$\int_{-7}^{7} \frac{x^4 \sin x}{x^6 + 2} dx$$

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**3.** Evaluate the integral

(a) $\int_{-\pi}^{\pi} f(x) \cos nx dx$

(b) $\int_{-\pi}^{\pi} f(x) \sin nx dx$

f : (1) f(x) is an even function ,(2) f(x) is an odd function.

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**4.** Calculate the integral $\int_{-5}^{5} \frac{x^5 \sin^2 x}{x^4 + 2x^2 + 1} dx$

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**5.** Compute the integral $\int_{-\pi}^{\pi} \frac{\sin 2x}{\cos^4 x + \sin^4 x}$ dx

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**6.** Prove the equality

$$\int_{-2}^{2} \cos x\, f(x^2)\, dx = 2\int_{0}^{2} \cos x\, f(x^2)\, dx$$

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**7.** Evaluate the following definite integral:

$$\int_{-\sqrt{2}}^{\sqrt{2}} \frac{2x^7 + 3x^6 - 10x^5 - 7x^3 - 12x^2 + x + 1}{x^2 + 2} dx$$

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**8.** The value of the integral

$$\int_{-1/2}^{1/2} \cos x \log\left(\frac{1+x}{1-x}\right) dx, \text{ is}$$

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**9.** Prove that the equality

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

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**10.** Prove that $\int_0^t f(x)g(t-x)dx = \int_0^t g(x)f(t-x)dx$

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**11.** Prove that the equality $\int_0^{\pi/2} \sin^m x dx = \int_0^{\pi/2} \cos^m$ x dx and apply the obtained result in computing the following integrals :

$\int_0^{\pi/2} \cos^2 \text{x dx and } \int_0^{\pi/2} \sin^2 x dx$

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**12.** Prove the equality

$$\int_0^{\pi} f(\sin x) dx = 2\int_0^{\pi/2} f(\sin x) dx$$

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**13.** Show that $\int_0^{\pi} fx(\sin x) dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x) dx \cdot$

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**14.** Using the equality $\frac{\sin\left(n+\frac{1}{2}\right)x}{2\sin\frac{\pi}{2}} = \frac{1}{2} + \cos x + \cos 2x + \ldots\ldots + \cos nx$

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## 6 6 Integration By Parts Reduction Formulas

**1.** Compute the integral $\int_0^1 xe^x dx$

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**2.** Compute the integral $I = \int_0^{\pi/h} e^{ux}$ sin bx dx

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**3.** Compute the integral $\int_1^0 In^3 x dx$

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**4.** Compute the integral $\int_0^{\frac{\pi^2}{4}} \sin\sqrt{x}$ dx

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**5.** Compute the integral $I = \int_0^1 \frac{\text{arc sin x}}{\sqrt{1-x^2}} dx$

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**6.** $\int_0^{\pi/2} x^2 \sin x \quad dx =$

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**7.** Compute the integral $I_n = \int_0^a (a^2 - x^2)^n$ dx , where n is a natural number.

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**8.** Using the result of the preceding problem obtain the following formula :

$$1 - \frac{C_n^1}{3} + \frac{C_n^2}{5} - \frac{C_n^3}{7} + \ldots\ldots + (-1)^n \frac{C_n^n}{2n+1} = \frac{(2n)!!}{(2n+1)!!},$$

where $C_n^k$ are binomial coefficients.

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**9.** Compute the integral

$H_m = \int_0^{\pi/2} \sin^m x dx = \int_0^{\pi/2} \cos^m$x dx (m a natural number)

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**10.** Compute the integral

$I = \int_0^x x \sin^m$ x dx` (m is natural number)

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**11.** Compute the integral $I_n = \int_0^1 x^m (Inx)^n dx, m > 0, n$ is a natural number

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**12.** Compute the integral $I_{m,n} = \int_0^1 x^m (! - x)^n$ dx , where m and n are non-negative integers.

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**13.** Compute the integrals :

$$\int_0^1 \arctan \sqrt{x} dx$$

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**14.** Compute the integrals :

$$\int (x-1)e^{-x} dx$$

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**15.** Compute the integrals :

$$\int_{\pi/4}^{\pi/3} \frac{x dx}{\sin^2 x}$$

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**16.** Compute the integrals :

$$\int_0^1 x arc \tan x dx$$

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**17.** Compute the integrals :

$$\int_0^1 x In\left(1+x^2\right)dx$$

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**18.** $\int_0^{\pi/4} \log(1+\tan x)dx = ?$

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**19.** Compute the integrals :

$$\int_0^{\pi/2} \sin\ \text{In}\ 2\ \text{x}\ \text{arc}\tan \quad (\sin x\, dx)$$

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20. Compute the integrals :

$$\int_1^{15} \text{arc tan} \sqrt{\sqrt{x}-1}\, \text{dx}$$

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21. Prove that

$$\int_0^1 (\text{arc cosx})^n dx = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)\int_0^1 (\text{arc cos x})^{n-1} dx (n > 1)$$

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22. Prove that if f'' is continuous on [a,b] then the following formula is valid

$$\int_a^b xf''(x)dx = [bf'(b) - f(b)] - [af'(a) - f(a)]$$

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# 6 7 Approximating Definite Integrals

**1.** Approximate the integral $I = \int_0^1 \frac{dx}{1+x}$ using the trapezoidal formula at n = 10

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**2.** Evaluate by Simpson's formula the integral $\int_{a.5}^{1.5} \frac{e^{1.1 \, . \, x}}{x}$ dx accurate to four decimal places.

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**3.** The river is 26 m wide. The table below shows the successive depths of the river measured across its section at steps of 2 m

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
| $y$ | 0.3 | 0.9 | 1.7 | 2.1 | 2.8 | 3.4 | 3.3 | 3.0 | 3.5 | 2.9 | 1.7 | 1.2 | 0.8 | 0.6 |

Here x denotes the distance from one bank and y, the corresponding depth (in metres). knowing that the mean rate of flow is 1.3 m/sec, determine the flowrate per second Q of the water in the river.

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**4.** Compute the following integrals :

(a) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{x}$ dx accurate to three decimal places, using Simpson's formula :

(b) $\int_0^1 e^{-x^2} dx$ accurate to three decimal places, by the trapezoidal formula

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**5.** By Simpson's formula, approximate the integral

$$I = \int_{1.05}^{1.36} f(x) \text{ dx}$$

If the integrand is defined by the following table :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 | 1.35 |
| $f(x)$ | 2.36 | 2.50 | 2.74 | 3.04 | 3.46 | 3.98 | 4.6 |

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## 6 8 Additional Problems

**1.** Given the function $f(x)=\begin{cases}1-x & \text{at} \quad 0\leq x\leq 1\\ 0 \quad \text{at} \quad 1<x\leq 2\\ (2-x)^2 \text{at} \quad 2<x\leq 3\end{cases}$

Check directly that the function

$F(x)=\int_0^x f(t)dt$

is continuous on the interval [0,3] and that its derivative at each interior point of this interval exists and is equal to f(x)

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**2.** Show that the function

$$f(x)=\begin{cases}\frac{xInx}{1-x} & \text{at} \quad 0<x<1\\ 0 & \text{at} \qquad x=0\\ -1 & \text{at} \qquad x=1\end{cases}$$

is integrable on the interval [0,1]

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**3.** A line tangent to the graph of the function y = f(x) at the point x = a forms an angle $\frac{\pi}{3}$ with the axis of abscissas and an angle $\frac{\pi}{4}$ at the point x = b

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