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## MATHS

# BOOKS - DISHA PUBLICATION MATHS (HINGLISH) 

## RELATIONS AND FUNCTIONS-2

## Jee Main 5 Years At A Glance

1. Let R be a relation on N defined by $R=\{(x, y): 2 x+y=10\}$, then domain of $R$ is
A. Both $R_{1}$ and $R_{2}$ are transitive relations
B. Both $R_{1}$ and $R_{2}$ are symmetric relations
C. Range of $R_{2}$ is $\{1,2,3,4\}$
D. Range of $R_{1}$ is $\{2,4,8\}$
2. The function $f: R \rightarrow\left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x)=\frac{x}{1+x^{2}}$ is
A. neither injective nor surjective
B. invertible
C. injective but not surjective
D. surjective but not injective

## Answer: D

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3. The function $f: N \rightarrow N$ defined by $f(x)=x-5\left[\frac{x}{5}\right]$ where N is a set of natural numbers, then
A. one-one and onto.
B. one-one but not onto.
C. onto but not one-one
D. neither one-one nor onto.

## Answer: D

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4. let $f(x)=2^{10} x+1$ and $g(x)=3^{10} x+1$. If $f o g(x)=x$, then $x$ is equal to
A. $\frac{3^{10}-1}{3^{10}-2^{10}}$
B. $\frac{2^{10}-1}{2^{10}-3^{10}}$
C. $\frac{1-3^{10}}{2^{10}-3^{10}}$
D. $\frac{1-2^{10}}{3^{10}-2^{10}}$

## Answer: D

## D Watch Video Solution

$f_{0}(x)=\frac{1}{1-x}$ and $f_{n+1}(x)=f_{0}\left(f_{n}(x)\right), n=0,1,2 \ldots$. Then the value of
$f_{100}+f_{1}\left(\frac{2}{3}\right)+f_{2}\left(\frac{3}{2}\right)$ is equal to
A. $\frac{8}{3}$
B. $\frac{4}{3}$
C. $\frac{5}{3}$
D. $\frac{1}{3}$

## Answer: C

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6. Let $A=\left\{x_{1}, x_{2}, x_{3} \ldots, x_{7}\right\}, B=\left\{y_{1} y_{2} y_{3}\right\}$. The total number of functions $f: A \rightarrow B$ that are onto and ther are exactly three elements x in A such that $f(x)=y_{2}$, is equal to
B. $16^{7} C_{3}$
C. $14^{7} C_{2}$
D. $12^{7} C_{2}$

## Answer: A

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7. If g is the inverse of a function f and $f^{\prime}(x)=\frac{1}{1+x^{5}}$, then $\mathrm{g}^{\prime}(\mathrm{x})$ is equal to
A. $\frac{1}{1+\{g(x)\}^{5}}$
B. $1+\{g(x)\}^{5}$
C. $1+x^{5}$
D. $5 x^{4}$

## Answer: B

8. Let $P$ be the relation defined on the set of all real numbers such that $P=\left\{(a, b): \sec ^{2} a-\tan ^{2} b=1\right\}$. Then P is
A. reflexive and symmetric but not transitive.
B. reflexive and transitive but not symmetric.
C. symmetric and transitive but not reflexive
D. an equivalence relation.

## Answer: D

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## Exercise 1 Concept Builder Topicwise Topic 1 Types Of Relations Inverse Of A Relation

1. Prove that a relation $R$ on a set $A$ is symmetric iff $R=R^{-1}$.
A. Reflexive
B. Symmetric
C. Transitive
D. None of these

## Answer: B

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2. If $R$ is an equivalence relation on a set $A$, then $R^{-1}$ is $A$. reflexive only $B$. symmetric but not transitive C. equivalence D. None of these
A. Reflexive only
B. Symmetric but not transitive
C. Equivalence
D. None of these

## Answer: C

3. $R$ is a relation from $\{11,12,13\}$ to $\{8,10,12\}$ defined by $y=x-3$. Then, $R^{-1}$ is (a) $\{(8,11),(10,13)\}$ (b) $\{(11,8),(13,10)\}$ (c) $\{(10,13),(8,11),(8,10)\}$ (d) none of these
A. $\{(11,8),(13,10)\}$
B. $\{(8,11),(10,13)\}$
C. $\{8,11),(9,12),(10,13)\}$
D. None of these

## Answer: B

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4. The relation $R$ is defined on the set of natural numbers as $\{(a, b): a=2 b\}$, the $R^{-1}$ is given by
A. $\{(2,1),(4,2),(6,3) . . .$.
B. $\{(1,2),(2,4),(3,6) \ldots . . .$.
C. $R^{-1}$ is not defined
D. None of these

## Answer: B

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5. Let $R=\left\{(x, y): x^{2}+y^{2}=1, x, y \in R\right\}$ be a relation in $R$. Then the relation $R$ is
A. Reflexive
B. Symmetric
C. Transitive
D. Anti-symmetric

## Answer: B

6. Let $S$ be the set of all real numbers. Then the relation $R=$
$\{(a, b): 1+a b>0\}$ on $S$ is
A. Reflexive and symmetric but not transitive
B. Reflexive and transitive but not symmetric
C. Symmetric, transitive but not reflexive
D. Reflexive, transitive and symmetric

## Answer: A

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7. Let $R$ be a relation on the set $N$ be defined by $\{(x, y) \mid x, y \varepsilon N, 2 x+y=41\}$. Then prove that the $R$ is neither reflexive nor symmetric and nor transitive.
A. Reflexive
B. Symmetric
C. Transitive
D. None of these

## Answer: D

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8. Let $R$ and $S$ be two non-void relations on a set A. Which of the following statements is false?
A. R and S transitive $\Rightarrow R \cup S$ is transitive
B. R and S transitive $\Rightarrow R \cap S$ is transitive
C. R and S symmetric $\Rightarrow R \cup S$ symmetric
D. R and S reflexive $\Rightarrow R \cup S$ reflexive

## Answer: A

9. Determine whether Relation $R$ on the set $Z$ of all integer defined as
$R=\{(x, y): y$ is divisible by $x\}$
A. Reflexive
B. Symmetric
C. Transitive
D. an equivalence relation

## Answer: D

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10. Show that the relation $R$ in the set $A=\{1,2,3,4,5\}$ given by $R=\{(a, b)$ : $|a b|$ is divisible by 2$\}$ is an equivalence relation. Write all the quivalence classes of R.
A. 5
B. 2
C. 3
D. 4

## Answer: A

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11. On the set $N$ of all natural numbers define the rational $R$ by $a R b$ iff the G.C.D. of $a$ and $b$ is 2 . Then $R$ is
A. reflexive, but not symmetric
B. symmetric only
C. reflexive and transitive
D. not reflexive, not symmetric, not transitive

## Answer: B

12. Let $A=\{1,2,3\}$ Then number of relations containing $(1,2) \operatorname{and}(1,3)$ which are reflexive and symmetric but not transitive is (A) 1 (B) 2 (C) 3 (D)

4
A. 1
B. 2
C. 3
D. 4

## Answer: A

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13. Let $R$ be a relation over the set $N \times N$ and it is defined by $(a, b) R(c, d) \Rightarrow a+d=b+c$. Then $R$ is
A. Reflexive only
B. Symmetric only
C. Transitive only
D. An equivalence relation

## Answer: D

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14. Which of one of the following relations on $R$ is equivalence relation
A. $a R_{1} b \Leftrightarrow|a|=|b|$
B. $a R_{2} b \Leftrightarrow a \geq b$
C. $a R_{3} b \Leftrightarrow a$ divides $b$
D. $a R_{4} b \Leftrightarrow a<b$

## Answer: A

Exercise 1 Concept Builder Topicwise Topic 2 Mappings Mapping Of Functions Kinds Of Mapping Of Functions

1. Let $f: R, \vec{R}$ where $f(x)=\frac{x^{2}+4 x+7}{x^{2}+x+1}$. Is $f(x)$ oneone?
A. one-one
B. many-one
C. one-manu
D. None of these

## Answer: B

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2. The set of parameter 'a' for which the functions $f: R \rightarrow R$ defined by $f(x)=a x+\sin x$ is bijective, is
A. $[-2 \infty]$
B. $(-\infty-1] \cup[1, \infty)$
C. $(-\infty,-2) \cup[2 / 3,8)$
D. $[-2,2 / 3]$

## Answer: B

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3. If $A=\{1,3,5,7\}$ and $B=\{1,2,3,4,5,6,7,8\}$, then the number of one-to-one functions from $A$ into $B$ is
A. 1340
B. 1680
C. 1430
D. 1880

## Answer: B

4. Which of the following is one-one function ?
A. $e^{x}$
B. $e^{x^{2}}$
C. $\sin x$
D. None of these

## Answer: A

5. $f: X \rightarrow Y$ is onto, if and only if
6. range of $f=Y$
7. range of $f \neq Y$
8. range of $f<Y$
9. range of $f \geq Y$
A. range of $f=Y$
B. range of $f \neq Y$
C. range of $f<Y$
D. range of $f \geq Y$

## Answer: A

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6. Let $A=\{1,2, \ldots, n\}$ and $B=\{a, b\}$. Then number of subjections from $A$ into $B$ is nP2 (b) $2^{n}-2$ (c) $2^{n}-1$ (d) nC2
A. $a p_{2}$
B. $2^{n}-2$
C. $2^{n}-1$
D. None of these

## Answer: B

7. Let $f: R \rightarrow R$ be function defined by $f(x)=\sin (2 x-3)$, then f is
A. injective
B. surjective
C. bijective
D. None of these

## Answer: D

## Watch Video Solution

8. On the set of integers Z , define $\mathrm{f}: Z \rightarrow Z$ as
$f(n)= \begin{cases}\frac{n}{2}, & \mathrm{n} \text { is even. } \\ 0, & \mathrm{n} \text { is odd. }\end{cases}$
Then, $f$ is
A. injective but not surjective
B. neither injective nor surjectives
C. surjective but not injective
D. bijective

## Answer: C

## - Watch Video Solution

9. If the function $f: R \vec{A}$ given by $f(x)=\frac{x^{2}}{x^{2}+1}$ is surjection, then find A.
A. $[0,1)$
B. $(0,1)$
C. $(0,1]$
D. $[0,1]$

## Answer: A

10. If $f: R \rightarrow R$ be a function such that
$f(x)=\{x|x|-4 ; x \in Q, x|x|-\sqrt{3} ; x \notin Q$ then $\mathrm{f}(\mathrm{x})$ is
A. one to one and onto
B. many to one and onto
C. one to one and into
D. many to one and into

## Answer: D

## - Watch Video Solution

11. Consider functions $f$ and $g$ such that composite gof is defined and is one-one.Are $f$ and $g$ both necessarily one-one.
A. neither fnor $g$ is one-one
B. $f$ and $g$ both are necessarily one-one
C. g must be one-one
D. None of the above

## Answer: D

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12. If $f(x)=|x-2|$, where x is a real number, then, which one of the followin is correct?
A. $f$ is Periodic
B. $f(x+y)=f(x)+f(y)$
C. $f$ is an odd function
D. f is not a one- one function
13. Let $\mathrm{f}: \mathrm{R}-\{\mathrm{n}\} \rightarrow \mathrm{R}$ be a function defined by $f(x)=\frac{x-m}{x-n}$, where $m \neq n$. Then,
A. $f$ is one-one onto
B. $f$ is one-one into
C. $f$ is many-one onto
D. $f$ is many one into

## Answer: B

## - Watch Video Solution

14. The function $f: R \rightarrow R$ is defined by $f(x)=(x-1)(x-2)(x-3)$ is
A. one-one but not onto
B. onto but not one-one
C. both one-one and onto
D. neither one-one nor onto

## Answer: B

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15. 

$A=\{1,2,3)$ and $B=\{a, b, c\}$, and $f=\{(1, a),(2, b),(P, c)\}$ be a function from $A$ to $B$. For the function $f$ to be one-one and onto the value of $P=$
A. 1
B. 2
C. 3
D. 4

## Answer: C

16. A function $f: X \rightarrow Y$ is said to be onto, if for every $y \in Y$ there exists an element x in X such that
A. $f(x)=y$
B. $f(y)=0$
C. $f(x)+y=0$
D. $f(y)+x=0$

## Answer: A

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17. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1,2,3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(X)=1, f(y) \neq 1 f(z) \neq 2$ determine $f^{-1}(1)$
A. $f(x)>f(y)>f(z)$
B. $f(x)<f(y)<f(z)$
C. $f(y)<f(x)<f(z)$
D. $f(y)<f(z)<f(x)$

## Answer: C

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18. The mapping $\mathrm{f}: N \rightarrow N$ given by $f(n)=1+n^{2}, n \in N$, where N is the set of natural numbers, is
A. one-one and onto
B. onto but not one-one
C. one-one but not onto
D. neither one-one nor onto

## Answer: C

19. The function $f: R \rightarrow R$ given by $f(x)=x^{2}+x$ is
A. one-one
B. onto
C. many-one
D. None of the above

## Answer: C

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Exercise 1 Concept Builder Topicwise Topic 3 Composite Function And Relation Inverse Of A Function Binary Operations

1. If $f(x)=\frac{x}{\sqrt{1+x^{2}}}$ then $\operatorname{fofof}(x)$
A. $\frac{3 x}{\sqrt{1+x^{2}}}$
B. $\frac{x}{\sqrt{1+3 x^{2}}}$
C. $\frac{3 x}{\sqrt{1-x^{2}}}$
D. None of these

## Answer: B

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2. If $f(x)=|x|$ and $g(x)=[x]$ then value of fog
$\left(-\frac{1}{4}\right)+g o f\left(-\frac{1}{4}\right)$ is
A. 0
B. 1
C. -1
D. $1 / 4$

## Answer: B

3. The inverse of the function $\frac{10^{x}-10^{-x}}{10^{x}+10^{-x}}$ is
A. $\frac{1}{3} \log _{10} \frac{1+x}{1-x}$
B. $\frac{1}{2} \log _{10} \frac{2+3 x}{2-3 x}$
C. $\frac{1}{3} \log _{10} \frac{2+3 x}{2-3 x}$
D. $\frac{1}{6} \log _{10} \frac{2-3 x}{2+3 x}$

## Answer: B

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4. Let $f:[4, \infty) \rightarrow[4, \infty)$ be defined by $f(x)=5^{x^{(x-4)}}$.Then $f^{-1}(x)$ is
A. $2-\sqrt{4+\log _{5} x}$
B. $2+\sqrt{3+\log _{5} x}$
C. $\left(\frac{1}{5}\right)^{x(x-4)}$
D. None of these

## Answer: B

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5. If the binary operation * on the set of integers $Z$, is defined by $a \cdot b=a+3 b^{2}$, then find the value of $2 \cdot 4$
A. 32
B. 40
C. 36
D. 35

## Answer: D

## D Watch Video Solution

6. If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(S o R)^{-1}=$
A. $S^{-1} o R^{-1}$
B. RoS
C. $R^{-1} o S^{-1}$
D. None of these

## Answer: C

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7. The binary operation * defined on $N$ by $a \cdot b=a+b+a b$ for all $a, b \in N$ is (a) commutative only (b) associative only (c) commutative and associative both (d) none of these
A. commutative only
B. associative only
C. both commutative and associative
D. None of these

## Answer: C

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8. If $f: R \rightarrow R, g: R \rightarrow R$ and $h: R \rightarrow R \quad$ is such that
$f(x)=x^{2}, g(x)=\tan x$ and $h(x)=\log x$ then the value of $[h o(g \circ f)] x$, if $x=\frac{\sqrt{\pi}}{2}$ will be
A. 0
B. 1
C. -1
D. 10

Answer: A
9. Let $f:\left[-\frac{\pi}{3}, \frac{2 \pi}{3}\right] \overrightarrow{0,4}$ be a function defined as $f(x)=\sqrt{3} \sin x-\cos x+2$. Then $f^{-1}(x)$ is given by $\sin ^{-1}\left(\frac{x-2}{2}\right)-\frac{\pi}{6} \sin ^{-1}\left(\frac{x-2}{2}\right)+\frac{\pi}{6} \frac{2 \pi}{3}+\cos ^{-1}\left(\frac{x-2}{2}\right)$
none of these
A. $\sin \left(\frac{x-2}{2}\right)-\frac{\pi}{6}$
B. $\sin \left(\frac{x-2}{2}\right)+\frac{\pi}{6}$
c. $\frac{2 \pi}{3}+\cos ^{-1}\left(\frac{x-2}{2}\right)$
D. None of these

## Answer: B

## - Watch Video Solution

10. If $f(x)=1+x+x^{2}+x^{3}+\ldots . \infty$ for $|x|<1$ then $f^{-1}(x)=$
A. $\frac{x}{1+x}$
B. $\frac{x}{1-x}$
C. $\frac{1-x}{x}$
D. $\frac{1}{x}$

## Answer: B

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11. Let $f$ be $a$ function with domain $X$ and range $Y$. Let $A, B \subseteq X$ and $C, D \subseteq Y$ Which of the following is not true?
A. $f(A \cup B)=f(A) \cup f(B)$
B. $f(A \cap B)=f(A) \cap f(B)$
C. $f^{-1}(C \cup D)=f^{-1}(C) \cup f^{-1}(D)$
D. $f^{-1}(C \cap D)=f^{-1}(C) \cap f^{-1}(D)$

## Answer: B

## D View Text Solution

12. If a binary operation * is defined by $a \cdot b=a^{2}+b^{2}+a b+1$, then $(2 \cdot 3) \cdot 2$ is equal to (a) 20 (b) 40 (c) 400 (d) 445
A. 20
B. 40
C. 400
D. 445

## Answer: D

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13. A binary operation * on the set $\{0,1,2,3,4,5\}$ is defined as: $a \cdot b=\{a+b a+b-6 \backslash \backslash \backslash \backslash$ if $\backslash a+b<6 \backslash \backslash \backslash$ if $a+b \geq 6 \quad$ Show that zero is the identity for this operation and each element a of the set is invertible with 6a, being the inverse of a.
A. 0
B. 1
C. 2
D. 3

## Answer: A

## - Watch Video Solution

14. Let * be a binary operation on $N$ given by $a \cdot b=H C F(a, b), a, b \in N$. Write the value of $22 \cdot 4$.
A. 1
B. 2
C. 3
D. 4

## Answer: B

15. Show that the total number of binary operation from set A to A is $n^{n^{2}}$.
A. $n^{n^{2}}$
B. $n^{n}$
C. $2^{n^{2}}$
D. $n^{2}$

## Answer: A

16. If $f Q \rightarrow Q f(x)=2 x, g, Q \rightarrow Q, g(x)=x+2$ then value of $(f o g)^{-1}(20)$ is
A. 5
B. -8
C. 4
D. 8

Answer: D

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17. If $g(x)=x-2$ is the inverse of the function $f(x)=x+2$, then graph of $g(x)$ is the image of graph of $f(x)$ about the line $y=k x$. Here $k=$
A. 1
B. 2
C. 3
D. 4

## Answer: A

18. Which of the following is not a binary operation on the indicated set?
A. On $Z^{+}, *$ defined by $a * b=a-b$
B. On $Z^{+}, *$ defined by $a * b=a b$
C. On $R, *$ defined by $a * b=a b^{2}$
D. None of above

## Answer: A

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19. 

$f(x)=-1+|x-1|,-1 \leq x \leq 3$ and $g(x)=2-|x+1|,-2 \leq x$ then find $f \circ g(x)$ and $g \circ f(x)$.
A. $\begin{cases}x+1 & -2 \leq x \leq 0 \\ x-1 & 0<x \leq 2\end{cases}$
B. $\begin{cases}x-1 & -2 \leq x \leq 0 \\ x+1 & 0<x \leq 2\end{cases}$
C. $\begin{cases}-x-1 & -2 \leq x \leq 0 \\ x-1 & 0<x \leq 2\end{cases}$
D. None of these

## Answer: D

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20. Let $f(x)=\frac{a x+b}{c x+d}$. Then the fof $(\mathrm{x})=\mathrm{x}$ provided that
A. $d=-a$
B. $d=a$
C. $a=b=c=d=1$
D. $a=b=1$

## Answer: A

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21. Let $A=\{1,2,3,4,5\}$ and functions $f: A \rightarrow$ and $g: A \rightarrow A$ to defined by $f(1)=3, f(2)=5, f(3)=3, f(4)=1, f(5)=2, g(1)=4$
A. $f o g=\{(1,1),(2,3),(3,2),(4,5)\}$
B. $\mathrm{fog}=\{(1,1),(2,3),(3,3),(4,5),(5,3)\}$
C. $g \circ f=\{(1,1),(2,3),(3,3),(4,4),(5,5)\}$
D. gof= $\{(2,2),(2,3),(3,1),(4,1),(5,1)\}$

## Answer: B

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22. Suppose that $f$ is an even function, $g$ is an odd function and both $f$ and $g$ are defined on the entire real line $R$. Which of the following wherever defined are odd function?
A. an even function
B. an odd function
C. neither even nor odd
D. a periodic function

## Answer: A

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23. If $f(x)=e^{x}$ and $g(x)=\log _{e} x$, hen which of the following is true?
A. $f\{g(x)\}=\{f(x)\}$
B. $f\{g(x)\}=g\{f(x)\}$
C. $f\{g(x)\}+g(\{f(x)\}=0$
D. $f\{g(x)\}-g\{f(x)\}=1$

## Answer: B

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24. The inverse of the function $f(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}+2$ is given by
A. $\log _{e}\left(\frac{x-3}{x-1}\right)^{1 / 2}$
B. $\log _{e}\left(\frac{x-1}{3-1}\right)^{1 / 2}$
C. $\log _{e}\left(\frac{x+2}{x-3}\right)^{1 / 2}$
D. $\log _{e}\left(\frac{x+1}{x-2}\right)^{1 / 2}$

## Answer: B

## ( Watch Video Solution

25. Show that if $f: R-\left\{\frac{7}{5}\right\} \rightarrow R-\left\{\frac{3}{5}\right\} \quad$ is defined by $f(x)=\frac{3 x+4}{5 x-7} \quad$ and $\quad g: R-\left\{\frac{3}{5}\right\} \rightarrow R-\left\{\frac{7}{5}\right\} \quad$ is define by $g(x)=\frac{7 x+4}{5 x-3} \quad$, then $\quad f \circ g=I_{A} \quad$ and $\quad g \circ f=I_{B} \quad$, where $A=R-\left\{\frac{3}{5}\right\}, B=R-\left\{\frac{7}{5}\right\} ; I_{A}(x)=x, \forall x \in A, I_{B}(x)=x, \forall x \in 1$ are called ide

$$
\text { A. } f o g=I_{A} \text { and } g o f=I_{A}
$$

B. $f o g=I_{A}$ and $g o f=I_{B}$
C. $f o g=I_{B}$ and $g o f=I_{B}$
D. $f o g=I_{B}$ and $g \circ f I_{A}$

## Answer: B

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26. Let $f: R \rightarrow$ be defined by $f(x)=3 x^{2}-5$ and $g: R \rightarrow R$ by $g(x)=\frac{x}{x^{2}}+1$ then gof is
A. $\frac{3 x^{2}-5}{9 x^{4}-30 x^{2}+26}$
B. $\frac{3 x^{2}-5}{9 x^{4}-6 x^{2}+26}$
C. $\frac{3 x^{2}}{x^{4}+2 x^{2}-4}$
D. $\frac{3 x^{2}}{9 x^{4}+30 x^{2}-2}$

## Answer: A

27. Let $f(x)=\left\{\begin{array}{ll}x^{3}-1, & x<2 \\ x^{2}+3, & x \geq 2\end{array}\right.$ Then
A. $f^{-1}(x)= \begin{cases}(x+1)^{1 / 3}, & x<2 \\ (x-3)^{1 / 2}+, & x \geq 2\end{cases}$
B. $f^{-1}(x)= \begin{cases}(x+1)^{1 / 3}, & x<7 \\ (x-3)^{1 / 2}+, & x \geq 7\end{cases}$
C. $f^{-1}(x)= \begin{cases}(x+1)^{1 / 3}, & x<1 \\ (x-3)^{1 / 2}+, & x \geq 7\end{cases}$
D. $f^{-1}(x)$ does not exist

## Answer: B

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## Exercise 2 Concept Applicator

1. If R be a relation from $A=\{1,2,3,4\}$ to $B=\{1,3,5\}$ i.e., $(a, b) \in R \Leftrightarrow a<b$, then Ro $R^{-1}$ is
B. $\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$
C. $\{(3,3),(3,5),(5,3),(5,5)\}$
D. $\{(3,3),(3,4),(4,5)\}$

## Answer: C

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2. Let $r$ be relation from $R$ (set of real numbers) to $R$ defined by $r=\{(a, b) \mid a, b \in R$ and $a-b+\sqrt{3}$ isan irrational number $\}$. The relation $r$ is
A. an equivalence relation
B. reflexive only
C. symmetric only
D. transitive only

## Answer: B

3. Let $R$ be a relation on the set of all real numbers defined by $x R y \Leftrightarrow|x-y| \leq \frac{1}{2}$ Then R is
A. reflexive and symmetric but not transitive
B. symmetric and transitive but not reflexive
C. transitive but neither reflexive nor symmetric
D. None of these

## Answer: A

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4. Let $f: R \rightarrow R$ be a function defined $\mathrm{by}, f(x)=\frac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$ then
A. $f$ is both one-one and onto
B. $f$ is one-one but not onto
C. $f$ is onto but not one-one
D. $f$ is neither one-one nor onto

## Answer: D

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5. If $X$ and $Y$ are two non-empty sets where $f: X \rightarrow Y$, is function is defined such that $f(c)=\{f(x): x \in C\}$ for $C \subseteq X$ and $f^{-1}(D)=\{x: f(x) \in D\} \quad$ for $\quad D \subseteq Y$,for $\quad$ any $A \subseteq Y$ and $B \subseteq Y$, then
A. $\left.f\left(f^{-1} B\right)\right)=B$
B. $f\left(f^{-1}(B)\right) \subset B$
C. $f^{-1}(f(A))=A$
D. $f^{-1}(f(A)) \subset A$
6. Let $f(x)=\sin z$ and $g(z)=\cos z$. If denotes a composition of functions, then $(f+i g) \cdot(f-i g)$ (where $i=\sqrt{-1})$ is
A. $i e^{-e^{-i z}}$
B. $i e^{-e^{i z}}$
C. $-i e^{-e^{-i z}}$
D. None of these

## Answer: B

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7. If $f: R \rightarrow R, g, R \rightarrow R$ be two funcitons, and $h(x)=2 \min \{f(x)-g(x), 0\}$ then $h(x)=$
A. $f(x)+g(x)-1|g(x)-f(x)|$
B. $f(x)+g(x)+|g(x)-f(x)|$
C. $f(x)-g(x)+|g(x)-f(x)|$
D. $f(x)-g(x)-|g(x)-f(x)|$

## Answer: B

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8. The relation R defined in $A=\{1,2,3\}$ by $a R b$ if $\left|a^{2}-b^{2}\right| \leq 5$. Which of the following is faise
A. $R=\{(1,1),(2,2),(3,3),(2,1),(1,2),(2,3),(3,2)\}$
B. $R^{-1}=R$
C. Domain of $\mathrm{R}=\{1,2,3\}$
D. Range of $R=\{5\}$

## Answer: D

9. Let $R=\left\{(x, y): x, y \in N\right.$ and $\left.x^{2}-4 x y+3 y^{2}=0\right\}$, where $N$ is the set of all natural numbers. Then the relation $R$ is
A. reflexive but neither symmetric nor transitive
B. symmetric and transitive.
C. reflexive and symmetric
D. reflexive and transitive

## Answer: D

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10. If $f(x)=\sin x+\cos x$ and $g(x)=x^{2}-1$, then $g(f(x))$ is invertible in the domain .
A. $\left[0, \frac{\pi}{2}\right]$
B. $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$
C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
D. $[0, \pi]$

## Answer: B

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11. If $f(x)=\frac{x}{x-1}$, then $($ fofof $\odot \ldots . . o f)(x)$ is equal to
A. $\frac{x}{x-1}$
B. $\left(\frac{x}{x-1}\right)^{19}$
C. $\frac{19 x}{x-1}$
D. $x$

Answer: A
12. Statement-1: If $f: R \rightarrow R$ and $g: R \rightarrow R$ be two functions such that $f(x)=x^{2}$ and $g(x)=x^{3}$, then fog $(\mathrm{x})=g \circ f(\mathrm{x})$.

Statement-2: The composition of functions is commulative.
A. $f(x)=x^{3}, g(x)=x+1$
B. $f(x)=\sqrt{x}, g(x)=\cos x$
C. $f(x)=x^{m}, g(x), m \neq n, m, n \in I$ (I is the st of all integers)
D. $f(x)=x-1, g(x)=x^{2}+1$

## Answer: C

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13. Which of the following functions is NOT one-one ?
14. $f: R \rightarrow R$ defined by $f(x)=6 x-1$
15. $f: R \rightarrow R$ defined by $f(x)=x^{2}+7$
16. $f: R \rightarrow R$ defined by $f(x)=x^{3}$
17. $f: R-\{7\} \rightarrow R$ defined by $f(x)=\frac{2 x+1}{x-7}$
A. $f: R \rightarrow R$ defined by $f(x)=6 x-1$
B. $f: R \rightarrow R$ defined by $f(x)=x^{2}+7$
C. $f: R \rightarrow R$ defined by $f(x)=x^{3}$
D. $f: R-\{7\} \rightarrow R$ defined by $f(x)=\frac{2 x+1}{x-7}$

## Answer: B

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14. If $f(x)=2 x+|x|, g(x)=\frac{1}{3}(2 x-|x|)$ and $\mathrm{h}(\mathrm{x})=\mathrm{f}(\mathrm{g}(\mathrm{x}))$, domain of $\underbrace{\sin ^{-1}(h(h(h(h \ldots h(x) \ldots))))}_{\mathrm{n} \text { times }}$ is
A. $[-1,1]$
B. $\left[-1,-\frac{1}{2}\right] \cup\left[\frac{1}{2}, 1\right]$
C. $\left[-1,-\frac{1}{2}\right]$
D. $\left[\frac{1}{2}, 1\right]$

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15. If $f(x)=\sqrt{3|x|-x-2}$ and $g(x)=\sin x$, then domain of $(f o g)(x)$ is
A. $\left\{2 n \pi+\frac{\pi}{2}\right\}, n \in I$
B. $\underset{n \in I}{\cup}\left(2 n \pi+\frac{7 \pi}{7}, 2 n \pi+\frac{11 \pi}{6}\right)$
C. $\left\{2 n \pi+\frac{7 \pi}{6}\right\}, n \in I$
D. $\left\{(14 m+)+\frac{\pi}{2}: m \in I\right\} \underset{n \in I}{\cup}\left[2 n \pi+\frac{7 \pi}{7}, 2 n \pi+\frac{11 \pi}{6}\right]$

## Answer: D

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16. Letf: $A \rightarrow B$ be a function then show that f is a bijection if and only if there exists a function $g: B \rightarrow A$ such that fog $=I_{B} \& g o f=I_{A} \&$ in this case $g=f^{-1}$
A. one-one
B. onto
C. one-one and onto
D. None of these

## Answer: C

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17. Let $A=N \times N$ and $*$ be the binary opertation on A defined by $(a, b) *(c, d)=(a+c, b+d)$. Show that $*$ is commutative and associative.
A. commutative
B. associative
C. Both (a) and (b)
D. None of these

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18. If the binary operation $\odot$ is defined on the set $Q^{+}$of all positive rational numbers by $a \odot b=\frac{a b}{4}$. Then, $3 \odot\left(\frac{1}{5} \odot \frac{1}{2}\right)$ is equal to $\frac{3}{160}$
(b) $\frac{5}{160}$ (c) $\frac{3}{10}$ (d) $\frac{3}{40}$
A. $\frac{3}{160}$
B. $\frac{5}{160}$
C. $\frac{3}{10}$
D. $\frac{3}{40}$

Answer: A

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19. Find the inverse of the function:
$f:(-\infty, 1] \frac{1}{2}, \infty$, where $f(x)=2^{x(x-2)}$
A. $1-\sqrt{1+\log _{2} x}$
B. $\sqrt{\log _{2} x}$
C. $\sqrt{\log _{2} x+1}$
D. $\log _{2} x^{2}$

## Answer: A

20. Let $f: N \rightarrow R$ be the function defined by $f(x)=\frac{2 x-1}{2}$ and $g: Q \rightarrow Q$ be another function defined by $g(x)=x+2$ then $(g \circ f)\left(\frac{3}{2}\right)$ is
A. 1
B. 0
C. $\frac{7}{2}$
D. 3

## Answer: D

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21. Let $f(x)=(x+1)^{2}-1, x \geq-1$. Then the set
$\left\{x: f(x)=f^{-1}(x)\right\}$ is $\left\{0,1, \frac{-3+i \sqrt{3}}{2}, \frac{-3-i \sqrt{3}}{2}\right\}$ (b) $\{0,1,-1$
$\{0,1,1\}$ (d) empty
A. $\left\{0,-1, \frac{-3+I \sqrt{3}}{2}, \frac{-3-i \sqrt{3}}{2}\right\}$
B. $\{0,1-1\}$
C. $\{0,-1\}$
D. empty

## Answer: C

22. Let $f(x)= \begin{cases}2 x+a, & x \geq-1 \\ b x^{2}+3, & x<-1\end{cases}$
and $g(x)= \begin{cases}x+4, & 0 \leq x \leq 4 \\ -3 x-2, & -2<x<0\end{cases}$
If $a=2$ and $b=3$, then the range of $g(f(x))$ is
A. $a=0, b>5$
B. $a=2, b>7$
C. $a=2, b>10$
D. $a=0, b \in R$

## Answer: A

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23. Let $f:(4,6) \cup(6,8)$ be a function defined by $f(x)=x+\left[\frac{x}{2}\right]$ where [.] denotes the greatest integer function, then $f^{-1}(x)$ is equal to
A. $x-\left[\frac{x}{2}\right]$
B. $-x-2$
C. $x-2$
D. $\frac{1}{x+\left[\frac{x}{2}\right]}$

## Answer: C

24. Let [ x$]$ denot the greatest integer $\leq x$. If $f(x)=[x]$ and $g(x)=|x|$ then the value of $f\left(g\left(\frac{8}{5}\right)\right)-g\left(f\left(-\frac{8}{5}\right)\right)$ is
A. 2
B. -2
C. 1
D. -1

Answer: D
25. Let $f(x)=a x+\operatorname{bandg}(x)=c x+d, a \neq 0$. Assume $a=1, b=2$. If $(f \circ g)(x)=(g \circ f)(x)$ for all $x$, what can you say about candd?
A. cand d both arbitary
B. $c=1, d$ arbitrary
C. c arbitrary, $\mathrm{d}=1$
D. $c=1, d=1$

## Answer: B

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26. Let $f(x)=x^{2}+3 x-3, x \geq 0$ if in points $x_{1}, x_{2}, x_{3}, \ldots x_{n}$ are so chosed on the $x$-axis such that
(i) $\frac{1}{n} \sum_{i=1}^{n} f^{-1}\left(x_{i}\right)=f\left(\frac{1}{n} \sum_{i=1}^{n} x_{i}\right)$
(ii) $\sum_{i=1}^{n} f^{-1}\left(x_{i}\right)=\sum_{i=1}^{n} x_{i}$ wehre $f^{-1}$ denots the inverser of $f$. then mean of $x_{1}, x_{2}, x_{3}, \ldots x_{n}$ is:
A. 1
B. 2
C. 3
D. 4

## Answer: A

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27. If $f: R \rightarrow R, g: R$ and $h: R \rightarrow R$ be three functions are given by $f(x)=x^{2}-1, g(x)=\sqrt{x^{2}+1}$ and $h(x)= \begin{cases}0 & x \leq 0 \\ x & x>0\end{cases}$ Then the composite functions (ho fog) ( x ) ) is given by
A. $x^{2}$
B. 0
C. x
D. None of these

## Answer: A

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28. Let $g(x)=1+x-[x]$ and $f(x)=\left\{\begin{array}{ll}-1, & x<0 \\ 0, & x=0 \\ 1, & x>0\end{array}\right.$ then for all $x$,
$f[g(x)]$ is equal to
A. $x$
B. 1
C. $f(x)$
D. $g(x)$

## Answer: C

29. Which of the following functions is the inverse of itself? $f(x)=\frac{1-x}{1+x}$ (b) $f(x)=5^{\log x} f(x)=2^{x(x-1)}$ (d) None of these
A. $f(x)=\frac{1-x}{1+x}$
B. $f(x)=3^{\log x}$
C. $f(x)=3^{x(x+1)}$
D. None of these

## Answer: A

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30. Let $f(x)=\sin \operatorname{xandg}(x)=(\log )_{e}|x|$. If the ranges of the composition functions fogandgofare $R_{1}$ and $R_{2}$, respectively, then ${ }^{\text {}} \mathrm{R}_{-} 1\{$ u :-1lt=u<1\},R_2=\{v :-oo
A. $R_{1}=\{u:-1 \leq u<1\}, R_{2}=\{v:-\infty<v<0\}$
B. $R_{1}=\{u:-\infty<u<0\}, R_{2}=\{v:-\infty<v<0\}$

$$
\begin{aligned}
& \text { C. } R_{1}=\{u:-1<u<1\}, R_{2}=\{v:-\infty<v<0\} \\
& \text { D. } R_{1}=\{u:-1 \leq u \leq 1\}, R_{2}=\{v:-\infty<v \leq 0\}
\end{aligned}
$$

