MATHS

BOOKS - JEE MAINS PREVIOUS YEAR

JEE Mains 2021

MATHEMATICS (SECTION -A)

1. The Boolen expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

A. $q \Rightarrow p$

B. $p \Rightarrow q$

C. $\sim q \Rightarrow p$

D. $p \Rightarrow \sim q$

Answer: B

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2. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx = 10e - 9$ where [x] is the greatest integer less than or equal to X. Then a equal to :

A. $10 - \log_e(1+e)$

B. $10 + \log_e 2$

C. $10 + \log_e 3$

D. $10 + \log_e(1+e)$

Answer: B

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3. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4,5 and 7, then remaining two observations are :

A. 10,11

B. 3,18

C. 8,13

D. 1,20

Answer: A

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4. The value of the integral $\int_{-1}^{1} \log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to :

A. $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$

B. $2\log_e 2+\frac{\pi}{4}-1$

C. $\log_e 2+\frac{\pi}{2}-1$

D. $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$

Answer: C

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5. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1))$ is equal to :

A. 56×3^{25}

B. 56×3^{24}

C. 52×3^{24}

D. 28×3^{25}

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6. Let A = $\begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

A. 36

B. 24

C. 45

D. 18

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7. If Z and ω are two complex numbers such that $|z\omega| = 1$ and $arg(z) - arg(\omega) = \frac{3\pi}{2}$, then arg $\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

A. $\frac{\pi}{4}$

B. $-\frac{3\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

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8. If a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

A. $10 + 6\sqrt{2}$

B. $8 + 2\sqrt{2}$

C. $6 + 8\sqrt{3}$

D. $4 + 2\sqrt{3}$

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9. Let [x] denote the greatest integer $\leq$ x, where $x \in R$. If the domian of the real valued function $f(x) = \sqrt{\frac{|[x]| - 2}{|[x]| - 3}}$ is $(-\infty, a) \cup [b, c), \cup [4, \infty), a < b < c$, then the value of a + b + c is:

A. 8

B. 1

C. -2

D. -3

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10. Let y = y(x) be the solution of the differential equation $\left(x\tan\left(\frac{y}{x}\right)\right)dy=\left(y\tan\left(\frac{y}{x}\right)-x\right)dx-1\leq x\leq 1,$ y $\left(\frac{1}{2}\right)=\frac{\pi}{6}.$

Then the area of the region bounded by the curves x = 0, x = $\frac{1}{\sqrt{2}}$ and $y=y(x)$ in the upper half plane is :

A. $\frac{1}{8}(\pi-1)$

B. $\frac{1}{12}(\pi-3)$

C. $\frac{1}{4}(\pi-2)$

D. $\frac{1}{6}(\pi-1)$

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11. The coefficient of x^{256} in the expansion of $(1-x)^{101}\left(x^2+x+1\right)^{100}$ is :

A. ${}^{100}C_{16}$

B. ${}^{100}C_{15}$

C. $-{}^{100}C_{16}$

D. $-{}^{100}C_{15}$

Answer: B

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12. Let A = $\left[a_{ij}\right]$ be a 3×3 matrix, where
$$a_{ij}=\begin{cases}1 & , \quad \text{if } \ i=j\\ -x & , \quad \text{if } \ |i-j|=1\\ 2x+1 & , \ \text{otherwise}\end{cases}$$

Let a function $f: R \rightarrow R$be defined as f(x) = det (A). Then the sum of maximum and minimum values of f on R is equal to:

A. $-\frac{20}{27}$

B. $\frac{88}{27}$

C. $\frac{20}{27}$

D. $-\frac{88}{27}$

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13. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ equals :

A. $\frac{2}{3}$

B. 4

C. 3

D. $\frac{3}{2}$

Answer: D

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14. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is :

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15. Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}dx+\left(\frac{y}{x}\right)dy=0, y(1)=-1$. Then the value of $(y(3))^2$ is equal to :

A. $1-4e^3$

B. $14e^6$

C. $1+4e^3$

D. $1-4e^6$

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16. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15, x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function g(x) $= ax^2 - 6x + 15, x \in R$ has a :

A. local maximum at x = -$\frac{3}{4}$

B. local minimum at $x = -\frac{3}{4}$

C. loca maximum at $x = \frac{3}{4}$

D. local minimum at $x = \frac{3}{4}$

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17. Let a function $f: R \to R$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if} \quad x \le 0 \\ a + [-x] & \text{if} \quad 0 < x < 1 \\ 2x - b & \text{if} \quad x \ge 1 \end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on R, then (a + b) is equal to:

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18. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the latter M appears at the fourth position in any such word is :

A. $\frac{1}{66}$

B. $\frac{1}{11}$

C. $\frac{1}{9}$

D. $\frac{2}{11}$

Answer: B

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19. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in R$, is :

A. $\frac{7}{36}$

B. $\frac{2}{9}$

C. $\frac{1}{6}$

D. $\frac{1}{4}$

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20. Let the tangent to the parabola $S: y^2 = 2x$ at the point P(2,-2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

A. $\frac{25}{2}$

B. $\frac{35}{2}$

C. $\frac{15}{2}$

D. 25

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21. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

A. $8(2+2\sqrt{3}+\sqrt{2})$

B. $8(\sqrt{6}+\sqrt{2}+2)$

C. $8(\sqrt{2}+2+\sqrt{3})$

D. $8(\sqrt{6}-\sqrt{2}+2)$

Answer: B

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22. Let $f(x)=3\sin^4 x+10\sin^3 x+6\sin^2 x-3$, $x\in\left[-\frac{\pi}{6},\frac{\pi}{2}\right]$.Then , f is :

A. increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

B. decreasing in $\left(0, \frac{\pi}{2}\right)$

C. increasing in $\left(-\frac{\pi}{6}, 0\right)$

D. decreasing in $\left(-\frac{\pi}{6}, 0\right)$

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23. Let S_n be the sum of the first n terms of an arithmetic progression .If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

A. 6

B. 4

C. 2

D. 8

Answer: A

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24. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$ and its foci is :

A. $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

B. $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

C. $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

D. $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Answer: A

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25. Let the vectors $(2 + a + b)\hat{i} + (a + 2b + c)\hat{j} - (b + c)\hat{k}, (1 + b)\hat{i} + 2b\hat{j} - b\hat{k}$ and $(2 + b)$

be co-planar. Then which of the following is true?

A. $2b = a + c$

B. $3c = a + b$

C. $a = b + 2c$

D. $2a = b + c$

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26. Let $: R \rightarrow R$ be defined as $f(x) = \begin{cases} \dfrac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)} & x < 2 \\ e^{\frac{\tan(x-2)}{x - [x]}} & x > 2 \\ \mu & x = 2 \end{cases}$ where

[x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda + \mu$ is equal to :

A. $e(-e + 1)$

B. $e(e - 2)$

C. 1

D. $2e - 1$

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27. The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is :

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{18}$

Answer: C

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28. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+\ldots+\frac{1}{a-nb}=\alpha n+\beta n^2+\lambda n^3$, then the value of λ is :

A. $\dfrac{a^2+b}{3a^3}$

B. $\dfrac{a+b}{3a^2}$

C. $\dfrac{b^2}{3a^3}$

D. $\dfrac{a+b^2}{3a^3}$

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29. Let y = y(x) be the solution of the differential equation $\dfrac{dy}{dx}=1+xe^{y-x}, -\sqrt{2}<x<\sqrt{2}, y(0)=0$ then , the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to :

A. $(2-\sqrt{3})-\log_e 2$

B. $(2+\sqrt{3})+\log_e 2$

C. $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$

D. $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$

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30. $(p \rightarrow q) \wedge (q \rightarrow \sim p)$ is equivalent to

A. $\sim q$

B. q

C. p

D. $\sim p$

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31. The area (in sq. units) of the region, given by the set $\{(x, y) \in R \times R \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

A. $\frac{8}{3}$

B. $\frac{17}{3}$

C. $\frac{13}{3}$

D. $\frac{7}{3}$

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32. $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$. Find sum of roots that lying in $[0, 2\pi]$

A. 8π

B. 11π

C. 12π

D. 9π

Answer: D

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33. Let $g: N \rightarrow N$ be defined as

$g(3n+1) = 3n+2,$

$g(3n+2) = 3n+3,$

$g(3n+3) = 3n+1$ for all $n \geq 0$. The which of the following statements is true ?

A. There exists a function $f: N \rightarrow N$ such that gof=f

B. There exists a one -one function $f: N \rightarrow N$ such that fog=f

C. gogog=g

D. There exists an onto function $f: N \rightarrow N$ such that fog=f

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34. Let $f: [0, \infty) \rightarrow [0, \infty)$ be defined as $f(x) = \int_0^x [y]$ dy where [x] is the greatest integer less than or equal to x. Which of the following is true?

A. f is continuous at every point in $[0, \infty]$ and differentiable except at the integer points.

B. f is both continuous and differentiable except at the integer points in $[0, \infty)$.

C. f is continuous everywhere except at the integer points in $[0, \infty)$.

D. f is differentiable at every point in $[0, \infty)$.

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35. The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution ,are :

A. $a = 3, b \neq 13$

B. $a \neq 3, b \neq 13$

C. $a \neq 3, b = 3$

D. $a = 3, b = 13$

Answer: A

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36. 9 different balls are to be arranged in 4 different boxes number B_1, B_2, B_3 and B_4 . If the probability that B_3 has exactly three balls is $k\left(\frac{3}{4}\right)^9$ then find k

A. $\{x \in R : |x-3| < 1\}$

B. $\{x \in R : |x-2| \leq 1\}$

C. $\{x \in R : |x-1| < 1\}$

D. $\{x \in R : |x-5| \leq 1\}$

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37. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin , respectively .If tangents are drawn from O(0,0) to the parabola P which meet P at S and R , then the area (in sq .units)of ΔSOR is equal to :

A. $16\sqrt{2}$

B. 16

C. 32

D. $8\sqrt{2}$

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38. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is :

A. 2

B. 4

C. 6

D. 1

Answer: A

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39. Let an ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$,passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$ .If a circle , centered at focus $F(\alpha, 0), \alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q ,then PQ^2 is equal to :

A. $\frac{8}{3}$

B. $\frac{4}{3}$

C. $\frac{16}{3}$

D. 3

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40. Let the foot of perpendicular from a point $P(1, 2, -1)$ to the straight line $L: \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ ne N. Let a line be drawn from P parallel to the plane x+y+2z =0 which meets L at point Q . If α is the acute angle between the lines PN and PQ , then $\cos\alpha$ is equal to ______.

A. $\frac{1}{\sqrt{5}}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{2\sqrt{3}}$

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MATHEMATICS (SECTION -B)

1. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. The $36\cos^2 2\theta$ is equal to ________ .

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2. Let A = $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3 if $B = [b_{ij}]$ then b_{13} is equal to ________ .

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3. Let P be a plane passing through the points (1,0,1),(1,-2,1) and (0,1,-2). Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P. perpendicular to $\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{a}.\left(\hat{i} + \hat{j} + 2\hat{k}\right)$. then $(\alpha - \beta + \gamma)^2$ equals ________ .

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4. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is ________ .

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5. If the shortest distance between the lines $\overrightarrow{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}-2\hat{k}\right), \lambda\in R, \alpha>0$ and $\overrightarrow{r_2}=-4\hat{i}-\hat{k}$

is 9, then α is equal to _______ .

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6. Let T be the tangent to the ellipse E : $x^2+4y^2=5$ at the point P(1,1). If the area of the region bounded by the tangent T, ellipse E,lines x = 1 and $x=\sqrt{5}$ is $\alpha\sqrt{5}+\beta+\gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to

_______ .

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7. Let a,b,c,d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2,$$

then value of λ^2 is equal to ______ .

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8. There are 15 playears in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 more wicketkeepers. The number of ways, a team of 11 players be selected from them so as to inculede at least 4 bowlers. 5 batsmen and I wicketkeeper is _____ .

A. 888

B. 555

C. 222

D. 777

Answer: D

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9. Let y = mx + c, m > 0 be the focal chord of $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then the value of $4\sqrt{2}$ (m + c) is equal to ______ .

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10. If the value of $\lim_{x \to 0} \left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a then a is equal to ______ .

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11. Let $y = y(x)$ be solution of the following differential equation

$$e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 = 0, y\left(\frac{\pi}{2}\right) = 0$$

If $y(0) = \log_e\left(\alpha + \beta e^{-2}\right)$, then $4(\alpha + \beta)$ is equal to ____.

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12. If the value of $\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots.\text{ upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots..\text{upto } \infty\right)}$ is l, then l^2 is equal to ______.

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13. Consider the following frequency distribution :

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then |alpha-beta|` is equal to _______.

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14. Let $\vec{p}=2\hat{i}+3\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+2\hat{j}+\hat{k}$ be two vectors .If a vector $\vec{r}=\left(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}\right)$ is prependicular to each of the vectors

$\left(\vec{p}+\vec{q}\right)$ and $\left(\vec{p}-\vec{q}\right)$, and $\left|\vec{r}\right|=\sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to ____.

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15. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is _____ .

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16. Let $M=\left\{A=\begin{pmatrix}a & b\\ c & d\end{pmatrix}: a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}\right.$.

Define $f: M \to Z$, as f (A) =det (A).for all $A \in M$, where Z is set of all integers .Then the number of $A \in M$ such that $f(X)=15$ is equal to _____.

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17. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to ______.

A. 250

B. 238

C. 150

D. 338

Answer: B

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18. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n , then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to ______.

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19. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$,where $x \in 0\,,1$ is equal to______.

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20. Let $S = \left\{n \in N \mid \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in R\right\}$,

where $i = \sqrt{-1}$ .Then the number of 2- digit number in the set S is ____.

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MATHEMATICS

1. For the natural number m, n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots + a_{m+n}y^{m+n}$ and $a_1 = a_2 = 10$,

then the value of (m+n) is equal to :

A. 100

B. 80

C. 83

D. 64

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2. Let y =y (x) satisfies the equation $\frac{dy}{dx} - |A| = 0, \text{ for all} x > 0, \text{ where}$
$$\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$$
If $y(\pi) = \pi + 2,$ then the vlaue of $y\left(\frac{\pi}{2}\right)$ is :

A. $\frac{3\pi}{2} - \frac{1}{\pi}$

B. $\frac{\pi}{2} + \frac{4}{\pi}$

C. $\frac{\pi}{2} - \frac{1}{\pi}$

D. $\frac{\pi}{2} - \frac{4}{\pi}$

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3. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point (–4,1) and having their centres on the circumference of the circle $x^2 + y^2 + 2x + 4y - 4 = 0$. If $\frac{r_1}{r_2} = a + b\sqrt{2}.$ then $a + b$ is equal to .

A. 5

B. 7

C. 3

D. 11

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4. Let $f: R - \left\{\frac{\alpha}{6}\right\} \rightarrow R$ be defined by $f(x) = \frac{3x+3}{6x-\alpha}$. Then the value of α for which (fof) $(x) = x$, for all $x \in R - \left\{\frac{\alpha}{6}\right\}$, is :

A. 6

B. 8

C. 5

D. No such α exists

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5. If $f: R \rightarrow R$is given $f(x) = x + 1$, then the value of $\lim_{b - \infty} \frac{1}{n}\left[f(0) + f\left(\frac{5}{n}\right) + f\left(\frac{10}{n}\right) + \ldots + f\left(\frac{5(n-1)}{n}\right)\right]$, is :

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{7}{2}$

D. $\frac{5}{3}$

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6. Let in a right angled triangle, the smallest angle θ If triangle formed by taking the reciprocal of its sides is also a right angled triangle, then sin θ is equal to :

A. $\frac{\sqrt{5}+1}{4}$

B. $\frac{\sqrt{2}-1}{2}$

C. $\frac{\sqrt{5}-1}{2}$

D. $\frac{\sqrt{5}-1}{4}$

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7. Let $g^{(t)} = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right) dx,$ where $f(x) = lg_e\left(x + \sqrt{x^2 + 1}\right), x \in R$. Then which one of following is correct ?

A. $g(1) = g(0)$

B. $g(1) + g(0 = 0)$

C. $g(1) + \sqrt{2}g(0)$

D. $\sqrt{2}g(1) = g(0)$

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8. If sum of the first 21 terms of series $\log_{\frac{1}{9^2}} x + \log_{\frac{1}{9^3}} x + \log_{\frac{1}{9^4}} x + \dots,$ where $x > 0 is 504,$ then x is equal to :

A. 9

B. 243

C. 7

D. 81

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9. The value of $k \in R$, for which the following system of linear equation

$3x - y + 4z = 3,$

$x + 2y - 3z = -2,$

$6x + 5y + kz = -3,$

Has infinitely many solutions, is:

A. 5

B. 3

C. -5

D. -3

Answer: C

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10. Let A,B and C be three events such that the probability that exactly one of A and B occurs is (1-k), the probability that exactly one of B and C occurs is (1 –2k), the probability that exactly one of C and A occurs is (1 – K) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is:

A. exactly equal to $\frac{1}{2}$

B. greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

C. greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

D. greater than $\frac{1}{2}$

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11. Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid- point of the point P and the foot of the perpendicular drawn from the point P to the line y =x is:

A. $(3x - y)^2 + (3 - 3y) + 2 = 0$

B. $2(3x - y)^2 + (x - 3y) + 2 = 0$

C. $2(x - 3y)^2 + (3x - y) + 2 = 0$

D. $(3x - y)^2 + 2(x - 3y) + 2 = 0$

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12. If the mean and variance of six observations 7,10, 11, 15 a, b are 10 and $\frac{20}{3}$, respectively, then value of $|a - b|$ is equal to :

A. 1

B. 11

C. 1

D. 9

Answer: C

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13. The sum of all the local minimum values of the twice differentiable function $f: R \rightarrow R$ defined by $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ is :

A. 0

B. 5

C. -22

D. -27

Answer: D

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14. If [X] denotes the gratest integer less than or equal to x, then the value of the integeral $\int_{-\pi/2}^{\pi/2} [[x] - \sin x] dx$ is equal to :

A. $-\pi$

B. 0

C. π

D. 1

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15. Consider the following three statements:

(A) If 3 + 3 = 7 then 4 + 3 = 8.

(B) If 5 + 3 = 8 then earth is flat.

(C) If both (A) and (B) are true then 5 + 6 =17.

Then, which of the following statements is correct?

A. (A) is true while (B) and (C) are false

B. (A) and (C) are true while (B) is false

C. (A) and (B) are false while (C) is true

D. (A) is false, but (B) and (C) are true

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16. The lines $x = ay - 1 = z - 2$ and $z = 3y - 2 = bz - 2, (ab \neq 0)$ are coplanar, if

A. $b = 1, a \in R - \{0\}$

B. $a = 2, b = 2$

C. $a = 1, b \in R - \{0\}$

D. $a = 2, b = 3$

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17. Consider the line L given by the equation $\frac{x-3}{2} = \frac{y-1}{1} = \frac{z-2}{1}$.

Let Q be the mirror image of the point (2,3 –1) with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is one the plane P?

A. $(-1, 1, 2)$

B. $(1, 1, 2)$

C. $(1, 1, 1)$

D. $(1, 2, 2)$

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18. If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1} is \frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral $\int_0^\theta$ sin xdx is equal to :

A. 1

B. 0

C. -1

D. 2

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19. The value of $\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to:

A. $\frac{151}{63}$

B. $\frac{220}{21}$

C. $\frac{-181}{69}$

D. $\frac{-291}{76}$

Answer: B

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20. In a triangle ABC, if $\left|\overrightarrow{BC}\right| = 3, \left|\overrightarrow{CA}\right| = 5$ and $\left|\overrightarrow{BA}\right| = 7,$ then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

A. $\frac{19}{2}$

B. $\frac{13}{2}$

C. $\frac{11}{2}$

D. $\frac{15}{2}$

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21. The number of solutions of the equation $\log_{(x+1)}\left(2x^2 + 7x + 5\right) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0,$ is ______.

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22. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right) is(\alpha, \beta)$ then $2(\alpha + \beta)$ is equal to _______.

Watch Video Solution

23. If $\lim_{x \to 0} \frac{\alpha x e^x - \beta \log_e(1 + x) + \gamma x^2 e^{-x}}{x^2 \sin x} = 10, \alpha, \beta, \gamma \in R,$ then the value of $\alpha + \beta + \gamma$ is _________.

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24. For $p > 0$ vector $\overrightarrow{v}_2 = 2\hat{i} + (p + 1)\hat{j}$ is obtained by rotating the vector $\overrightarrow{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \left(\frac{\alpha\sqrt{3} - 2}{4\sqrt{3} + 3}\right)$, then the value of α is equal to

____

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25. Let a curve y = y (x) be given by the solution of differential equation $\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)dx = \sqrt{e^{2x}-1}dy$ If it intersects y – axis at y = –1, and the intersection point point of the curve with x – axis is (α, θ), then e^{α} is equal to ________.

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26. Let a function $g:[0,4] \to R$ be defined as

$$g(x) = \begin{cases} \max\limits_{0 \le t \le x} \{t^3 - 6t^3 + 9t - 3\}, & 0 \le x \le 3 \\ 4 - x & 3 < x \le 4 \end{cases}$$

then the number of points in the interval (0,4) where g(x) is NOT differentiable, is __________.

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27. For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots.(\alpha+20)} = \sum_{k=0}^{20}\frac{A_k}{\alpha+k}$, where $a > 0.$ Then the value of $1000\left(\frac{A_{14}+A_{15}}{A_{13}}\right)$ is equal to ________.

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28. Let $\{a_n\}_{n-1}^{\infty}$ be a sequene such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n-1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to __________.

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29. Let $A = \{a_{ij}\} bea3 \times 3$ matrix, where

$$a_{ij}\begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 \text{ if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then det $\left(3Adj\left(2A^{-1}\right)\right)$ is equal to __________

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30. Consider a triangle having vertices $A(-2, 3), B(1, 9)$ and $C(3, 8)$.

If a line L passing through the circume centre of triangle ABC, bisects line

BC, and intersects y – axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is __________.

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MATHEMATICS (SECTION-A)

1. Let L be the line of intersection of planes $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=2$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)=2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1,2,0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :

A. 101

B. 119

C. 143

D. 134

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2. Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10} = 530, S_5 = 140$ then $S_{20} - S_6$ is equal to :

A. 1862

B. 1842

C. 1852

D. 1872

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3. Let $f: R \rightarrow R$ be defined as

$$f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3x & , \quad x > 0 \\ 3xe^x & , \quad x \leq 0 \end{cases}$$

. Then f is increasing function in the interval

A. $\left(-\frac{1}{2}, 2\right)$

B. (0, 2)

C. $\left(-1, \frac{3}{2}\right)$

D. $(-3, -1)$

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4. Let $y = y(x)$ be the solution of the differential equation $\operatorname{cosec}^2 x dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x dx$, with $y\left(\frac{\pi}{4}\right) = 0$. Then the value of $(y(0) + 1)^2$ is equal to :

A. $e^{1/2}$

B. $e^{-1/2}$

C. e^{-1}

D. e

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5. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2×2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :

A. $\frac{45}{162}$

B. $\frac{23}{81}$

C. $\frac{22}{81}$

D. $\frac{43}{162}$

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6. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $\left|\vec{a}\right| = \sqrt{10}$. Then a possible value of $\left[\vec{a}\ \vec{b}\ \vec{c}\right] + \left[\vec{a}\ \vec{b}\ \vec{d}\right] + \left[\vec{a}\ \vec{c}\ \vec{d}\right]$ is equal to :

A. -42

B. -40

C. -29

D. -38

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7. If $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}, \alpha \in R$ where [x] is the greatest integer less than or equal to x, then the value of α is :

A. $200\left(1 - e^{-1}\right)$

B. $100(1 - e)$

C. $50(e - 1)$

D. $150\left(e^{-1} - 1\right)$

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8. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}$ and $\left|\vec{a}\right| = 2$. Then which one of the following is not true ?

A. $\vec{a} \times \left(\left(\vec{b} + \vec{c}\right) \times \left(\vec{b} \times \vec{c}\right)\right) = \vec{0}$

B. Projection of $\vec{a}$ on $\left(\vec{b} \times \vec{c}\right)$ is 2

C. $\left[\vec{a} \ \vec{b} \ \vec{c}\right] + \left[\vec{c} \ \vec{a} \ \vec{b}\right] = 8$

D. $\left|3\vec{a} + \vec{b} - 2\vec{c}\right|^2 = 51$

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9. The values of λ and μ such that the system of equations $x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$ has no solution, are :

A. $\lambda = 3, \mu = 5$

B. $\lambda = 3, \mu \neq 0$

C. $\lambda \neq\; = 2, \mu = 0$

D. $\lambda = 2, \mu \neq 10$

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10. If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3), \lambda \in R$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

A. 3

B. 2

C. 5

D. -1

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11. Which of the following Boolean expressions is not a tautology ?

A. $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

B. $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

C. $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

D. $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

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12. Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for i = 1, 2, 3. Then, the sum of all entries of the matrix A^3 is equal to :

A. 2

B. 1

C. 3

D. 9

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13. Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$

A. $\left[0, \frac{1}{e}\right)$

B. $[\log_e 2, \log_e 3)$

C. $[1, e)$

D. $[0, \log_e 2)$

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14. Let the circle $S: 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

A. $\frac{25}{9} < C < \frac{13}{3}$

B. $100 < C < 165$

C. $81 < C < 156$

D. $100 < C < 156$

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15. If 'n' is the number of solution of $z^2 + 3\bar{z} = 0$ where $z \in C$ then find

$$\sum_{k=0}^{\infty} \frac{1}{n^k}$$

A. 1

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. 2

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16. The number of solutions of $\sin^7 x + \cos^7 x = 1, x \in [0, 4\pi]$ is equal to

A. 11

B. 7

C. 5

D. 9

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17. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

A. $\frac{3}{2}$

B. 2

C. $\frac{1}{2}$

D. 1

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18. Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , \quad x \neq 0 \\ \alpha & , \quad x = 0 \end{cases}.$$

If f is continuous at x = 0, then α is equal to :

A. 1

B. 3

C. 0

D. 2

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19. Let a line $L: 2x + y = k, k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to

A. 12

B. -12

C. 24

D. -24

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20. If $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. E_2 is an ellipse which touches E_1 at the ends of major axis of E_1 and end of major axis of E_1 are the focii of E_2 and the eccentricity of both the ellipse are equal then find e

A. $\dfrac{-1+\sqrt{5}}{2}$

B. $\dfrac{-1+\sqrt{8}}{2}$

C. $\dfrac{-1+\sqrt{3}}{2}$

D. $\dfrac{-1+\sqrt{6}}{2}$

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21. The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3}+3^{1/4}\right)^{12}$ is :

A. 89

B. 27

C. 35

D. 43

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22. The first of the two samples have 100 items with mean 15 and S.D.3. If the whole group has 250 items with mean 15.6 and $S.D. = \sqrt{13.44}$ then S.D. of the second group is

A. 8

B. 6

C. 4

D. 5

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23. Let $f(x) = \begin{cases} \int_0^x (5 + |1 - t|)dt, & \text{if } x > 2 \\ 5x + 1, & \text{if } x \leq 2 \end{cases}$ then the function is

A. f(x) is not continuous at x = 2

B. f(x) is everywhere differentiable

C. f(x) is continuous but not differentiable at x = 2

D. f(x) is not differentiable at x=1

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24. Find the greatest value of the term independent of x in the expansion of $\left(x\sin\alpha + \frac{\cos\alpha}{x}\right)^{10}$, where $\alpha \in R$.

A. -1

B. 1

C. -2

D. 2

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25. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following:

A. The match will not be played and weather is not good and ground is wet.

B. If the match will not be played, then either weather is not good or ground is wet.

C. The match will be played and weather is not good or ground is wet.

D. The match will not be played or weather is good and ground is not wet .

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26. Find the Value of $\frac{\cot \pi}{24}$

A. $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

B. $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

C. $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

D. $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

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27. Which of the following value is just greater than $\left[1 + \frac{1}{10^{100}}\right]^{10^{100}}$

A. 3

B. 4

C. 2

D. 1

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28. The value of the integral

$$\int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx \text{ is}$$

A. 2

B. 0

C. -1

D. 1

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29. Let a, b and c be distinct non-negative numbers. If vectos $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, then c is

A. $\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$

B. $\dfrac{a+b}{2}$

C. $\dfrac{1}{a}+\dfrac{1}{b}$

D. $\sqrt{ab}$

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30. If [x] be the greatest integer less than or equal to x then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to :

A. 0

B. 4

C. -2

D. 2

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31. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq t\frac{\pi}{4}$ is

A. 4

B. 1

C. 2

D. 3

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32. If $\left|\vec{a}\right| = 2\left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, then $\left|\vec{a} \cdot \vec{b}\right|$ is equal to :

A. 6

B. 4

C. 3

D. 5

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33. The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is:

A. 2

B. 3

C. 1

D. 4

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34. Consider function $f: A \rightarrow B$ and $g: B \rightarrow C(A, B, \subseteq R)$ such that (gof)$^{-1}$ exists then

A. f and g both are one-one

B. f and g both are onto

C. f is one-one and g is onto

D. f is onto and g is one - one

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35. If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is "

A. $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

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36. Let x be a random variable such that the probability function of a distribution is given by $P(X=0)=\frac{1}{2}, P(X=J)=\frac{1}{3^j}(j=1,2,3,\ldots..,,\infty)$ then the mean of the distribution and P(X is positive and even) respectively are:

A. $\frac{3}{8}$ and $\frac{1}{8}$

B. $\frac{3}{4}$ and $\frac{1}{8}$

C. $\frac{3}{4}$ and $\frac{1}{9}$

D. $\frac{3}{4}$ and $\frac{1}{16}$

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37. If a tangent to the ellipse $x^2+4y^2=4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

A. $(\sqrt{3},0)$

B. $(\sqrt{2}, 0)$

C. $(1, 1)$

D. $(-1, 1)$

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38. Let the equation of the pair of lines, y = px and y = qx, can be written as (y – px) (y – qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

A. $x^2 - 3xy + y^2 = 0$

B. $x^2 - 3xy + y^2 = 0$

C. $x^2 + 3xy - y^2 = 0$

D. $x^2 - 3xy - y^2 = 0$

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39. If ${}^{n}P_{r} = {}^{n}P_{r+1}$ and ${}^{n}C_{r} = {}^{n}C_{r-1}$ then the value of r is equal to

A. 1

B. 4

C. 2

D. 3

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40. Let y = y (x) be the solution of the differential equation $xdy = \left(y + x^3 \cos x\right)dx$with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to

A. $\frac{\pi^2}{4} + \frac{\pi}{2}$

B. $\frac{\pi^2}{2} + \frac{\pi}{4}$

C. $\frac{\pi^2}{2} - \frac{\pi}{4}$

D. $\frac{\pi^2}{4} - \frac{\pi}{2}$

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41. If the mean and variance of the following data: 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to

A. 24

B. 12

C. 32

D. 16

Answer: D

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42. $\lim_{n \to \infty} \frac{1}{n} \sum_{j=0}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$

A. $5 + \log_e\left(\frac{3}{2}\right)$

B. $2 - \log_e\left(\frac{2}{3}\right)$

C. $3 + 2\log_e\left(\frac{2}{3}\right)$

D. $1 + 2\log_e\left(\frac{3}{2}\right)$

Answer: D

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43. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $\left(\vec{a} + \vec{b}\right) \times \left(\left(\vec{a} \times \left(\left(\vec{a} - \vec{b}\right) \times \vec{b}\right)\right) \times \vec{b}\right)$ is equal to

A. $5\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

B. $7\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

C. $7\left(30\hat{i} - 5\hat{j} + 7\hat{k}\right)$

D. $5\left(30\hat{i} - 5\hat{j} + 7\hat{k}\right)$

Answer: B

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44. The value of the definite integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x+\cos^4 x)}$

is equal to

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2\sqrt{2}}$

C. $-\frac{\pi}{4}$

D. $\frac{\pi}{\sqrt{2}}$

Answer: B

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45. Let C be the set of all complex numbers. Let $S_i =$ {z in C||z-3-2i|^(2)=8},

S_2= $\{z \in C \mid |z-\bar{z}| \geq 8\}$ and S_3= $\{z \in C \mid Re(z) \geq 5\}$. Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

A. 1

B. 0

C. 2

D. Infinite

Answer: A

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46. If the area of the bounded region $R = \left\{(x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$ is $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to

A. 8

B. 2

C. 4

D. 1

Answer: B

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47. A ray of light through (2,1) is reflected at a point P on the y-axis and then passes through the point (5,3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be

A. $11x+7y+8=0$ or $11x+7y-15=0$

B. $11x-7y-8=0$ or $11x+7y+15=0$

C. $2x-7y+29=0$ or $2x-7y-7=0$

D. $2x-7y-39=0$ or $2x-7y-7=0$

Answer: C

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48. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b\neq 0$, are equal, then the value of b is equal to

A. 2

B. -1

C. 1

D. -2

Answer: C

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49. The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to

A. $P \vee Q$

B. $P \wedge \sim Q$

C. $\sim (P \Rightarrow Q)$

D. $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$

Answer: D

Watch Video Solution

50. If $\sin\theta + \cos\theta = \frac{1}{2}$, then $16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to

A. 23

B. -27

C. -23

D. 27

Answer: C

Watch Video Solution

51. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A, \alpha, \beta \in R$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to

A. 5

B. $\frac{8}{3}$

C. 2

D. 4

Answer: D

Watch Video Solution

52. Let $f:\left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \rightarrow R$ be defined as

$$f(x) = \begin{cases} (1+|\sin x|)^{\frac{3a}{|\sin x|}}, & -\frac{\pi}{4} < x < 0 \\ b, & x = 0 \\ e^{\cot 4x / \cot 2x}, & 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at x = 0, then the value of $6a + b^2$ is equal to

A. $1 - e$

B. $e - 1$

C. $1 + e$

D. e

Answer: C

Watch Video Solution

53. Let y= y(x) be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with y(0)=0. If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to

A. $-\frac{1}{4}$

B. $\frac{1}{4}$

C. 2

D. $-\frac{1}{2}$

Answer: A

Watch Video Solution

54. Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of $a + b + c$ is equal to

A. 3

B. 8

C. 5

D. 4

Answer: D

Watch Video Solution

55. Two tangents are drawn from the point $P(-1, 1)$ to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangen tstouch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to

A. 2

B. $(3\sqrt{2} + 2)$

C. 4

D. $3(\sqrt{2} - 1)$

Answer: C

Watch Video Solution

56. Let $f: R \to R$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then, the value of $\lim_{x \to 2} \frac{x^2 f(2) - 4f(x)}{x - 2}$ is equal to

A. 4

B. 8

C. 16

D. 12

Answer: D

Watch Video Solution

57. Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P,Q} is equal to

A. $\{(4, 0), (0, 6)\}$

B. $\{(2 + 2\sqrt{2}, 3 - \sqrt{5}), (2 - 2\sqrt{2}, 3 + \sqrt{5})\}$

C. $\{(2 + 2\sqrt{2}, 3 + \sqrt{5}), (2 - 2\sqrt{2}, 3 - \sqrt{5})\}$

D. $\{(-1, 5), (5, 1)\}$

Answer: D

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58. Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0.$

Then $\alpha^8 + \beta^8$ is equal to

A. 10

B. 100

C. 50

D. 160

Answer: C

Watch Video Solution

59. The probability that a randomly selected 2-digit number belongs to the set $\{n \in N: (2^n - 2)$ is a multiple of 3$\}$ is equal to

A. $\frac{1}{6}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{3}$

Answer: C

Watch Video Solution

60. Let

$A = \{(x, y) \in R \times R \mid 2x^2 + 2y^2 - 2x - 2y = 1\}, B = \{(x, y) \in R \times R$

. Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

A. $\frac{3+\sqrt{10}}{2}$

B. $\frac{2+\sqrt{10}}{2}$

C. $\frac{3+2\sqrt{5}}{2}$

D. $1+\sqrt{5}$

Answer: C

Watch Video Solution

61. The point P (a,b) undergoes the following three transformations successively :

(a) reflection about the line y = x.

(b) translation through 2 units along the positive direction of x-axis.

(c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a - b$ is equal to :

A. 13

B. 9

C. 5

D. 7

Answer: B

Watch Video Solution

62. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{\left(\frac{1}{8}\right)\log_3^{\left(5^{x-1}+1\right)}}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3^{\left(5^{x-1}+1\right)}}$ is equal to 180 is

A. 0

B. -1

C. 2

D. 1

Answer: D

Watch Video Solution

63. For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1}=\frac{y-1}{2}=\frac{z-1}{3}$ and $\frac{x-4}{\beta}=\frac{y-6}{3}=\frac{z-7}{3}$, lies on the plane $x+2y-z=8$ then $\alpha-\beta$ is equal to

A. 5

B. 9

C. 3

D. 7

Answer: D

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64. Let $f: R \to R$ be defined as

$f(x+y)+f(x-y)=2f(x)f(y), \left(\frac{1}{2}\right) = -$. Then, the value of $\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$ is equal to

A. $\cos ec^2(21)\cos(20)\cos(2)$

B. $\sec^2(1)\sec(21)\cos(20)$

C. $\cos ec^2(1)\cos ec(21)\sin(20)$

D. $\sec^2(21)\sin(20)\sin(2)$

Answer: C

Watch Video Solution

65. Let $\mathbb{C}$ be the set of all complex numbers . Let

$S_1 = \{ \in \mathbb{C}: |z-2| \leq 1\}$ and

$S_2 = \{z \in \mathbb{C} z(1+i) + \bar{z}(1-i) \geq 4\}$

Then, the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to

A. $\frac{3+2\sqrt{2}}{4}$

B. $\frac{5+2\sqrt{2}}{2}$

C. $\frac{3+2\sqrt{2}}{2}$

D. $\frac{5+2\sqrt{2}}{4}$

Answer: D

Watch Video Solution

66. A student appeared in an examination consisting of 8 true - false . The student guesses the answers with equal probability . The smallest value of n, so that the probability of guessing at least 'n' correct answer is less than $\frac{1}{2}$ is

A. 5

B. 6

C. 3

D. 4

Answer: A

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67. If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to

A. 4

B. 3

C. 0

D. 1

Answer: C

Watch Video Solution

68. Let the mean and variance of the frequency distribution

$x:$	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
$f:$	4	4	α	β

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be:

A. 4

B. 5

C. $\frac{17}{3}$

D. $\frac{16}{3}$

Answer: C

Watch Video Solution

69. The area of the region bounded by y – x = 2 and $x^2 = y$ is equal to

A. $\frac{16}{3}$

B. $\frac{2}{3}$

C. $\frac{9}{2}$

D. $\frac{4}{3}$

Answer: C

Watch Video Solution

70. Let y = y (x) be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx, x > 2$. If y (3) = 3 then y (4) is equal to :

A. 4

B. 12

C. 8

D. 16

Answer: B

Watch Video Solution

71. The value of $\lim_{x \to 0}\left(\frac{x}{8\sqrt{1-\sin x}-8\sqrt{1+\sin x}}\right)$ is equal to :

A. 0

B. 4

C. -4

D. -1

Answer: C

Watch Video Solution

72. Two sides of a parallelogram are along the lines 4x + 5y = 0 and 7x + 2y = 0. If the equation of one of the diagonals of the parallelogram is 11x + 7y = 9, then other diagonal passes through the point :

A. (1,2)

B. (2,3)

C. (2,1)

D. (1,3)

Answer: B

Watch Video Solution

73. Let $\alpha = \max_{x \subset R} \left\{ 8^{2\sin 3x} \cdot 4^{4\cos 3x} \right\}$ and $\beta = \min_{x \subset R} \left\{ 8^{2\sin 3x} \cdot 4^{4\cos 3x} \right\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$ then the value of $c - b$ is equal to :

A. 42

B. 47

C. 43

D. 50

Answer: A

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74. Let $f:[0,\infty)\rightarrow[0,3]$ be a function defined by

$$f(x)=\begin{cases}\max\{\sin t : 0\leq t\leq x\}\ , & 0\leq x\leq\pi\\ 2+\cos x, & x>\pi\end{cases}$$

Then which of the following is true ?

A. f is continuous everywhere but not differentiable exactly at one point in $(0,\infty)$

B. f is differentiable everywhere in $(0,\infty)$

C. f is not continuous exactly at two points in $(0,\infty)$

D. f is continuous everywhere but not differentiable exactly at two points in $(0,\infty)$

Answer: B

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75. Let N be the set of natural numbers and a relation R on N be defined by

$R = \{(x, y) \in N \times N : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}$. Then the relation R is :

A. symmetric but neither reflexive nor transitive

B. reflexive but neither symmetric nor transitive

C. reflexive and symmetric, but not transitive

D. an equivalence relation

Answer: B

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76. Which of the following is the negation of the statement "for all $M > 0$, there exists $x \in S$ such that $x \geq M$?

A. there exists $M > 0$, such that $x <$ M for all $x \in S$

B. there exists $M > 0$, there exists $x \in S$ such that $x \geq M$

C. there exists $M > 0$, there exists $x \in S$ such that $x < M$

D. there exists $M > 0$, such that $x \geq M$ for all $x \in S$

Answer: A

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77. Consider a circle C which touches the y-axis at (0,6) and cuts off an intercept $6\sqrt{5}$ on the x - axis. Then the radius of the circle C is equal to

A. $\sqrt{53}$

B. 9

C. 8

D. $\sqrt{82}$

Answer: B

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78. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times \left(\vec{b} \times \vec{c}\right)$.

If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to :

A. $\sqrt{3} + 1$

B. 2

C. 1

D. $\frac{\sqrt{3} + 1}{\sqrt{3}}$

Answer: B

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79. Let A and B be two 3×3 real matrices such that $\left(A^2 - B^2\right)$ is invertible matrix . If $A^5 = B^5$ and $A^3B^2 = A^2B^3$, then the value of the determinant of the matrix $A^3 + B^3$ is equal to :

A. 2

B. 4

C. 1

D. 0

Answer: D

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80. Let $f:(a, b) \rightarrow R$ be twice differentiable function such that

$$f(x) = \int_a^x g(t)dt$$

for a differentiable function g(x) . If $f(x) = 0$ has exactly five distinct roots in (a, b), then $g(x)g'(x) = 0$ has at least

A. twelve roots in (a,b)

B. five roots in (a,b)

C. seven roots in (a,b)

D. three roots in (a,b)

Answer: C

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MATHEMATICS (SECTION-B)

1. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f: A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

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2. If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to ______.

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3. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set $\{1, 2, 3, 4, 5\}$ and satisfying AB = BA is ______.

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4. Consider the following frequency distribution :

Class :	$0-6$	$6-12$	$12-18$	$18-24$	$24-30$
Frequency :	a	b	12	9	5

If mean $= \frac{309}{22}$ and median = 14, then the value $(a-b)^2$ is equal to ________ .

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5. The sum of all the elements in the set $\{n \in \{(1, 2, \ldots\ldots, 100\} \mid H.C.F. \text{ of n and 2040 is 1}\}$ is equal to _______ .

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6. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0, y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is ______.

Watch Video Solution

7. Let $f: R \to R$ be a function defined as

$$f(x) = \begin{cases} 3\left(1 - \frac{|x|}{2}\right) & \text{if } |x| \leq 2 \\ 0 & \text{if } |x| \geq 2 \end{cases}$$

Let $g: R \to R$ be given by $g(x) = f(x+2) - f(x-2)$. If n and m denote the number of points in R where g is not continuous and not differentiable, respectively, then n + m is equal to ______.

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8. If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _________ .

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9. Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)}+(y+1)\right)dx=(x+2)dy,\ y(1)=1$. If the domain of y = y(x) is an open interval (α, β), then $|\alpha+\beta|$ is equal to ______ .

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10. The number of elements in the set $\{n \in \{1, 2, 3, \ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is ______ .

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11. Let n $\in$ N and [x] denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^nC_0, 3.\ {}^nC_1, 5.\ :{}^nC_2, \&.\ {}^nC_3$ is equal to 2^{100} . 101 then $2\left[\frac{n-1}{2}\right]$ is equal to ______

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12. Consider the function $\begin{array}{ll} f(x)\frac{P(X)}{\sin(x-2)} & x \neq 2 \\ = 7 & x = 2 \end{array}$

Where P(x) is a polynomial such that P' ' (x) is always a constant and P(3) =

9. If f(x) is continuous at x = 2, then P(5) is equal to _______.

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13. Equation of circle $Re(z^2) + 2(img(z))^2 + 2Re(z) = 0$ where $z = x + iy$ .A line passes through the vetex of parabola $x^2 - 6x + y + 13 = 0$ and center of circle, then the y intercept of the line is _____?

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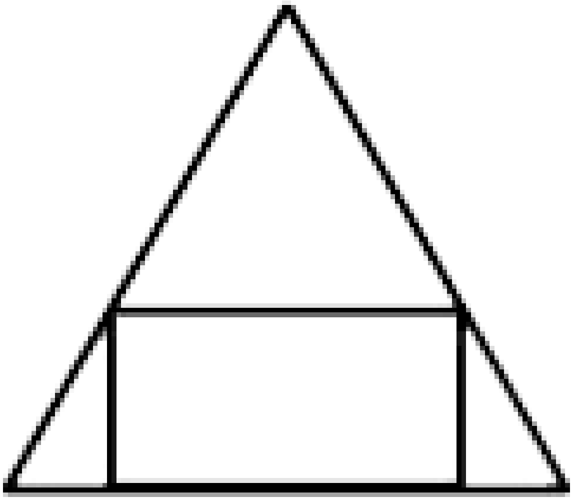
14. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is______.



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15. If $\left(\vec{a}+3\vec{b}\right)$ is perpendicular to $\left(7\vec{a}-5\vec{b}\right)$ and $\left(\vec{a}-4\vec{b}\right)$ is perpendicular to $\left(7\vec{a}-2\vec{b}\right)$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _____

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16. Let a curve y = f(x) pass through the point $\left(2, (\log_e 2)^2\right)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to______.

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17. If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to ____

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18. A coin is tossed n times, if the probability of getting at least one head is atlest 99% then the minimum value of n is

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19. If the coefficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal then n =?

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20. If the lines $\frac{x-k}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x+1}{3}=\frac{y+2}{2}=\frac{z+3}{1}$ are co - planar , then value of k is _____

Watch Video Solution

21. For real numbers α and β, consider the following system of linear equations: $x+y-z=2, x+2y+\alpha z=1, 2x-y+z=\beta$. If the system has infinite solutions, then $\alpha+\beta$ is equal to ____

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22. Let $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{b}$ and $\overrightarrow{c} = \hat{j} - \hat{k}$ be three vectors such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c}$ and $\overrightarrow{a} . \overrightarrow{b} = 1$. If the length of projection vector of the vector $\overrightarrow{b}$ on the vector $\overrightarrow{a} \times \overrightarrow{c}$ is l, then the vlaue of $3l^2$ is equal to ____

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23. If $(\log)_3 2, (\log)_3 (2^x - 5) and (\log)_3 \left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, determine the value of $x \cdot$

Watch Video Solution

24. Find the domain of function $f(x) = (\log)_4 \left[(\log)_5 \left\{(\log)_3 \left(18x - x^2 - 77\right\}\right]\right.$

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25. Let $f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$. Then the maximum value of f(x) is equal to ____

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26. Let $F: [3, 5] \to R$ be a twice differentiable function on (3,5) such that $F(x) = e^{-x}\int_3^x \left(3t^2 + 2t + 4F'(t)\right)dt$. If $F'(4) = \frac{\alpha e^{\beta} - 224}{\left(e^{\beta} - 4\right)^2}$, then $\alpha + \beta$ is equal to ____

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27. Let a plane p passes through the point $(3, 7, -7)$ and contain the line , $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$, or distance of the plane p from the origin is d then d^2 is

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28. Let $S = \{1, 2, 3, 45, 6, 7\}$. Then the number of possible functions $f: S \to S$ such that f(m.n)= f(m).f(n) for every $m, n \in S$ and $m. n \in S$ is equal to ____

Watch Video Solution

29. If $y = y(x), y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation $\sec y \frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with y(0)= 0, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _____

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30. Let $f: [0, 3] \to R$ be defined by $f(x) = \min\{x - [x], 1 + [x] - x\}$ where [x] is the greatest integer less than or equal to x. Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to_____

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31. Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$ then $\left(\vec{a} \times \vec{b}\right) \cdot \vec{c}$ is equal to ____

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32. Find the distance of the point $P3, 4, 4)$ from the point where the line joining the points $A(3, -4, -5) and B(2, -3, 1)$ intersects the plane $2x + y + z = 7$.

Watch Video Solution

33. If the real part of the complex number $z = \dfrac{3 + 2i\cos\theta}{1 - 3i\cos\theta}, \theta \in \left(0, \dfrac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to ____

Watch Video Solution

34. Let E be an ellipse whose axes are parallel to the co-ordinates axes, having its center at (3, – 4), one focus at (4, – 4) and one vertex at (5, – 4). If mx – y = 4, $m > 0$ is a tangent to the ellipse E, then the value of $5m^2$ is equal to ____

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35. If $\int_0^{\pi} (\sin^3 x) e^{-\sin^2 x} dx = \alpha - \frac{\beta}{e} \int_0^1 \sqrt{t} e^t \text{ dt , then }$ $\alpha + \beta$ is equal to ____

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36. Find number of real roots of equation $e^{4x} + e^{3x} - 4e^{2x} + e^x + 1 = 0$ is

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37. Let $y = y(x)$ be the solution of the differential equation dy $= e^{\alpha . x + y} dx, \alpha \in N$. If $y(\log_e 2) = \log_e 2$ and $y(0) = \log_e\left(\frac{1}{2}\right)$, then the value of α is equal to ____

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38. Let n be a non - negative integer . Then the number of divisors of the form $''4n + 1''$ of the number $(10)^{10} \cdot (11)^{11} \cdot (13)^{13}$ is equal to ____

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39. Let $A = \{n \in N \mid n^2 \leq n + 10, 000\} B = \{3k + 1 \mid k \in N\}$ and $C = \{2k \mid k$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ______

Watch Video Solution

40. If A $= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $M = A + A^2 + A^3 + \ldots . + A^{20}$ then the sum of all the elements of the matrix M is equal to ____

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Mathematics Section A

1. The range of the function

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3}{}\right.\right.$$

is :

A. $[0, 2]$

B. $(0, \sqrt{5})$

C. $[-2, 2]$

D. $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

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2. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$.

Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4), x_1 < x_2,$ such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4), x_3 < x_4,$ such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

A. (S1) is false and (S2) is true

B. both (S1) and (S2) are true

C. (S1) is true and (S2) is false

D. both (S1) and (S2) are false

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3. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to :

A. $\frac{25}{2}$

B. $\frac{75}{8}$

C. $\frac{125}{16}$

D. $\frac{15}{2}$

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4. Let

$S_n = 1(n-1) + 2.(n-2) + 3.(n-3) + \ldots + (n-1).1, n \geq 4.$

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to :

A. $\frac{e-2}{6}$

B. $\frac{e}{3}$

C. $\frac{e-1}{3}$

D. $\frac{e}{6}$

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5. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6z + 1 = 0$ be the plane P. Then which of the following points lies on P ?

A. $\left(3, 1, -\frac{1}{2}\right)$

B. $\left(-2, 0, -\frac{1}{2}\right)$

C. $(4, 0, -2)$

D. $(0, 2, -4)$

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6. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

A. $3\pi - 11$

B. $4\pi - 11$

C. $3\pi + 1$

D. $4\pi - 9$

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7. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1, x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :

A. $(3, 5\pi/3)$

B. $(3, 13\pi/9)$

C. $(2, 8\pi/9)$

D. $(2, 2\pi/3)$

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8. Which of the following is equivalent to the Boolean expression $P \wedge \sim q$?

A. $\sim p \rightarrow \sim q$

B. $\sim(q \rightarrow p)$

C. $\sim(p \rightarrow q)$

D. $\sim(p \rightarrow \sim q)$

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9. Let $J_{n,m} = \int_0^{1/2} \frac{x^n}{x^m - 1}, \forall n > m$ and $n, m \in N$. Consider a matrix $A = [a_{ij}]_{3 \times 3}$ where

$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i < j \\ 0, & i > j \end{cases}$. Then $|adjA^{-1}|$ is :

A. $(105)^2 \times 2^{38}$

B. $(105)^2 \times 2^{36}$

C. $(15)^2 \times 2^{42}$

D. $(15)^2 \times 2^{34}$

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10. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2

be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

A. $n(S_1) = 0, n(S_2) = 2$

B. $n(S_1) = 2, n(S_2) = 0$

C. $n(S_1) = 2, n(S_2) = 2$

D. $n(S_1) = 1, n(S_2) = 0$

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11. The number of pairs (a, b) of real numbers, such that whenever a is a root of the equation $x^2 + ax + b = 0, \alpha^2 - 2$ is also a root of this equation, is :

A. 4

B. 6

C. 8

D. 2

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12. If y = y (x) is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right) dx = 0, x > 0, \text{ and } y(1) = 1, \text{ then } y\left(\frac{1}{2}\right)$ is equal to

A. $3 + e$

B. $\frac{3}{2} - \frac{1}{\sqrt{e}}$

C. $3 - e$

D. $3 + \frac{1}{\sqrt{e}}$

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13. The function f(x), that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x . \cos y f(y) dy$, is :

A. $x + \frac{\pi}{2} \sin x$

B. $x + (\pi - 2) \sin x$

C. $x + \frac{2}{3}(\pi - 2) \sin x$

D. $x + (\pi + 2) \sin x$

Watch Video Solution

14. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{1} = 1$ and the circle $x^2 + y^2 = 3$ at their point of intersection in the first quadrat. Then $\tan\theta$ is equal to :

A. $\frac{4}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{5}{2\sqrt{3}}$

D. 2

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15. The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$

A. $4(\sqrt{2} - 1)$

B. $2\sqrt{2}(\sqrt{2} - 1)$

C. $2\sqrt{2 + 1}$

D. $2\sqrt{2}(\sqrt{2} + 1)$

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16. Let $a_1, a_2, \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. if the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

A. 36

B. 48

C. 72

D. 57

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17. Let $P_1, P_2 \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

A. 12

B. 455

C. 443

D. 419

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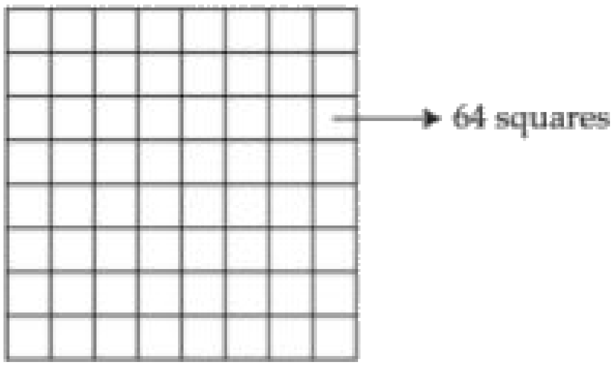
18. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :



A. $\frac{1}{9}$

B. $\frac{1}{18}$

C. $\frac{2}{7}$

D. $\frac{1}{7}$

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19. Let f: $R \to R$ be a continuous function. Then $\lim_{x \to \pi/4} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to :

A. $f(2)$

B. $4f(2)$

C. $2f(\sqrt{2})$

D. $2f(2)$

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20. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, -1, 6) is :

A. $2\sqrt{6}$

B. $2\sqrt{5}$

C. $\sqrt{26}$

D. $4\sqrt{2}$

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21. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If f(0) = 0, f(1) = 1 and f(2) = 2, then :

A. $f''(x) = 0$ for some $x \in (0, 2)$

B. $f'(x) = 0$ for some $x \in [0, 2]$

C. $f''(x) > 0$ for all $x \in (0, 2)$

D. $f''(x) = 0$ for all $x \in (0, 2)$

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22. Let $f: N \rightarrow N$ be a function such that $f(m+n) = f(m) + f(n)$ for every m, $n \in N$. If $f(6) = 18$, then $f(2) . f(3)$ is equal to :

A. 18

B. 6

C. 54

D. 36

Watch Video Solution

23. If $\alpha = \lim_{x \rightarrow \pi/4} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$ and $\beta = \lim_{x \rightarrow 0} (\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

A. $(1, -3)$

B. $(-1, 3)$

C. $(1, 3)$

D. $(-1, -3)$

Watch Video Solution

24. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies

$$\vec{a} \times \left\{\left(\vec{r} - \vec{b}\right) \times \vec{a}\right\} + \vec{b} \times \left\{\left(\vec{r} - \vec{c}\right) \times \vec{b}\right\} + \vec{c} \times \left\{\left(\vec{r} - \vec{a}\right)\right.$$

, then $\vec{r}$ is equal to :

A. $\frac{1}{3}\left(\vec{a} + \vec{b} + \vec{c}\right)$

B. $\frac{1}{3}\left(2\vec{a} + \vec{b} - \vec{c}\right)$

C. $\frac{1}{2}\left(\vec{a} + \vec{b} + 2\vec{c}\right)$

D. $\frac{1}{2}\left(\vec{a} + \vec{b} + \vec{c}\right)$

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25. An angle of intersection of the curves , $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab, a > b$, is :

A. $\tan^{-1}(2\sqrt{ab})$

B. $\tan^{-1}\left(\frac{1+b}{\sqrt{ab}}\right)$

C. $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$

D. $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

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26. If [x] is the greatest integer $\leq x$, then $\pi^2 \int_0^2 \left(\sin\frac{\pi x}{2}\right)(x - [x])^{[x]} dx$ is equal to :

A. $4(\pi - 1)$

B. $4(\pi + 1)$

C. $2(\pi + 1)$

D. $2(\pi - 1)$

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27. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

A. no solution

B. a unique solution

C. infinitely many solutions

D. exactly two solutions

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28. The sum of the roots of the equations,

$x + 1 - 2\log_2(3 + 2^x) + 2\log_4\left(10 - 2^{-x}\right) = 0$, is

A. $\log_2 11$

B. $\log_2 12$

C. $\log_2 14$

D. $\log_2 13$

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29. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \leq x \leq \frac{\pi}{4}$ is :

A. 1

B. 3

C. 0

D. 2

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30. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

A. $\sim p \wedge q \wedge r$

B. $\sim p \wedge q \wedge \sim r$

C. $p \wedge q \wedge r$

D. $p \wedge \sim q \wedge \sim r$

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31. Let A be the set of all points (α, β) such that the area of triangle formed by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is

A. $\frac{8}{\sqrt{5}}$

B. $\frac{12}{\sqrt{5}}$

C. $\frac{4}{\sqrt{5}}$

D. $\frac{16}{\sqrt{5}}$

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32. The locus of mid-points of the segments joining (-3, -5) and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

A. $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

B. $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

C. $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

D. $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

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33. Let $a_1, a_2, a_3,$ .....be in A.P. If $\frac{a_1 + a_2 + \dots + a_{10}}{a_1 + a_2 + \dots + a_p} = \frac{100}{p^2}, p \neq 10,$ then $\frac{a_{11}}{a_{10}}$ is equal to :

A. $\frac{21}{19}$

B. $\frac{100}{121}$

C. $\frac{19}{21}$

D. $\frac{121}{100}$

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34. If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right], x \geq 0, \phi > 0,$ y(1) = -1, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

A. $\phi(1)$

B. $4\phi(1)$

C. $4\phi(2)$

D. $2\phi(1)$

Watch Video Solution

35. The distance of the point (-1, 2, -2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is :

A. $\frac{\sqrt{34}}{2}$

B. $\frac{\sqrt{42}}{2}$

C. $\frac{5}{2}$

D. $\frac{1}{\sqrt{2}}$

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36. If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z-(3+3i)|$ is :

A. $2\sqrt{2}$

B. $3\sqrt{2}$

C. $2\sqrt{2}-1$

D. $6\sqrt{2}$

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37. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

A. $\frac{112}{5}$

B. $\frac{134}{5}$

C. $\frac{92}{5}$

D. $\frac{536}{25}$

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38. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is :

A. $\frac{1}{15}$

B. $\frac{1}{5}$

C. $\frac{1}{30}$

D. $\frac{1}{10}$

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39. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{3x^2 + x - 1}{(x - 1)^2}\right) + \cos^{-1}\left(\frac{x - 1}{x + 1}\right) \text{ is :}$$

A. $\left[0, \frac{1}{4}\right]$

B. $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$

C. $\left[0, \frac{1}{2}\right]$

D. $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

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40. If $\frac{dy}{dx} = \frac{2^x y + 2^y . 2^x}{2^x + 2^{x+y} \log_e 2}$, $y(0) = 0$, then for y = 1, the value of x lies in the interval :

A. $\left(0, \frac{1}{2}\right]$

B. $\left(\frac{1}{2}, 1\right]$

C. (1, 2)

D. $(2, 3)$

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Mathematics Section B

1. Let the points of intersections of the lines $x - y + 1 = 0, x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is _______ .

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2. If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is 924 b. 792 c. 1594 d. none of these

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3. Let X be a random variable with distribution.

x	-2	-1	3	4	6
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to :

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4. Let [t] d enote the greatest integer $< t$. The number of points where the function $f(x) = [x]|x^2 - 1| + \sin\left(\frac{\pi}{[x] + 3}\right) - [x + 1], x \in (-2, 2)$ is not continuous is ________.

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5. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ________ .

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6. If for the complex numbers z satisfying $|z - 2 - 2i| < 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to _______.

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7. Let $\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\overrightarrow{v}$ be in the plane containing $\overrightarrow{a}$ and $\overrightarrow{b}$. If $\overrightarrow{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\overrightarrow{a}$ is 19 units, then $\left|2\overrightarrow{v}\right|^2$ is equal to ________.

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8. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3, x \in R$. Then the n natural number for which $\lim_{x \to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$ is ______.

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9. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to ________.

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10. A man starts walking from the point P(-3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2 + (RQ)^2\right)$ is equal to ________ .

Watch Video Solution

11. If the line y = mx bisects the area enclosed by the lines x = 0, y = 0, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to ________.

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12. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ________ .

Watch Video Solution

13. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \ldots$.., then 160 S is equal to _______ .

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14. A tangent line L is drawn at the point (2, -4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to

________.

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15. Let f(x) be a cubic polynomial with f(1) = -10, f(-1) = 6, and has a local minima at x = 1, and f'(x) has a local minima at x = -1. Then f(3) is equal to ________.

Watch Video Solution

16. Suppose the line $\frac{x-2}{\alpha}=\frac{y-2}{-5}=\frac{z+2}{2}$ lies on the plane $x+3y-2z+\beta=0$. Then $(\alpha+\beta)$ is equal to _______.

Watch Video Solution

17. If

$$\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx=\alpha \log_e|1+\tan x|+\beta \log_e\left|1-\tan x+\tan^2 x\right|+\gamma \text{ta}$$

, when C is constant of integration, then the value of $18(\alpha+\beta+\gamma^2)$ is

__________.

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18. The number of elements in the set

$$\left\{A=\begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1) \text{ and } (I-A)^3 = I - A^3\right\},$$

where I is 2×2 identity matrix, is __________.

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19. Let B the centre of the circle $x^2+y^2-2x+4y+1=0$. Let the tangents at two points P and Q on the circle intersect at the point A(3, 1). Then $8.\left(\frac{\text{area } \Delta APQ}{\text{area } \Delta BPQ}\right)$ is equal to ________.

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20. If the coefficient of a^7b^8 in the expansion of $(a+2b+4ab)^{10}$ is $K.2^{16}$, then K is equal to __________.

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Mathematics (Section A)

1. If $U_n = \left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots\ldots\ldots\ldots \left(1+\frac{n^2}{n^2}\right)^n$ m then $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to

A. $\frac{4}{e}$

B. $\frac{16}{e^2}$

C. $\frac{e^2}{16}$

D. $\frac{4}{e^2}$

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2. A tangent and a normal are drawn at the point P(2,-4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q (a,b) is a point such that AQBP is a square , then 2a+b is equal to :

A. -12

B. -20

C. -18

D. -16

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3. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line whose direction ratios are 2,3,-6 is

A. 3

B. 5

C. 1

D. 2

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4. When a certain biased die is rolled, a particular face occurs with probablitiy $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die . If $0 < x < \frac{1}{6}$, and the probability of obtaining total sum $=7$, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is

A. $\frac{1}{8}$

B. $\frac{1}{9}$

C. $\frac{1}{16}$

D. $\frac{1}{12}$

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5. Let A be a fixed point (0,6) and B be a moving point (2t,0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the y-axis at C. The locus of the mid-point P of MC is :

A. $2x^2 - 3y + 9 = 0$

B. $3x^2 + 2y - 6 = 0$

C. $3x^2 - 2y - 6 = 0$

D. $2x^2 + 3y - 9 = 0$

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6. If $x^2 + 9y^2 - 4x + 3 = 0, x, y \in \mathbb{R}$, then x and y respectively lie in the intervals

A. $[1, 3]$ and $[1, 3]$

B. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

C. $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

D. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$

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7. Let us consider a curve, y=f(x) passing through the point (-2,2) and the slope of the tangent to the curve at any point (x,f(x)) is given by $f(x) + xf'(x) = x^2$. Then :

A. $x^3 + xf(x) + 12 = 0$

B. $x^2 + 2xf(x) + 4 = 0$

C. $x^2 + 2xf(x) - 12 = 0$

D. $x^2 - 3xf(x) - 4 = 0$

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8. $\sum_{k=0}^{20} \left(.^{20} C_k\right)^2$ is equal to

A. $.^{40} C_{19}$

B. $.^{41} C_{20}$

C. $.^{40} C_{21}$

D. $.^{40} C_{20}$

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9. If for x,y $\in R, \quad x > 0, \quad y = \log_{10} x + \log_{10} x^{1/3} + \log_{10} x^{1/9} +$ ....upto ∞ terms and $\frac{2 + 4 + 6 + \ldots\ldots. + 2y}{3 + 6 + 9 + \ldots\ldots + 3y} = \frac{4}{\log_{10} x}$, then the ordered pair (x,y) is equal to :

A. $\left(10^2, 3\right)$

B. $\left(10^6, 6\right)$

C. $\left(10^6, 9\right)$

D. $\left(10^4, 6\right)$

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10. If $S = \left\{ z \in \mathbb{C} : \frac{z - i}{z + 2i} \in \mathbb{R} \right\}$, then

A. S is a straight line in the complex plane

B. S is a circle in the complex plane

C. S contains exactly two elements

D. S contains only one element

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11. A wire of length 20m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the

length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum is

A. $\dfrac{10}{3+2\sqrt{3}}$

B. $\dfrac{5}{2+\sqrt{3}}$

C. $\dfrac{10}{2+3\sqrt{3}}$

D. $\dfrac{5}{3+\sqrt{3}}$

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12. $\int_0^{16} \dfrac{\log_e x^2}{\log_e x^2 + \log_e(x^2 - 44x + 484)} dx$ is equal to

A. 5

B. 6

C. 10

D. 8

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13. If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 +$ ....is equal to

A. $\frac{1+x}{1-x} + \log_e(1-x)$

B. $\frac{1-x}{1+x} + \log_e(1-x)$

C. $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

D. $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

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14. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A\left(A^3 + 3I\right) = 2I$, then then value of K is

A. -1

B. $-\frac{1}{2}$

C. 1

D. $\frac{1}{2}$

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15. If α, β are the distinct roots of $x^2 + bx + c = 0$, then $\lim_{x \to \beta} \frac{e^{2\left(x^2 + bx + c\right)} - 1 - 2\left(x^2 + bx + c\right)}{(x - \beta)^2}$ is equal to

A. $b^2 + 4c$

B. $2\left(b^2 + 4c\right)$

C. $2\left(b^2 - 4c\right)$

D. $b^2 - 4c$

Watch Video Solution

16. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to

A. $3e^{\pi^2}$+5

B. e^{π^2}+5

C. $7e^{\pi^2}$+5

D. $2e^{\pi^2}$+5

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17. Let $\frac{\sin A}{\sin B} = \frac{\sin(A - C)}{\sin(C - B)}$, where A , B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a,b,c respectively, then

A. $b^2 - a^2 = a^2 + c^2$

B. a^2, b^2, c^2 are in A.P.

C. c^2, a^2, b^2 are in A.P.

D. b^2, c^2, a^2 are in A.P.

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18. The statement $(p \wedge (p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow r$ is

A. equivalent to $p \rightarrow \sim r$

B. a fallacy

C. equivalent to $q \rightarrow \sim r$

D. a tautology

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19. If $\left(\sin^{-1}x\right)^2 - \left(\cos^{-1}x\right)^2 = a$, $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is

A. $\cos\left(\frac{2a}{\pi}\right)$

B. $\sin\left(\frac{4a}{\pi}\right)$

C. $\sin\left(\frac{2a}{\pi}\right)$

D. $\cos\left(\frac{4a}{\pi}\right)$

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20. Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin which contains the line of intersection of the planes $x - y - z - 1 = 0$ and $2x + y - 3z + 4 = 0$, is

A. $3x - 4z + 3 = 0$

B. $4x - y - 5z + 2 = 0$

C. $3x - 6 - 5z + 2 = 0$

D. $-x + 2y + 2z - 3 = 0$

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21. Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is :

A. 1

B. $\frac{5}{8}$

C. $\frac{5}{16}$

D. $\frac{1}{8}$

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22. The set of all values of $k > -1$ for which the equation

$$\left(3x^2 + 4x + 3\right)^2 - (k+1)\left(3x^2 + 4x + 3\right)\left(3x^2 + 4x + 2\right) + k\left(3x^2 + 4x +$$

has real roots, is :

A. $\left(1, \frac{5}{2}\right]$

B. $\left[-\frac{1}{2}, 1\right)$

C. $[2, 3)$

D. $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

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23. Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x + y + z = 4, 3x + 2y + 5z = 3, 9x + 4y + (28 + [\lambda])z = [\lambda]$ has a solution is :

A. R

B. $[-9, -8)$

C. $(-\infty, -9) \cup (-9, \infty)$

D. $(-\infty, -9) \cup [-8, \infty)$

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24. Two poles, AB of length a metres and CD of length $a+b(b \neq a)$ metres are erected at the same horizontal level with bases at B and D. If BD =x and tan $\underline{|ACB} = \frac{1}{2}$, then:

A. $x^2 + 2(a+2b)x + a(a+b) = 0$

B. $x^2 - 2ax + a(a+b) = 0$

C. $x^2 - 2ax + b(a+b) = 0$

D. $x^2 + 2(a+2b)x - b(a+b) = 0$

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25. If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots\ldots.$, then the vlaue of e^{1+y} at $x = \frac{1}{2}$ is :

A. $\frac{1}{2}\sqrt{e}$

B. 2e

C. $2e^2$

D. $\frac{1}{2}e^2$

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26. The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x-axis is :

A. 6

B. 10

C. 9

D. 4

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27. If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is :

A. -1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. 0

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28. Let $A = \begin{pmatrix} [\mathrm{x+1}] & [\mathrm{x+2}] & [\mathrm{x+3}] \\ [\mathrm{x}] & [\mathrm{x+3}] & [\mathrm{x+3}] \\ [\mathrm{x}] & [\mathrm{x+2}] & [\mathrm{x+4}] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A)=192, then the set of values of x is the interval :

A. [65, 66)

B. [68, 69)

C. [62, 63)

D. [60, 62)

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29. Let $\mathbb{Z}$ be the set of all integers,

$A = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + y^2 \leq 4\right\}$,

$B = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \leq 4\right\}$ and

$C = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : (x-2)^2 + (y-2)^2 \leq 4\right\}$

If the total number of relations from $A \cap B$ to $A \cap C$ is 2^p, then the value of pis :

A. 9

B. 16

C. 25

D. 49

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30. Let M and m respectively be the maximum and minimum values of the function

$f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the vlaue of $\tan(M - m)$ is equal to :

A. $2 - \sqrt{3}$

B. $3 - 2\sqrt{2}$

C. $2+\sqrt{3}$

D. $3+2\sqrt{2}$

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31. The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to :

A. $(q \wedge r) \Rightarrow (p \wedge q)$

B. $(p \wedge q) \Rightarrow (r \vee q)$

C. $(p \wedge q) \Rightarrow (r \wedge q)$

D. $(p \wedge r) \Rightarrow (p \wedge q)$

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32. The angle between the straight lines, whose direction cosines are given by the equations $2l + 2m - n = 0$ and $mn + nl + lm = 0$, is :

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\pi - \cos^{-1}\left(\frac{4}{9}\right)$

D. $\cos^{-1}\left(\frac{8}{9}\right)$

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33. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$ and $\vec{r}.\left(2\hat{i} + 3\hat{j} - \hat{k}\right) + 4 = 0$ and parallel to x-axis.

A. $\vec{r} \cdot \left(\hat{j} - 3\hat{k}\right) + 6 = 0$

B. $\vec{r} \cdot \left(\hat{j} - 3\hat{k}\right) - 6 = 0$

C. $\overrightarrow{r} \cdot \left(\hat{j} + 3\hat{k}\right) + 6 = 0$

D. $\overrightarrow{r} \cdot \left(\hat{i} - 3\hat{k}\right) + 6 = 0$

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34. If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is :

A. $x + 4 = 0$

B. $x + 2 = 0$

C. $x + 3 = 0$

D. $x + 1 = 0$

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35. A box open from top is made from a rectangular sheet of dimension a $\times$ b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to :

A. $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$

B. $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$

C. $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

D. $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

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36. If $\lim_{x \to \infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is :

A. $\left(1, \frac{1}{2}\right)$

B. $\left(-1, -\frac{1}{2}\right)$

C. $\left(1,\ -\frac{1}{2}\right)$

D. $\left(-1,\ \frac{1}{2}\right)$

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37. Let A(a, 0), B(b, 2b +1) and C(0, b), b $\neq$ 0, |b| $\neq$ 1, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

A. $\frac{2b}{b+1}$

B. $\frac{-2b^2}{b+1}$

C. $\frac{-2b}{b+1}$

D. $\frac{2b^2}{b+1}$

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38. The value of the integral $\int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$ is :

A. $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$

B. $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{6}\right)$

C. $\frac{\pi}{8}\left(1-\frac{\sqrt{3}}{6}\right)$

D. $\frac{\pi}{4}\left(1-\frac{\sqrt{3}}{2}\right)$

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39. A differential equation representing the family of parabolas with axis parallel to y-axis and whose length of latus rectum is the distance of the point (2, -3) form the line 3x+4y=5, is given by :

A. $11\frac{d^2x}{dy^2}=10$

B. $10\frac{d^2x}{dy^2}=11$

C. $10\frac{d^2y}{dx^2}=11$

D. $11\frac{d^2y}{dx^2} = 10$

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40. If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and $(2, \beta)$, then β is a root of the equation :

A. $y^5 - 2y - 2 = 0$

B. $2y^5 - y^2 - 2 = 0$

C. $y^5 - y^2 - 1 = 0$

D. $2y^5 - 2y - 1 = 0$

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41. If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$ then y(1) is equal to

A. $\log_2(1+e^2)$

B. $\log_2(2e)$

C. $\log_2(1+e)$

D. $\log_2(2+e)$

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42. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|2\vec{a}+3\vec{b}\right|=\left|3\vec{a}+\vec{b}\right|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $\left|\vec{b}\right|$ is equal to :

A. 5

B. 6

C. 4

D. 8

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43. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

A. $8\sqrt{10}$

B. $6\sqrt{10}$

C. $12\sqrt{15}$

D. $12\sqrt{10}$

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44. Let $\cdot, \square \varepsilon\{\wedge, \vee)$ be such that the Boolean expression $(p \cdot \sim q) \rightarrow (p \square q)$ is a tautology. Then :

A. $* = \wedge, \ \square = \vee$

B. $* = \vee, \ \square = \vee$

C. $* = \wedge, \ \square = \wedge$

D. $* = \vee, \ \square = \wedge$

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45. If $p \& q$ are lengths of perpendicular from the origin $x \sin \alpha + y \cos \alpha = a \sin \alpha \cos \alpha$ and $x \cos \alpha - y \sin \alpha = a \cos 2\alpha$, then $4p^2 + q^2$

A. $p^2 + 2q^2$

B. $4p^2 + q^2$

C. $p^2 + 4q^2$

D. $2p^2 + q^2$

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46. $\underset{x \to 0}{I\ im}\left(\frac{\sin^2(\pi\cos^4 x)}{x^4}\right)$ is equal to .

A. 4π

B. $4\pi^2$

C. $2\pi^2$

D. π^2

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47. Let f be a non-negative function in [0, 1] and twice differentiate in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt, 0 \le x \le 1$ and $f(0) = 0$ then the value of $\lim_{x \to 0} \int_0^x \frac{f(t)}{x^2}dt$ is

A. equals $\frac{1}{2}$

B. does not exist

C. equals 0

D. equals 1

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48. Let the equation of the plane, that passes through the point (1, 4, - 3) and contains the line of intersection of the planes 3x - 2y + 4z -7 = 0 and x + 5y - 2z + 9 = 0, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

A. -23

B. -15

C. 15

D. 23

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49. The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots \text{ is}$$

A. $\frac{120}{121}$

B. 1

C. $\frac{99}{100}$

D. $\frac{143}{144}$

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50. If the function f(x)= $\begin{cases} \frac{1}{x} \frac{\log_e\left(1+\frac{x}{a}\right)}{1-\frac{x}{b}} & x < 0 \\ k & x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & x > 0 \end{cases}$

is continuous at x=0 then $\frac{1}{a} + \frac{1}{b} + \frac{4}{k}$ is equal to

A. 5

B. -4

C. 4

D. -5

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51. If $a^r = (\cos 2r\pi + I\sin 2r\pi)^{1/9}$, then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a^6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

A. $a_1a_9 - a_3a_7$

B. $a_2a_6 - a_4a_8$

C. a_9

D. a_5

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52. The function f(x)=$\left|x^2-2x-3\right|.\,e^{\left|9x^2-12x+4\right|}$ is not differentiable at exactly

A. three points

B. one point

C. four points

D. two points

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53. If the following system of linear equations

2x + y + z = 5

x - y +z = 3

x + y + az = b

has no solution, then :

A. $a \neq \frac{1}{3}, b = \frac{7}{3}$

B. $a = -\frac{1}{3}, b = \frac{7}{3}$

C. $\neq = -\frac{7}{3}, b = \frac{7}{3}$

D. $a = \frac{1}{3}, b \neq \frac{7}{3}$

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54. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^{x} + 1 = 0$ is :

A. 2

B. 0

C. 4

D. 1

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55. $\cos ec18^{\circ}$ is a root of the equation :

A. $4x^2 + 2x - 1 = 0$

B. $x^2 + 2x - 4 = 0$

C. $x^2 - 2x + 4 = 0$

D. $x^2 - 2x - 4 = 0$

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56. The integral $\int \frac{1}{4\sqrt{(x-1)^3(x+2)^5}}$ dx is equal to

(where c is a constant of integration)

A. $\frac{4}{3}\frac{x-1}{(x+2)^{\frac{5}{4}}} + C$

B. $\frac{4}{3}\frac{x-1}{(x+2)^{\frac{1}{4}}} + C$

C. $\frac{3}{4}\frac{x+1}{(x-2)^{\frac{1}{4}}} + C$

D. $\frac{3}{4}\frac{x+2}{(x-1)^{\frac{1}{4}}} + C$

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57. P is parabola, whose vertex and focus are on the positive x axis at distances a and a' from the origin respectively, then $(a' > a)$. Length of latus ractum of P will be

A. 2(S-R)

B. 4(S+R)

C. 2(S+R)

D. 4(S-R)

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58. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then r^2 - d is equal to :

A. $7-7\sqrt{3}$

B. $7+\sqrt{3}$

C. $7+3\sqrt{3}$

D. $7-\sqrt{3}$

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59. Which of the following is not correct for relation R on the set of real numbers ?

A. $(x, y)\varepsilon R \to |x|-|y| \leq 1$ is reflexive but not symmetric

B. $(x, y)\varepsilon R \to 0<|x|-|y| \leq 1$ is neither transitive nor symmetric

C. $(x, y)\varepsilon R \to |x-y| \leq 1$ is reflexive and symmetric

D. $(x, y)\varepsilon R \to \ <|x-y| \leq 1$ is symmetric and transitive

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60. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves ?

A. $144x^2 + 25y^2 = 3600$

B. $25x^2 + 12y^2 = 3600$

C. $x^2 + y^2 = 60$

D. $x^2 + y^2 = 169$

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61. The range of the function

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3}{\cdot}\right.\right.$$

is :

A. $[0, 2]$

B. $(0, \sqrt{5})$

C. $[-2, 2]$

D. $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

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62. The function $f(x) = x^3 - 6x^2 + ax + b$ is such that $f(2) = f(4) = 0$. Consider two statements.

(S1) there exists $x_1, x_2 \in (2, 4), x_1 < x_2,$ such that $f'(x_1) = -1$ and $f'(x_2) = 0$.

(S2) there exists $x_3, x_4 \in (2, 4), x_3 < x_4,$ such that f is decreasing in $(2, x_4)$, increasing in $(x_4, 4)$ and $2f'(x_3) = \sqrt{3}f(x_4)$.

A. (S1) is false and (S2) is true

B. both (S1) and (S2) are true

C. (S1) is true and (S2) is false

D. both (S1) and (S2) are false

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63. Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$ and the directrix $y = \frac{1}{2}$. Let P be the point where the parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q, then $(PQ)^2$ is equal to :

A. $\frac{25}{2}$

B. $\frac{75}{8}$

C. $\frac{125}{16}$

D. $\frac{15}{2}$

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64. Let

$S_n = 1(n-1) + 2.(n-2) + 3.(n-3) + \ldots + (n-1).1, n \geq 4.$

The sum $\sum_{n=4}^{\infty} \left(\frac{2S_n}{n!} - \frac{1}{(n-2)!} \right)$ is equal to :

A. $\frac{e-2}{6}$

B. $\frac{e}{3}$

C. $\frac{e-1}{3}$

D. $\frac{e}{6}$

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65. Let the acute angle bisector of the two planes $x - 2y - 2z + 1 = 0$ and $2x - 3y - 6 + 1 = 0$ be the plane P. Then which of the following points lies on P ?

A. $\left(3, 1, -\frac{1}{2}\right)$

B. $\left(-2, 0, -\frac{1}{2}\right)$

C. $(4, 0, -2)$

D. $(0, 2, -4)$

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66. $\cos^{-1}(\cos(-5)) + \sin^{-1}(\sin(6)) - \tan^{-1}(\tan(12))$ is equal to :

(The inverse trigonometric functions take the principal values)

A. $3\pi - 11$

B. $4\pi - 11$

C. $3\pi + 1$

D. $4\pi - 9$

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67. If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)-1\right)=1, x\in[0,\pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is :

A. $(3, 5\pi/3)$

B. $(3, 13\pi/9)$

C. $(2, 8\pi/9)$

D. $(2, 2\pi/3)$

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68. Which of the following is equivalent to the Boolean expression $P \wedge \sim q$?

A. $\sim p \rightarrow \sim q$

B. $\sim(q \rightarrow p)$

C. $\sim(p \rightarrow q)$

D. $\sim(p \rightarrow \sim q)$

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69. Let $J_{n,m} = \int_0^{1/2} \frac{x^n}{x^m - 1}, \forall n > m$ and $n, m \in N$. Consider a matrix $A = [a_{ij}]_{3 \times 3}$ where

$$a_{ij} = \begin{cases} J_{6+i,3} - J_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$$. Then $|adjA^{-1}|$ is :

A. $(105)^2 \times 2^{38}$

B. $(105)^2 \times 2^{36}$

C. $(15)^2 \times 2^{42}$

D. $(15)^2 \times 2^{34}$

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70. Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

A. $n(S_1) = 0, n(S_2) = 2$

B. $n(S_1) = 2, n(S_2) = 0$

C. $n(S_1) = 2, n(S_2) = 2$

D. $n(S_1) = 1, n(S_2) = 0$

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71. The number of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0, \alpha^2 - 2$ is also a root of this equation, is :

A. 4

B. 6

C. 8

D. 2

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72. If y = y (x) is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right)dx = 0, x > 0, \text{ and } y(1) = 1, \text{ then } y\left(\frac{1}{2}\right)$ is equal to

A. $3 + e$

B. $\frac{3}{2} - \frac{1}{\sqrt{e}}$

C. $3 - e$

D. $3 + \frac{1}{\sqrt{e}}$

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73. The function f(x), that satisfies the condition $f(x) = x + \int_0^{\pi/2} \sin x . \cos y f(y) dy$, is :

A. $x + \frac{\pi}{2} \sin x$

B. $x + (\pi - 2) \sin x$

C. $x + \frac{2}{3}(\pi - 2) \sin x$

D. $x + (\pi + 2) \sin x$

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74. Let θ be the acute angle between the tangents to the ellipse $\frac{x^2}{9}+\frac{y^2}{1}=1$ and the circle $x^2+y^2=3$ at their point of intersection in the first quadrat. Then $\tan\theta$ is equal to :

A. $\frac{4}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{5}{2\sqrt{3}}$

D. 2

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75. The area enclosed by the curves $y=\sin x+\cos x$ and $y=|\cos x-\sin x|$ over the interval $\left[0,\frac{\pi}{2}\right]$

A. $4(\sqrt{2}-1)$

B. $2\sqrt{2}(\sqrt{2}-1)$

C. $2\sqrt{2+1}$

D. $2\sqrt{2}\left(\sqrt{2}+1\right)$

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76. Let $a_1, a_2, \ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. if the sum of this AP is 189, then $a_6 a_{16}$ is equal to :

A. 36

B. 48

C. 72

D. 57

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77. Let $P_1, P_2 \ldots, P_{15}$ be 15 points on a circle. The number of distinct triangles formed by points P_i, P_j, P_k such that $i + j + k \neq 15$, is :

A. 12

B. 455

C. 443

D. 419

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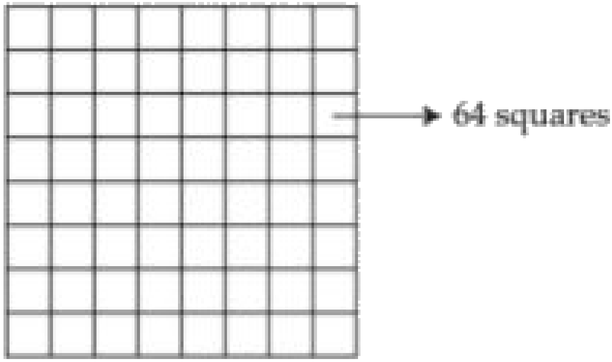
78. Two squares are chosen at random on a chessboard (see figure). The probability that they have a side in common is :



A. $\frac{1}{9}$

B. $\frac{1}{18}$

C. $\frac{2}{7}$

D. $\frac{1}{7}$

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79. Let f: $R \rightarrow R$ be a continuous function. Then $\lim_{x \rightarrow \pi/4} \frac{\frac{\pi}{4}\int_2^{\sec^2 x} f(x)dx}{x^2 - \frac{\pi^2}{16}}$ is equal to :

A. $f(2)$

B. $4f(2)$

C. $2f(\sqrt{2})$

D. $2f(2)$

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80. The distance of line $3y - 2z - 1 = 0 = 3x - z + 4$ from the point (2, -1, 6) is :

A. $2\sqrt{6}$

B. $2\sqrt{5}$

C. $\sqrt{26}$

D. $4\sqrt{2}$

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Mathematics (Section B)

1. Let the equation $x^2 + y^2 + px + (1-p)y = 0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S = \{q : q = p^2$ and q is an integer $\}$ is ________.

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2. If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to ________

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3. Let n be an odd natural number such that the variance of 1,2,3,4,...., n is 14. Then n is equal to ________.

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4. If the system of linear equations

$2x + y - z = 3$

$x - y - z = \alpha$ has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is

$3x + 33y - \beta z = 3$

equal to _____.

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5. If $A = \{x \in R : |x - 2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2 - 3} > 1\right\}$, $C = \{x \in R : |x - 4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is ________.

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6. If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to __________.

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7. A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55 is______.

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8. Let $\overrightarrow{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}, \overrightarrow{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\overrightarrow{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $\left|\overrightarrow{b} \times \overrightarrow{c}\right| = 5\sqrt{3}$ and $\overrightarrow{a}$ is perpendicular to $\overrightarrow{b}$. Then the greatest amongst the values of $\left|\overrightarrow{a}\right|^2$ is __________.

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9. If $\int \frac{dx}{\left(x^2+x+1\right)^2} = a\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b\left(\frac{2x+1}{x^2+x+1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9\left(\sqrt{3}a + b\right)$ is equal to ________.

Watch Video Solution

10. The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is ________.

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11. The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$, is a positive integer,is _____

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12. Let the mean and variance of four numbers 3, 7, x and y $(x > y)$ be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x - y is ___ _

Watch Video Solution

13. Let $a_1, a_2, \ldots, a_{10}$ be an AP with common difference - 3 and $b_1, b_2, \ldots$, b_{10} be a GP with 10 common ratio 2. Let $c_k = a_k + b_k, k = 1, 2, \ldots, 10$. If

$c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} ck$ is equal to ____

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14. Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of the equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to ________

Watch Video Solution

15. Let A be a 3×3 real matrix. If det(2 Adj(2 Adj(Adj (2A)))) = 2^{41}, then the value of det (A^2) equals ______

Watch Video Solution

16. Let Q be the foot of the perpendicular from the point P(7, - 2, 13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and

$$\frac{x-1}{3}=\frac{y-2}{5}=\frac{z-3}{7}.$$

Then $(PQ)^2$, is equal to ____

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17. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is ___________.

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18. Let $\binom{n}{k}$ denote ${}^nC_k$ and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise.} \end{cases}$$

If $A_k = \sum_{i=0}^{9} \binom{9}{i} \left[\begin{array}{c} 12 \\ 12-k+i \end{array} \right] + \sum_{i=0}^{8} \binom{8}{i} \left[\begin{array}{c} 13 \\ 13-k+i \end{array} \right]$ and $A_4 - A_3 = 190p$,then p is equal to

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19. If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to ______

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20. Let a and b respectively be the points of local maximum and local minimum of the function

$f(x) = 2x^3 - 3x^2 - 12x.$

If A is the total area of the region bounded by y = f (x), the x-axis and the lines x = a and x = b, then 4 A is equal to ________.

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21. If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}} dx = \frac{1}{14}\left(ux + v\log_e\left(4e^x + 7e^{-x}\right)\right) + C$, where C is a constant of integration, then $u + v$ is equal to ______.

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22. Let S={1, 2,3, 4,5, 6, 9}. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3} is _________.

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23. The probability distribution of random variable X is given by :

X	1	2	3	4	5
P(X)	K	2K	2K	3K	K

Let $p = P(1 < X < 4 | X < 3)$. If $5p = \lambda K$, then λ is equal to _______.

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24. Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to _______.

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25. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder _______.

Watch Video Solution

26. Two circles each of radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is $4x + 3y = 10$, and $C_1(\alpha, \beta)$ and $C_2(\gamma, \delta), C_1 \neq C_2$ are their centres, then $|(\alpha + \beta)(\gamma + \delta)|$ is equal to ______.

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27. An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to ________.

Watch Video Solution

28. Let $A(\sec\theta, 2\tan\theta)$ and $B(\sec\phi, 2\tan\phi)$, where $\theta + \phi = \pi/2$ be two points on the hyperbola $2x^2 - y^2 = 2$. If (α, β) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ________.

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29. Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane $2x - y + z + 3 = 0$ and let R $(3, 5, \gamma)$ be a point of this plane. Then the square of the length of the line segment SR is ________.

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30. Let z_1 and z_2 be two complex numbers such that arg $(z_1 - z_2) = \frac{\pi}{4}$ and z_1, z_2 satisfy the equation $|z - 3| = Re(z)$. Then the imaginary part of $z_1 + z_2$ is equal to _______.

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31. The mean of 10 number

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14...$ is _____

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32. Let [t] denote the greatest integer $\leq$ t. Then the value of 8

$\int_{-\frac{1}{2}}^{1} [2x] + |x| dx$ is __________

Watch Video Solution

33. If yR' is the least value of 'a' such that the function f(x) = $x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of ' a' such that the function f(x) = $x^2 + ax + 1$ is decreasing on [1, 2], then the value of |R — S| is___________

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34. If $\frac{3^6}{4^4}$ k is the term, independent of x in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$ then k is equal to __________

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35. An instrument consists of two units. Each unit must function for the instrument to operate.The reliability of the first unit is 0.9 and that of the second unit is 0.8. The instrument is tested & fails. The probability that only the first unit failed & the second unit is sound is

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36. If $x\phi(x) = \int_5^x \left(3t^2 - 2\phi(t)\right)dt,\ x > -2$ and $\phi(0) = 4$ then $\phi(2)$ is __________

Watch Video Solution

37. A point z moves in the complex plane such that art $\frac{z-2}{z+2} = \frac{\pi}{4}$ then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to ___________

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38. The square of the distance of the point of intersection of the line and the plane $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane 2x-y+z=6 from the point (- 1, -1, 2) is________

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39. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is__________

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40. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is________

Watch Video Solution

41. Let the points of intersections of the lines $x - y + 1 = 0, x - 2y + 3 = 0$ and $2x - 5y + 11 = 0$are the mid points of the sides of a triangle ABC. Then the area of the triangle ABC is ______ .

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Watch Video Solution

42. If the sum of the coefficients in the expansion of $(x+y)^n$ is 4096, then the greatest coefficient in the expansion is _______.

Watch Video Solution

43. Let X be a random variable with distribution.

x	-2	-1	3	4	6
$P(X=x)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is $\sigma^2, \text{ then } 100\sigma^2$ is equal to :

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44. Let [t] denote the greatest integer $< t$. The number of points where the function

$f(x) = [x]|x^2 - 1| + \sin\left(\frac{\pi}{[x] + 3}\right) - [x + 1], x \in (-2, 2)$ is not continuous is ________.

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45. All the arrangements, with or without meaning, of the word FARMER are written excluding any word that has two R appearing together. The arrangements are listed serially in the alphabetic order as in the English dictionary. Then the serial number of the word FARMER in this list is ________ .

Watch Video Solution

46. If for the complex numbers z satisfying $|z - 2 - 2i| < 1$, the maximum value of $|3iz + 6|$ is attained at $a + ib$, then $a + b$ is equal to _______.

Watch Video Solution

47. Let $\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{i} + 2\hat{j} - \hat{k}$. Let a vector $\overrightarrow{v}$ be in the plane containing $\overrightarrow{a}$ and $\overrightarrow{b}$. If $\overrightarrow{v}$ is perpendicular to the vector $3\hat{i} + 2\hat{j} - \hat{k}$ and its projection on $\overrightarrow{a}$ is 19 units, then $\left|2\overrightarrow{v}\right|^2$ is equal to ________.

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48. Let $f(x) = x^6 + 2x^4 + x^3 + 2x + 3, x \in R$. Then the n natural number for which $\lim_{x \to 1} \frac{x^n f(1) - f(x)}{x - 1} = 44$ is ______.

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49. Let f(x) be a polynomial of degree 3 such that $f(k) = -\frac{2}{k}$ for $k = 2, 3, 4, 5$. Then the value of $52 - 10f(10)$ is equal to ________.

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50. A man starts walking from the point P(-3, 4), touches the x-axis at R, and then turns to reach at the point Q(0, 2). The man is walking at a constant speed. If the man reaches the point Q in the minimum time, then $50\left((PR)^2 + (RQ)^2\right)$ is equal to ________ .

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Mathematics (Section A)

1. Consider the two statements :

$(S1): (p \rightarrow q) \vee (\sim q \rightarrow p)$ is a tautology

$(S2): (p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy

Then :

A. both (S1) and (S2) are true.

B. both (S1) and (S2) are false.

C. only (S1) is true.

D. only (S2) is true.

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2. A circle C touches the line x = 2y at the point (2, 1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5$= 0 at two points P and Q such that PQ is a diameter of C^1. Then the diameter of C is :

A. $7\sqrt{5}$

B. $4\sqrt{15}$

C. $\sqrt{285}$

D. 15

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3. The locus of the mid-point of the chords of the hyperbola $x^2 - y^2 = 4$, that touches the parabola $y^2 = 8x$ is

A. $y^3(x-2)=x^2$

B. $x^2(x-2)=y^3$

C. $x^3(x-2)=y^2$

D. $y^2(x-2)=x^3$

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4. The domain of the function $\cos ec^{-1}\left(\frac{1+x}{x}\right)$ is :

A. $\left(-\frac{1}{2},\infty\right)-\{0\}$

B. $\left[-\frac{1}{2},0\right)\cup[1,\infty)$

C. $\left[-\frac{1}{2},\infty\right)-\{0\}$

D. $\left(-1,-\frac{1}{2}\right]\cup(0,\infty)$

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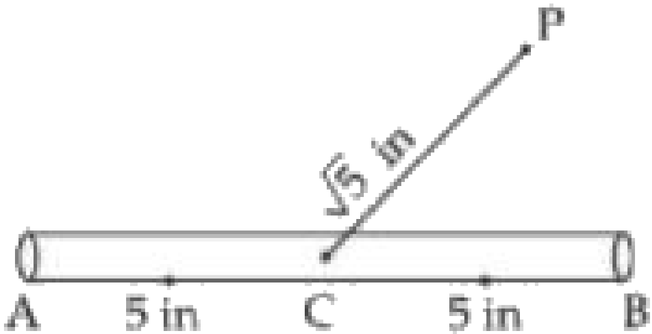
5. A 10 inches long pencil AB with mid pointC and a small eraser P are placed on the horizontal top of a table such that PC = $\sqrt{5}$ inches and $\angle$ PCB = $\tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :



A. $\tan^{-1}(1)$

B. $\tan^{-1}\left(\frac{3}{5}\right)$

C. $\tan^{-1}\left(\frac{4}{3}\right)$

D. $\tan^{-1}\left(\frac{1}{2}\right)$

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6. $\dfrac{\lim}{x \to 2}\left(\dfrac{\sum^{9}}{n=1}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$

A. $\dfrac{7}{36}$

B. $\dfrac{1}{5}$

C. $\dfrac{5}{24}$

D. $\dfrac{9}{44}$

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7. Let P be the plane passing through the point (1, 2, 3) and the line of intersection of the planes

$\overrightarrow{r}.\left(\hat{i}+\hat{j}+4\hat{k}\right)=16 \text{ and } \hat{r}.\left(-\hat{\imath}+\hat{j}+\hat{k}\right)=6.$

Then which of the following points does NOT lie on P ?

A. (4,2,2)

B. (-8,8,6)

C. (3,3,2)

D. (6,-6,2)

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8. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$,where $\alpha, \beta \in R, 5\alpha + 6\beta = 0, \text{ and } [x]$denotes the greatest integer less than or equal to x, then the value of $(\alpha + \beta)^2$ is equal to :

A. 16

B. 25

C. 36

D. 100

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9. Let [t] denote the greatest integer less than or equal to t.

Let f (x) = x-[x], g(x) = 1-x+[x], and h(x) = min{f(x), g(x)}, $x \in [-2, 2]$.

Then h is :

A. not continuous at exactly four points in [-2,2]

B. Continous in [- 2, 2] but not differentiable at exactly three poionts in (- 2, 2)

C. not continuous at exactly thr'ee points in [- 2,2]

D. continuous in [- 2, 2] but not differentiable at more than four points in (- 2, 2)

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10. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0,$ x>0. If y(e) = 1, then y(1) is equal to :

A. $\log_e(2e)$

B. $\log_e 2$

C. 2

D. 0

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11. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$ is :

A. $\frac{1}{4\sqrt{2}}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{8}\sqrt{2}$

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12. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)$ dx is

A. $\frac{5\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

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13. Let A = $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 1 & 0 & 0\end{pmatrix}$. Then $A^{2025}-A^{2020}$ is equal to :

A. A^5-A

B. A^5

C. A^6-A

D. A^6

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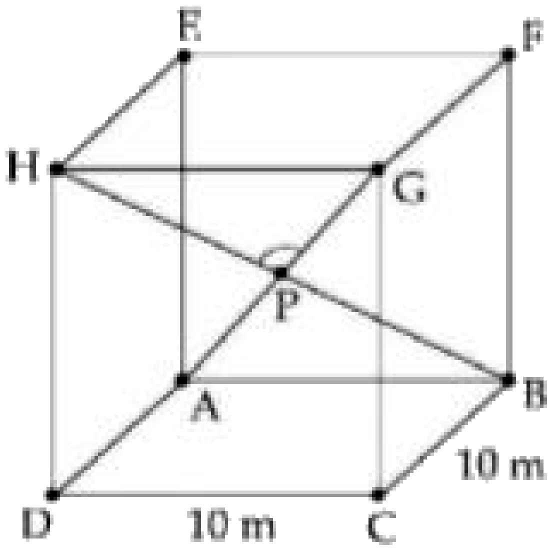
14. A hall has a square floor of dimension 10m $\times$ 10 m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\left(\frac{1}{5}\right)$, then the height of th e hall (in meters) is :



A. $5\sqrt{3}$

B. 5

C. $2\sqrt{10}$

D. $5\sqrt{2}$

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15. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to :

A. $6\sqrt{3}$

B. $3\sqrt{6}$

C. $4\sqrt{3}$

D. 6

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16. The local maximum value of the function

$f(x) = \left(\frac{2}{x}\right)^{x^2}$, $x > 0$, is

A. $((e))^{\frac{2}{e}}$

B. 1

C. $(2\sqrt{e})^{\frac{1}{e}}$

D. $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

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17. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5X \mid > 2)$ is :

A. $\frac{25}{36}$

B. $\frac{125}{216}$

C. $\frac{11}{36}$

D. $\frac{5}{6}$

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18. If $\sum_{r=1}^{50} \frac{\tan^{-1} 1}{2r^2} = p$, then the value of tan p is:

A. 100

B. $\frac{50}{51}$

C. $\frac{101}{102}$

D. $\frac{51}{50}$

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19. If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation:

A. $x^2 - \left(\sqrt{3}-1\right)x - \sqrt{3} = 0$

B. $x^2+(\sqrt{3}+1)x+\sqrt{3}=0$

C. $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

D. $x^2+(\sqrt{3}-1)x-\sqrt{3}=0$

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20. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

x+ y+z =5

x+2y +3z= μ

x + 3y+λz = 1

is constructed. If p is the probability that the system. has a unique solution and q is the probability that the system has no solution, then :

A. $p=\frac{5}{6}$ and $q=\frac{1}{36}$

B. $p=\frac{5}{6}$ and $q=\frac{5}{36}$

C. $p=\frac{1}{6}$ and $q=\frac{5}{36}$

D. $p = \frac{1}{6}$ and $q = \frac{1}{36}$

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