MATHS

BOOKS - JEE ADVANCED PREVIOUS YEAR

JEE ADVANCED 2021

Question

1. Consider a triangle Δ whose two sides lies on the x-axis and the line $x+y+1=0$. If the orthocenter of Δ is (1,1) , then the equation of the circle passing through the vertices of the triangle is

A. $x^2+y^2-3x+y=0$

B. $x^2+y^2+x+3y=0$

C. $x^2+y^2+2y-1=0$

D. $x^2 + y^2 + x + y = 0$

Answer:

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2. The area of the region $\left\{(x, y): 0 \leq x \leq \frac{9}{4}, 0 \leq y \leq 1, x \geq 3y, x + y \geq 2\right\}$ is

A. $\frac{11}{32}$

B. $\frac{35}{96}$

C. $\frac{37}{96}$

D. $\frac{13}{32}$

Answer:

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3. Consider three sets $E_1 = \{1, 2, 3\}, F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1, and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement, from the set G_2 and let S_3 denote the set of these chosen elements. Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is

A. $\frac{1}{5}$

B. $\frac{3}{5}$

C. $\frac{1}{2}$

D. $\frac{2}{5}$

Answer:

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4. Let $\theta_1, \theta_2, \ldots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \ldots + \theta_{10} = 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}, z_k = z_{k-1}e^{i\theta_k}$ for $k = 2, 3, \ldots, 10$, where $i = \sqrt{-1}$.

Consider the statements ? and ? given below:

$P: |z_2 - z_1| + |z_3 - z_2| + \ldots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

$Q: |z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \ldots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$

Then

A. P is TRUE and Q id FALSE

B. Q is TRUE and P id FALSE

C. both P and Q are TRUE

D. both P and Q are FALSE

Answer:

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5. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let P_1 be the probability that the maximum of chosen numbers is at least 81 and P_2 be the probability that the minimum of chosen numbers is at most 40.

then the vaue of $\frac{625}{4} P_1$ is

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6. Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \ldots, 100\}$. Let P_1 be the probability that the maximum of chosen numbers is at least 81 and P_2 be the probability that the minimum of chosen numbers is at most 40.

then the vaue of $\frac{125}{4} P_2$ is

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7. Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

The value of $|M|$ is _____

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8. Let α, β and γ be real numbers such that the system of linear equations

$x + 2y + 3z = \alpha$

$4x + 5y + 6z = \beta$

$7x + 8y + 9z = \gamma - 1$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P.

The value of D is ______

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9. Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S' . Let D be the square of the distance between R' and S'.

The value of λ^2 is

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10. Consider the lines L_1 and L_2 defined by

$L_1 : x\sqrt{2} + y - 1 = 0$ and $L_2 : x\sqrt{2} - y + 1 = 0$

For a fixed constant λ, let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2. The line $y = 2x + 1$ meets C at two points R and S, where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S' . Let D be the square of the distance between R' and S'.

The value of D is

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11. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let

$$E = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix}, P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ and } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3, then which of the following statements is (are) TRUE ?

A. $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\left|EQ + PFQ^{-1}\right| = |EQ| + \left|PFQ^{-1}\right|$

C. $\left|(EF)^3\right| > |EF|^2$

D. Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Answer:

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12. Let $f: R \rightarrow R$ be definded by

$$f(x) = \frac{x^2 - 3x - 6}{x^2 + 2x + 4}$$

Then which of the following statements is(are) TRUE?

A. f is decreasing in the interval $(-2, -1)$

B. f is increasing in the interval (1,2)

C. f is onto

D. Range of f is $\left[-\frac{3}{2}, 2\right]$

Answer:

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13. Let E,F and G be three events having probabilities

$P(E) = \frac{1}{8}, P(F) = \frac{1}{6}$ and $P(G) = \frac{1}{4}$, and let

$P(E \cap F \cap G) = \frac{1}{10}$

For any event H, if H^c denotes its complement, then which of the following statements is (are) TRUE ?

A. $P(E \cap F \cap G^c) \leq \frac{1}{40}$

B. $P(E^c \cap F \cap G) \leq \frac{1}{15}$

C. $P(E \cup F \cup G) \leq \frac{13}{24}$

D. $P(E^c \cap F^c \cap G^c) \leq \frac{5}{12}$

Answer:

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14. For any 3×3 matrix M, let $|M|$ denote the determinant of M. Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is (are) TRUE ?

A. $|FE| = |I - FE||FGE|$

B. $(1 - FE)(1 + FGE) = I$

C. $EFG = GEF$

D. $(I - FE)(I - FGE) = I$

Answer:

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15. For any positive integer n . let $S_n : (0, \infty) \to R$ be defined by

$$S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1 + k(k+1)x^2}{x}\right)$$

where for any $x \in R, \cot^{-1} x \in (0, \pi)$ and $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.Then which of the following statement is(are TRUE ?

A. $S_{10}(x) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1 + 11x^2}{10x}\right)$ for all $x > 0$

B. $\lim_{n \to \infty} \cot(S_n(x)) = x$ for all $x > 0$

C. The equation $S_3(x) = \frac{\pi}{4}$ has a root in $(0, \infty)$

D. $\tan(S_n(x)) \leq \frac{1}{2}$ for all $n \geq 1$ and $x \geq 0$

Answer:

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16. For any complex number $w = c + id$ let $arg(w) \in (-\pi, \pi], wherei = \sqrt{-1}$. Let α and β be real number such that all complex number $z = x + iy$ satisfying $arg\left(\frac{z+\alpha}{z+\beta}\right) = \frac{\pi}{4}$, the ordered pair (x,y) lies on the circle $\left(x^2 + y^2 + 5x - 3y + 4 = 0\right)$

Then which of the following statement is(are) TRUE?

A. $\alpha = -1$

B. $\alpha . \beta = 4$

C. $\alpha . \beta = -4$

D. $\beta = 4$

Answer:

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17. For $x \in R$ the number of real roots of the equation

$3x^2 - 4\left|x^2 - 1\right| + x - 1 = 0$ is ______

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18. In a triangle ABC, let $AB = \sqrt{23}$, $BC = 4$ and $CA = 5$. Then the value of $\frac{\cot A + \cot B}{\cot C}$ is

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19. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors in three-dimensional space, where $\vec{u}$ and $\vec{v}$ are unit vectors which are not perpendicular to each other and $\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1, \vec{w} \cdot \vec{w} = 4$

If the volume of the parallelepiped, whose adjacent sides are represented by the vectors $\vec{u}, \vec{v}$ and $\vec{w} is \sqrt{2}$ then the value of $\left|3u + 5\vec{v}\right|$ is

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20. Let $s_1 = \{(i, j, k) : i, j, k \in \{1, 2, \ldots, 10\}\}$

$, s_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1, 2, \ldots, 10\}\},$

$s_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1, 2, \ldots, 10\}\}$

and

$s_4 = \{(i, j, k, l) : i, j, k$ and l are distinct elements in $\{1, 2, \ldots, 10\}\}$. If the total number of elements in the set s_r is $n_r, r = 1, 2, 3, 4,$ then which of the following statements is (are) TRUE ?

A. n_1 = 1000

B. n_2 = 44

C. n_3 = 220

D. $\frac{n_4}{12}$ = 420

Answer:

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21. Consider a triangle PQR having sides of lengths p,q,r opposite to the angles P,Q,R respectively. Then which of the following statements is (are) true?

A. $\cos P \geq 1 - \frac{p^2}{2qr}$

B. $\cos R \geq \left(\frac{q-r}{p+q}\right)\cos P + \left(\frac{p-r}{p+q}\right)\cos Q$

C. $\frac{q+r}{p} < 2\frac{\sqrt{\sin Q \sin R}}{\sin P}$

D. if $p < q$ and $p < r$, then $\cos Q > \frac{p}{r}$ and $\cos R > \frac{p}{q}$

Answer:

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22. Let $f:\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R$ be a continuous function such that $f(0) = 1$ and $\int_0^{\frac{\pi}{3}} f(t)dt = 0$

Then which of the following statement is(are) TRUE?

A. The equation $f(x) - 3\cos 3x = 0$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

B. The equation $f(x) - 3\cos 3x = -\frac{6}{\pi}$ has at least one solution in $\left(0, \frac{\pi}{3}\right)$

C. $\lim_{x \to 0} x \frac{\int_0^x f(t)dt}{1 - e^{x^2}} = -1$

D. $\lim_{x \to 0} \frac{\sin x \left(\int_0^x f(t)dt\right)}{x^2} = -1$

Answer:

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23. For any real number α and β , let $y_{\alpha,\beta}(x) : \alpha, \beta \in R\}$ Then which of the following functions belong(s) to the set S ?

A. $f(x) = \frac{x^2}{2}e^{-x} + \left(e - \frac{1}{2}\right)e^{-x}$

B. $f(x) = -\frac{x^2}{2}e^{-x} + \left(e + \frac{1}{2}\right)e^{-x}$

C. $f(x) = \frac{e^x}{2}\left(x - \frac{1}{2}\right) + \left(e - \frac{e^2}{4}\right)e^{-x}$

D. $f(x) = \frac{e^x}{2}\left(\frac{1}{2} - x\right) + \left(e + \frac{e^2}{4}\right)e^{-x}$

Answer:

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24. Let O be the origin and $\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + \hat{k}, \overrightarrow{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}\left(\overrightarrow{OB} - \lambda\overrightarrow{OA}\right)$ for same $\lambda > 0. If \left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \frac{9}{2}$, then which of the following is(are) TRUE ?

A. Projection of $\overrightarrow{OC} on \overrightarrow{OA}$ is $-\frac{3}{2}$

B. Area of triangle OAB is $\frac{9}{2}$

C. Area of triangle ABC is $\frac{9}{2}$

D. The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{t}(OA)$ and $\overrightarrow{OC} is \frac{\pi}{3}$

Answer:

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25. Let E denote the parabola $y^2 = 8x$. Let $P = (-2, 4)$, and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE ?

A. The triangle PFQ is a right-angled triangle

B. The triangle QPQ' is a right-angled triangle

C. The distance between P and F is $5\sqrt{2}$

D. F lies on the line joining Q and Q`

Answer:

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26. Consider the region $R = \{(x, y) \in R \times R : x \geq 0 \text{ and } y^2 \leq (4 - x)\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

The radius of the circle C is ___

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27. Consider the region $R = \{(x, y) \in R \times R : x \geq 0 \text{ and } y^2 \leq (4 - x)\}$. Let F be the family of all circles that are contained in R and have centers on the x-axis. Let C be the circle that has largest radius among the circles in F Let (α, β) be a point where the circle C meets the curve $y^2 = 4 - x$.

The value of α is ___ .

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28. Let $f_1 : (0, \infty) \to R$ and $f_2 : (0, \infty) \to R$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21} \left((t - j)^j\right) dt, x > 0$ and $f_2(x) = 98(x - 1)^{50} - 600(x - 1)^{49} + 2450, x > 0$, where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n, \prod_{i=1}^{n} (a_i)$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of

points of local maxima of function $f_i, i = 1, 2,$ in the interval $(0, \infty)$

The value of $2m_1 + 3n_1 + m_1n_1$ is ___.

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29. Let $f_1 : (0, \infty) \to R$ and $f_2 : (0, \infty) \to R$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21} \left((t-j)^j\right) dt, x > 0$ and $f_2(x) = 98(x-1)^{50} - 600(x-1)^{49} + 2450, x > 0$, where, for any positive integer n and real numbers $a_1, a_2, \ldots, a_n, \prod_{i=1}^{n} (a_i)$ denotes the product of $a_1, a_2, \ldots, a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function $f_i, i = 1, 2,$ in the interval $(0, \infty)$

The value of $6m_2 + 4n_2 + 8m_2n_2$ is ___.

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30. Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R, i = 1, 2,$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be function such that

$g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$ for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$ Define

$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).\, g_i(x) dx, i = 1, 2$

The value of $\frac{16 S_1}{\pi}$ is

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31. Let $g_i : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R, i = 1, 2,$ and $f : \left[\frac{\pi}{8}, \frac{3\pi}{8}\right] \to R$ be function such that

$g_1(x) = 1, g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$ for all $x \in \left[\frac{\pi}{8}, \frac{3\pi}{8}\right]$ Define

$S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).\, g_i(x) dx, i = 1, 2$

The value of $\frac{48 S_2}{\pi}$ is

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32. Consider M with $r = \frac{1025}{513}$. Let k be the number of all those circle C_n that are inside M. Let l be the maximum possible number of circle amaong these k circles such that no two circle intersect .Then

A. $k + 2l = 22$

B. $2k + l = 26$

C. $2k + 3l = 34$

D. $3k + 2l = 40$

Answer:

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33. Consider M with $r = \frac{(2^{199}-1)\sqrt{2}}{2^{198}}$. The number of all those circle D_n that are inside M is

A. 198

B. 199

C. 200

D. 201

Answer:

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34. Let

$\psi_1 : [0, \infty] \to R, \psi_2 : [0, \infty) \to R, f : [0, \infty) \to R$ and $g : [0, \infty) \to R$

be functions such that $f(0) = g(0) = 0$

$\psi_1(x) = e^{-x} + x, x \geq 0$,

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2} dt, x > 0$

$g(x) = \int_{0}^{x^2} (\sqrt{t})e^{-t} dt, x > o$

Which of the following statements is TRUE

A. $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$

B. For every $x > 1$ there exists an $\alpha \in (1, x)$ such that

$\psi_1(x) = 1 + ax$

C. For every $x > 0$ there exists $\alpha\beta \in (0, x)$ such that

$\psi_2(x) = 2x(\psi_1(\beta) - 1)$

D. f is an increasing function on the interval $\left[0, \frac{3}{2}\right]$

Answer:

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35. Let

$\psi_1 : [0, \infty] \to R, \psi_2 : [0, \infty) \to R, f : [0, \infty) \to R$ and $g : [0, \infty) \to R$

be functions such that $f(0) = g(0) = 0$

$\psi_1(x) = e^{-x} + x, x \geq 0$,

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$f(x) = \int_{-x}^{x} (|t| - t^2) e^{-t^2} dt, x > 0$

$g(x) = \int_0^{x^2} (\sqrt{t}) e^{-t} dt, x > o$

Which of the following statements is TRUE

A. $\psi_1(x) \leq 1$ for all $x > 1$

B. $\psi_2(x) \leq 0$ for all $x > 1$

C. $f(x) \geq 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all $x \in \left(0, \frac{1}{2}\right)$

D. $g(x) \leq \frac{2}{3}x^3 - \frac{2}{5}x^5 + \frac{1}{7}x^7$, for all $x \in \left(0, \frac{1}{2}\right)$

Answer:

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36. A number is chosen at random from the set $\{1, 2, 3, \dots 2000\}$ Let p be the probability that the chosen number is a multiple of 3 or a multiple of 7 Then the value of $500p$ is

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37. Let E be the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. For For any three distinct points P, Q and Q' on E, let $M(P, Q)$ be the mid-point of the line segment joining P and Q, and $M(P, Q')$ be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between $M(P, Q)$ and $M(P, Q')$, as P, Q and Q' vary on E, is ___ .

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38. For any real number x, let $[x]$ denote the largest integer less than or equal to x. If

$\int_0^{10}\left[\sqrt{\frac{10x}{x+1}}\right]dx$ then the value of $9I$ is

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