MATHS

BOOKS - JEE ADVANCED PREVIOUS YEAR

MOCK TEST 2022

Question

1. if z is a complex number belonging to the set $S = \{z : |z - 2 + i| \geq \sqrt{5}\}$ and $z_0 \in S$ such that $\frac{1}{|z_0 - 1|}$ is maximum then arg $\left(\frac{4 - z_0 - \bar{z}_0}{z_0 - \bar{z}_0 + 2i}\right)$ is

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{4}$

Answer:

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2. $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ ar real numbers and I is an identity matric of 2×2

if $\alpha^* =$ min of set $\{\alpha(\theta) : \theta \in [0.2\pi)\}$

and $\beta^* =$ min of set $\{\beta(\theta) : \theta \in [0.2\pi)\}$

Then value of $\alpha^* + \beta^*$ is

A. $-\frac{37}{16}$

B. $-\frac{31}{16}$

C. $-\frac{29}{16}$

D. $-\frac{17}{16}$

Answer:

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3. A line y=mx+1 meets the circle $(x-3)^2+(y+2)^2=25$ at point P and Q. if mid point of PQ has abscissa of $-\frac{3}{5}$ then value of m satisfies

A. $-3 \leq m < -1$

B. $2 \leq m < 4$

C. $4 \leq m < 6$

D. $6 \leq m < 8$

Answer:

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4. The area of the region $\{(x, y): xy \leq 8, 1 \leq y \leq x^2\}$ is

A. $16 \log_e 2 - \frac{14}{3}$

B. $8 \log_e 2 - \frac{14}{3}$

C. $16 \log_e 2 - 6$

D. $8 \log_e 2 - \frac{7}{3}$

Answer:

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5. Let α and β be the roots of $x^2 - x - 1 = 0$, with $\alpha > \beta$.

For all positive integers n , define $a_n = \dfrac{\alpha^n - \beta^n}{\alpha - \beta}, n \geq 1$

$b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \geq 2.$

Then which of the following options is/are correct ?

A. $a_1 + a_2 + a_3 + \ldots.. + a_n = a_{n+2} - 1$ for all $n \geq 1$

B. $\sum_{n=1}^{\infty} \dfrac{a_n}{10^n} = \dfrac{10}{89}$

C. $b_n = \alpha^n + \beta^n$ for all $n \geq 1$

D. $\sum_{n=1}^{\infty} \dfrac{b_n}{10^n} = \dfrac{8}{89}$

Answer:

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6. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and adj $M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$

where a and b are real numbers. Which of the following options is/are correct ?

A. $a + b = 3$

B. $(adjM^{-1}) + adjM^{-1} = -M$

C. det (adj M^2) = 81

D. If M $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

Answer:

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7. There are three bags B_1, B_2, B_3, B_1 contians 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls, bags B_1, B_2 and B_3 have probabilities 3/10, 3/10, and 4/10

respectively of bieng chosen. A bag is selected at randon and a ball is randomly chosen from the bag. then which of the following options is/are correct?

A. Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

B. Probability that the chosen ball is green equals $\frac{39}{80}$

C. Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{4}{13}$

D. Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

Answer:

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8. In a non right angled triangle ΔPQR, let p, q, r denote the lengths of the sides opposite to the angle P,Q,R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. if $p = \sqrt{3}, q = 1$ and the radius of the circumcircle of the ΔPQR equals to 1, then which of the followign options is/are correct?

A. Length of RS $= \frac{\sqrt{7}}{2}$

B. Area of $\Delta SOE = \frac{\sqrt{3}}{12}$

C. Length of OE $= \frac{1}{6}$

D. Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2 - \sqrt{3})$

Answer:

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9. Define the collection $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles as follows:

$E_1 = \frac{x^2}{9} + \frac{y^2}{4} = 1$,

R_1 : rectangle of largest area, with sides parallel to the axes, inscribed in E_1 ,

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest are inscribed in $R_{n-1}, n > 1.$

then which of the following options is/are corrct?

A. The eccentricities of E_{18} and E_{19} are NOT equal

B. $\sum_{n=1}^{N}$ (area of R_n) $<$ 24, for each positive integer N

C. The length of latus rectum of E_9 is $\frac{1}{6}$

D. The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

Answer:

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10. Let $f: R \to R$ be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1 & x < 0 \\ x^2 - x + 1 & 0 \leq x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & 1 \leq x < 3 \\ (x-2)\log_e(x-2) - x + \frac{10}{3} & x \geq 3 \end{cases}$$

Then which of the following options is/are correct?

A. f is increasing on $(-\infty, 0)$

B. f' has a local maximum at x = 1

C. f is onto

D. f' is NOT differentiable at x = 1

Answer:

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11. let T denote a curve $y = f(x)$ which is in the first quadrant and let the point (1,0) lie on it. Let the tangent to T at a point P intersect the y-axis at Y_P and PY_P has length 1 for each poinit P on T. then which of the following option may be correct?

A. $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

B. $xy' + \sqrt{1+x^2} = 0$

C. $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

D. $xy' - \sqrt{1-x^2} = 0$

Answer:

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12. Let L_1 and L_2 denote the lines $\vec{r} = \vec{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in R$ and $\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in R$

Respectively if L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe(s) L_3?

A. $\vec{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

B. $\vec{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

C. $\vec{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

D. $\vec{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

Answer:

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13. What $\omega \neq 1$ be a cube root of unity. Then minimum value of set $\left\{\left|a + b\omega + c\omega^2\right|^2\right.$, a,b,c are distinct non zero intergers) equals

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14. Let AP(a, d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference $d > 0$. If $AP(1, 3) \cap AP(2, 5) \cap AP(3, 7) = AP(a, d)$ then a + d equals____

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15. Let S be the set of matrices of order 3×3 such that all elemtns of the matrix belong to $\{0, 1\}$

let $E_1 = \{A \in S : |A| = 0\}$ where |A| denotes determinant of matrix A

$E_2 = \{A \in S:$ sum of elements of $A = 7\}$ find $P(E_1 / E_2)$

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16. let the point B be the reflection of the point A(2,3) with respect to the line $8x - 6y - 23 = 0$. let T_A and T_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles T_A and T_B such that both the circles are on the same side of T. if C is the point of intersection of T and the line passing through A and B then the length of the line segment AC is

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17. If $I = \frac{2}{\pi} \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{\sin x})(2 - \cos 2x)}$ then $27I^2$ equals

__________ .

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18. Three lines are given by

$r = \lambda \hat{i}, \lambda \in R$

$r = \mu\left(\hat{i} + \hat{j}\right), \mu \in R$

and $r = v\left(\hat{i} + \hat{j} + \hat{k}\right), v \in R$

Let the lines cut the plane $x + y + z = 1$ at the poitns A, B, and C respecitvely . If the area of the tiangle ABC is Δ then the value of $(6\Delta)^2$ equals..............

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