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India's Number 1 Education App

## MATHS

# BOOKS - KVPY PREVIOUS YEAR 

## MOCK TEST 1

Exercise

1. The locus of the centre of a circle,which touches the circles $\left|z-z_{1}\right|=a$ and $\left|z-z_{2}\right|=b$ externally can be
A. an ellipse
B. a hyperbola
C. a circle
D. parabola
2. Which of the following number (s) is/are rational?
A. $\sin 15^{\circ}$
B. $\cos 15^{\circ}$
C. $\sin 15^{\circ} \cos 15^{\circ}$
D. $\sin 15^{\circ} \cos 75^{\circ}$

## Answer:

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3. From a fixed point $A$ on the circumference of a circle of radius $r$, the perpendicular $A Y$ falls on the tangent at $P$. Find the maximum area of triangle $A P Y$.
A. $r^{2}$
B. $\frac{3 \sqrt{3}}{4} r^{2}$
C. $\frac{3 \sqrt{3}}{8} r^{2}$
D. $\sqrt{3} r^{2}$

## Answer:

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4. Let $f: R \rightarrow R$ be a function defined by $f(x)=\min \{x+1,|x|+1\}$, then which of the following is true?
A. $f(x)$ is differentiable everywhere
B. $f(x)$ is not differentiable at $x=0$
C. $f(x) \geq 1$ for all $x \in R$
D. $f(x)$ is not differentiable at $x=1$

## Answer:

5. Let $f: \mathbb{R} \rightarrow \mathbb{R}, g: \mathbb{R} \rightarrow \mathbb{R}$ and $h: \mathbb{R} \rightarrow \mathbb{R}$ be differentiable functions such that $f(x)=x^{3}+3 x+2, g(f(x))=x$ and $h(g(g(x)))=x$ for all $x \in \mathbb{R}$. Then,
A. $g^{\prime}(2)=1 / 15$
B. $h^{\prime}(1)=666$
C. $h(g(3))=36$
D. None of these

## Answer:

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6. Two points $P$ and $Q$ are taken on the line joining the points $A(0.0)$ and $B(3 a, 0)$ such that $A P=P Q=Q B$. Circles are drawn on $A P, P Q$ and $Q B$ as diameters. The locus of the point S from which, the
sum ofsquares of the lengths of the tangents to the three circles is equal to $b^{2}$ is
A. $x^{2}+y^{2}-3 a x+2 a^{2}-b^{2}=0$
B. $3\left(x^{2}+y^{2}\right)-9 a x+8 a^{2}-b^{2}=0$
C. $x^{2}+y^{2}-5 a x+6 a^{2}-b^{2}=0$
D. $x^{2}+y^{2}-a x-b^{2}=0$

## Answer:

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7. 

$P=\{\theta: \sin \theta-\cos \theta=\sqrt{2} \cos \theta\}$ and $Q=\{\theta: \sin \theta+\cos \theta=\sqrt{2} \sin \theta\}$ be two sets. Then
A. $P \subset Q$ and $Q-P \neq \phi$
B. $Q \subset P$
C. $P \subset Q$
D. $P=Q$

## Answer:

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8. If the focus of the parabola $(y-k)^{2}=4(x-h)$ always lies between the lines $x+y=1$ and $x+y=3$, then
A. Olth+klt2
B. Olth +klt 1
C. 1lth+klt2
D. 1lth+klt3

## Answer:

9. Let us define a region $R$ is xy-plane as a set of points ( $x, y$ ) satisfying $\left[x^{2}\right]=[y]$ (where $[\mathrm{x}]$ denotes greatest integer $\leq x$ ), then the region R defines
A. a parabola whose axis is horizontal
B. a parabola whose axis is vertical
C. integer point of the parabolay $=x^{2}$
D. None of these

## Answer:

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10. The chance that doctor $A$ will diagnose disease $X$ correctly is $60 \%$.The chance that a parient of doctor A dies after correct treatment is $75 \%$ while it is $80 \%$ after wrong diagnosis.A patient of doctor $A$ having disease X dies.The probability that his disease is correctly diagnosed is
A. $\frac{8}{17}$
B. $\frac{9}{17}$
C. $\frac{11}{17}$
D. $\frac{6}{17}$

## Answer:

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11. Let $f:\{x, y, z\} \rightarrow\{1,2,3\}$ be a one-one mapping such that only one of the following three statements is true and remaining two are false:
$f(x) \neq 2, f(y)=3, f(z) \neq 1$,then-
A. $f(x) \operatorname{gtf}(y) \operatorname{gtf}(z)$
B. $\mathrm{f}(\mathrm{x}) \operatorname{ltf}(\mathrm{y}) \operatorname{ltf}(\mathrm{z})$
C. $\mathrm{f}(\mathrm{y}) \operatorname{ltf}(\mathrm{x}) \operatorname{ltf}(\mathrm{z})$
D. $f(y) \operatorname{lt} f(z) \mid \operatorname{tf}(x)$

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12. For hyperbola $\frac{x^{2}}{\cos ^{2} \alpha}-\frac{y^{2}}{\sin ^{2} \alpha}=1$ which of the following remains constant with change in $\alpha$
A. abscissa of vertices
B. abscissa of foci
C. eccentricity
D. directrix

## Answer:

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13. The equation ${ }^{`} 2 \cos ^{\wedge} 2 x / 2 \sin ^{\wedge} 2 x=x^{\wedge} 2+x^{\wedge}(-2) ; 0$
A. no real solution
B. one real solution
C. more than one solution
D. none of these

## Answer:

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14. The value of $\int_{\pi}^{2 \pi}[2 \sin x] d x$ where $[$.$] represents the greatest integer$ function is
A. $\frac{-5 \pi}{3}$
B. $-\pi$
C. $\frac{5 \pi}{3}$
D. $-2 \pi$

## (D) Watch Video Solution

15. If $f(x)=x^{3}+3 x^{2}+6 x+2 \sin x$ then the equation
$\frac{1}{x-f(1)}+\frac{2}{x-f(2)}+\frac{3}{x-f(3)}=0$ has
A. No real roots
B. 1 real roots
C. 2 real roots
D. More than 2 real roots

## Answer:

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16. If f and g are two continuous functions being even and and odd, respectively, then $\int_{-a}^{a} \frac{f(x)}{b^{g(x)+1}} d x$ is equal to (a being any non-zero number and b is positive real number, $b \neq 1$ )
A. independent of $f$
B. independent of $g$
C. independent of both $f$ and $g$
D. none of these

## Answer:

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17. A determinant of the second order is made with the elements 0 and 1 . If $\frac{m}{n}$ be the probability that the determinant made is non negative, where $m$ and $n$ are relative primes, then the value of $n-m$ is
A. 4
B. 3
C. 5
D. 8

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18. Let $f:[0, \infty) \overrightarrow{0, \infty}$ and $:[0, \infty) \overrightarrow{0, \infty}$ be non-increasing and nondecreasing functions, respectively, and $h(x)=g(f(x))$. If $f a n d g$ are differentiable functions, $h(x)=g(f(x))$. If fand $g$ are differentiable for all points in their respective domains and $h(0)=0$, then show $h(x)$ is always, identically zero.
A. $h(x)=0 \forall x \geq 0$
B. $h(x)>0 \forall x>\neq 0$
C. $h(x)<0 \forall x>\neq 0$
D. None of these

## Answer:

19. The number of such points $(a+1, \sqrt{3} a)$, where $\mathbf{a}$ is any integer, lying inside the region bounded by the circles $x^{2}+y^{2}-2 x-3=0$ and $x^{2}+y^{2}-2 x-15=0$, is
A. 2
B. 1
C. 3
D. 0

## Answer:

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20. If $x^{4}+p x^{3}+q x^{2}+r x+5=0$ has four positive roots,then the minimum value of pr is equal to
A. 5
B. 25
C. 80
D. 100

## Answer:

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21. If the area bounded by the curve $y=f(x), x$-axis and the ordinates $x=1$ and $\mathrm{x}=\mathrm{b}$ is $(\mathrm{b}-1) \sin (3 \mathrm{~b}+4)$, then-
A. $(x-1) \cos (3 x+4)$
B. $\sin (3 x+4)$
C. $\sin (3 x+4)+3(x-1) \cos (3 x+4)$
D. None of these

## Answer:

22. Let $n$ be a positive integer such that $\frac{\sin \pi}{2 n}+\frac{\cos \pi}{2 n}=\frac{\sqrt{n}}{2}$. Then $6 \leq n \leq 8$ (b) 4
A. $6 \leq n \leq 8$
B. $4<n \leftarrow 8$
C. $4 \leq n \leq 8$
D. $4<n<8$

## Answer:

23. The value of $L t_{x \rightarrow 0}\left\{\frac{\int_{0}^{x^{2}} \sec ^{2} t d t}{x \sin x}\right\}$ is (A) 0 (B) 3 (C) 2 (D) 1
A. 0
B. 3
C. 2
D. 1

Answer:

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24. Tangents are drawn to the circle $x^{2}+y^{2}=50$ from a point ' $P$ ' lying on the x-axis. These tangents meet the y -axis at points $P_{1}{ }^{\prime}$ and $P_{2}{ }^{\prime}$. Possible coordinates of ' P ' so that area of triangle $P P_{1} P_{2}$ is minimum, is/are
A. $(10,0)$
B. $(10 \sqrt{2}, 0)$
C. $(-10 \sqrt{2}, 0)$
D. None of these

## Answer:

25. LEt $F: R \rightarrow R$ is a differntiable function
$f(x+2 y)=f(x)+f(2 y)+4 x y$ for all $x, y \in R$
A. $f^{\prime}(1)=f^{\prime}(0)+1$
B. $f^{\prime}(1)=f^{\prime}(0)-1$
C. $f^{\prime}(0)=f^{\prime}(1)+2$
D. $f^{\prime}(0)=f^{\prime}(1)-2$

## Answer:

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26. If $|z-1|+|z+3| \leq 8$, then the range of values of $|z-4|$ is
A. $1(0,7)$
B. $(1,8)$
C. $(1,9)$
D. $(2,5)$

## Answer:

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27. Statement-1: If $a_{1}, a_{2}, a_{3}, \ldots . . a_{n}$ are positive real numbers, whose product is a fixed number $c$, then the minimum value of $a_{1}+a_{2}+\ldots .+a_{n-1}+2 a_{n}$ is $n(2 C)^{\frac{1}{n}}$

Statement-2 :A.M. $\geq$ G.M.
A. $n(2 c)^{1 / n}$
B. $(n+1) c^{1 / n}$
C. $2 n c^{1 / n}$
D. $(n+1)(2 c)^{1 / n}$

## Answer:

28. Let $S_{n}=\sum_{k=1}^{4 n}(-1)^{\frac{k(k+1)}{2}} k^{2}$. Then $S_{n}$ can take values
A. 1056
B. 1088
C. 1120
D. None of these

## Answer:

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29. The triangle $P Q R$ is inscribed in the circle $x^{2}+y^{2}=25$. If Q and R have co-ordinates( 3,4 ) and $(-4,3)$ respectively, then $\angle Q P R$ is equal to
A. $\frac{\pi}{2}$
B. $\frac{\pi}{3}$
C. $\frac{\pi}{4}$
D. $\frac{\pi}{6}$

## Answer:

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30. The ellipse $E_{1}: \frac{x^{2}}{9}+\frac{y^{2}}{4}=1$ is inscribed in a rectangle $R$ whose sides are parallel to the coordinate axes. Another ellipse $E_{2}$ passing through the point $(0,4)$ circumscribes the rectangle $R$. The eccentricity of the ellipse $E_{2}$ is $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$
A. $\frac{\sqrt{2}}{2}$
B. $\frac{\sqrt{3}}{2}$
C. $\frac{1}{2}$
D. $\frac{3}{4}$

## Answer:

31. The smaller radius of the sphere passing through ( $1,0,0$ ),( $0,1,0$ ) and (0,0, 1)is:
A. $\sqrt{\frac{3}{5}}$
B. $\sqrt{\frac{3}{8}}$
C. $\sqrt{\frac{2}{3}}$
D. $\sqrt{\frac{5}{12}}$

## Answer:

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32. The number of real values of the parameter $k$ for which $\left(\log _{16} x\right)^{2}-(\log )_{16} x+(\log )_{16} k=0$ with real coefficients will have exactly one solution is 2 (b) 1 (c) 4 (d) none of these
A. 0
B. 2
C. 1
D. 4

## Answer:

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33. Let $S$ be the set of complex number $a$ which satisfyndof $\log _{\frac{1}{3}}\left\{\log _{\frac{1}{2}}\left(|z|^{2}+4|z|+3\right)\right\}<0$, then S is (where $i=\sqrt{-1} 1$ )
A. $[-1,3]$
B. $\{z: \operatorname{Re}(z) \geq 1\}$
C. $\{z: i(z) \leq 2\}$
D. All ofthese

## Answer:

34. $(\lim )_{x \overrightarrow{0}}\left[\min \left(y^{2}-4 y+11\right) \frac{\sin x}{x}\right]($ where $[$.$] de \neg$ esthe greatest integer function is 5 (b) 6 (c) 7 (d) does not exist
A. 5
B. 6
C. 7
D. does not exist

## Answer:

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35. If $P(x)$ is a polynomial of the least degree that has a maximum equal to 6 at $x=1$, and a minimum equalto 2 at $x=3$, then $\int_{0}^{1} P(x) d x$ equals:
A. $\frac{17}{4}$
B. $\frac{13}{4}$
C. $\frac{19}{4}$
D. $\frac{5}{4}$

## Answer:

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36. if $10!=2^{p} 3^{q} 5^{r} 7^{s}$ then
A. $p=7$
B. $q=4$
C. $r=3$
D. $\mathrm{s}=2$

## Answer:

37. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius $R$ is $\frac{2 R}{\sqrt{3}}$.
A. $2 \mathrm{a} / 3$
B. $\frac{2 a}{\sqrt{3}}$
C. $a / 3$
D. $a / 5$

## Answer:

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38. If $(\vec{a} \times \vec{b})^{2}+(\vec{a} \cdot \vec{b})^{2}=676$ and $|\vec{b}|=2$, then $|\vec{a}|$ is equal to
A. 13
B. 26
C. 39
D. None of these

## Answer:

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39. The equation $\frac{x^{2}}{1-r}-\frac{y^{2}}{1+r}=1, r>1$, represents an ellipse (b) a hyperbola a circle (d) none of these
A. an ellipse
B. a hyperbola
C. a circle
D. None of these

## Answer:

40. The graph of the function, $\cos x \cos (x+2)-\cos ^{2}(x+1)$ is
A. A straight line passing through $(0,0)$
B. A straight line passing through $\left(\frac{\pi}{2},-\sin ^{2} 1\right)$ and paralles to $x$ axis
C. A straight line passing through ( $0, \sin ^{2} 1$ )
D. not a straight line

## Answer:

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41. If $f(x)=\cos \left\{\frac{\pi}{2}[x]-x^{3}\right\}, 1<x<2$, and [ x$]$ denotes the greatest integer less than or equal to $x$, then the value of $f^{\prime}\left(\sqrt[3]{\frac{\pi}{2}}\right)$, is
A. 0
B. 1
C. $\frac{1}{2}$
D. $\frac{1}{\sqrt{2}}$

## Answer:

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42. The number of real solutions of the equation $1+\left|e^{x}-1\right|=e^{x}\left(e^{x}-2\right)$ is:
A. 0
B. 1
C. 2
D. infinitely many

## Answer:

43. If $2 x-y+1=0$ is a tangent to the hyperbola $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{16}=1$ then which of the following CANNOT be sides of a right angled triangle? $a, 4,2$
(b) $a, 4,12 a, 4,1$ (d) $2 a, 8,1$
A. a, 4, 1
B. a,4, 2
C. $2 \mathrm{a}, 8,1$
D. $2 \mathrm{a}, 4,1$

## Answer:

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44. If the equation $\frac{x^{2}}{3}-4 x+13=\sin \left(\frac{a}{x}\right)$ has a solution then a is equal to
A. $(2 n+1) \frac{\pi}{2}$
B. $3(4 n+1) \frac{\pi}{2}$
C. $3(1+4 n) \pi$
D. None of these

## Answer:

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45. If $m$ be the slope of common tangent of $y=x^{2}-x+1$ and $y=x^{2}-3 x+1$. Then $m$ is equal to
A. 2
B. -1
C. $\frac{1}{2}$
D. -2

## Answer:

46. If $S_{n}={ }^{n} C_{0} \cdot{ }^{n} C_{1}+{ }^{n} C_{1} \cdot{ }^{n} C_{2}+\ldots . .+{ }^{n} C_{n-1} \cdot{ }^{n} C_{n} \quad$ and if $\frac{S_{n+1}}{S_{n}}=\frac{15}{4}$, then the sum of all possible values of n is (A) 2 (B) 4 (C) 6 (D) 8
A. 3
B. 6
C. 7
D. 5

## Answer:

47. The value of integrals $\int_{-2}^{2} \max \{x+|x|, x-[x]\} d x$ where [.] represents the greatest integer function is
A. 4
B. 5
C. $\frac{7}{2}$
D. $\frac{9}{4}$

## Answer:

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48. One hundred identical coins, each with probability ' $p$ ' of showing heads are tossed once. If $0<p<1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of $p$ is
A. $\frac{1}{2}$
B. $\frac{49}{101}$
C. $\frac{50}{101}$
D. $\frac{51}{101}$

## Answer:

49. If $\omega$ is a complex nth root of unity, then $\sum_{r=1}^{n}(a+b) \omega^{r-1}$ is equal to $\frac{n(n+1) a}{2}$ b. $\frac{n b}{1+n}$ c. $\frac{n a}{\omega-1}$ d. none of these
A. $\frac{n(n+1) a}{2 \omega}$
B. $\frac{n b}{1-n}$
C. $\frac{n a}{\omega-1}$
D. None of these

## Answer:

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50. If $f(n+1)=\frac{1}{2}\left\{f(n)+\frac{9}{f(n)}\right\}, n \in N, \quad$ and
$f(n)>0 f$ or al $\ln \in N$, then find $(\lim )_{n \infty} \vec{\infty} f(n)$.
A. 3
B. $\frac{3}{2}$
C. $\frac{1}{2}$
D. not finite

## Answer:

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51. An arch way is in the shape of a semi-ellipse, the road level being the major axis. If the breadth of the arch way is 30 feet and a man 6 feet tall just touches the top when 2 feet from the side, find the greatest height of the arch.
A. 10
B. 8
C. 6
D. 5

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52. Let $f(x)$ be positive, continuous, and differentiable on the interval $(a, b) \operatorname{and}(\lim )_{x \vec{a}^{+}} f(x)=1,(\lim )_{x \vec{b}^{-}} f(x)=3^{\frac{1}{4}} \dot{I} f f^{\prime}(x) \geq f^{3}(x)+\frac{1}{f(x)}$ then the greatest value of $b-a$ is $\frac{\pi}{48}$ (b) $\frac{\pi}{36} \frac{\pi}{24}$ (d) $\frac{\pi}{12}$
A. 1
B. $3^{1 / 4}$
C. $\left(3^{1 / 4}-1\right) \frac{\pi}{24}$
D. $\frac{\pi}{24}$

## Answer:

