MATHS

BOOKS - KVPY PREVIOUS YEAR QUESTION PAPER 2013

Part I Mathematics

1. Let x y, z be three non-negative integers such that $x + y + z = 10$. The maximum possible value of $xyz + xy + yz + zx$ is

A. 52

B. 64

C. 69

D. 73

Answer: C

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2. If a, b are natural numbers such that $2013 + a^2 = b^2$, then the minimum possible value of ab is: (A). 671 (B). 668 (C). 658 (D). 645

A. 671

B. 668

C. 658

D. 645

Answer: C

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3. The number of values of b for which there is an isoscles triangle with sides of length $b+5$, $3b-2$, and $6-b$ is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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4. Let a, b be non-zero real numbers. Which of the following statements about the quadratic equation

$$ax^2 + (a+b)x + b = 0$$

is neccesarily true ?

(I) It has at least one negative root

(II) It has at least one positive root.

(III) Both its roots are real.

A. (I) and (II) only

B. (I) and (III) only

C. (II) and (III) only

D. All of them

Answer: B

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5. Let x, y, z be non-zero real numbers such that

$\frac{x}{y}+\frac{y}{z}+\frac{z}{x}=7$and $\frac{y}{x}+\frac{z}{y}+\frac{x}{z}=9,$

then $\frac{x^3}{y^3}+\frac{y^3}{z^3}+\frac{z^3}{x^3}-3$ is equal to

A. 152

B. 153

C. 154

D. 155

Answer: C

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6. In a triangle ABC with $\angle A < \angle B < \angle C$, points D, E, F are on the interior of segments BC, CA, AB, respectively. Which of the following triangles CANNOT be similar to ABC?

A. Triangle ABD

B. Triangle BCE

C. Triangle CAF

D. Triangle DEF

Answer: A

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7. Tangents to a circle at points P and Q on the circle intersect at a point R. If PQ= 6 and PR= 5 then the radius of the circle is

A. $\frac{13}{3}$

B. 4

C. $\frac{15}{4}$

D. $\frac{16}{5}$

Answer: C

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8. In an acute-angled triangle ABC, the altitudes from A, B, C when extended intersect the circumcircle again at points A_1, B_1, C_1, respectively. If $\angle ABC = 45^\circ$ then $\angle A_1B_1C_1$ equals

A. 45°

B. 60°

C. 90°

D. 135°

Answer: C

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9. In a rectangle ABCD, points X and Y are the midpoints of AD and DC, respectively. Lines BX and CD when extended intersect at E, lines BY and AD when extended intersect at F. If the area of ABCD is 60 then the area of BEF is

A. 60

B. 80

C. 90

D. 120

Answer: C

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10. In the figure given below, ABCDEF is a regular hexagon of side length 1,

$AFPS$ and $ABQR$ are squares. Then the ratio $\frac{Area(APQ)}{Area(SRP)}$ equals

A. $\frac{\sqrt{2}+1}{2}$

B. $\sqrt{2}$

C. $\frac{3\sqrt{3}}{4}$

D. 2

Answer: D

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11. A person X is running around a circular track completing one round every 40 seconds. Another person Y running in the opposite direction meets X every 15 second. The time, expressed in seconds, taken by Y to complete one round is

A. 12.5

B. 24

C. 25

D. 55

Answer: B

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12. The least positive integer n for which $\sqrt{n+1}-\sqrt{n-1}<0.2$ is

A. 24

B. 25

C. 26

D. 27

Answer: C

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13. How many natural numbers n are there such that $n! + 10$ is a perfect square?

A. 1

B. 2

C. 4

D. infinitely many

Answer: A

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14. Ten points lie in a plane so that no three of them are collinear. The number of lines passing through exactly two of these points and dividing the plane into two regions each containing four of the remaining points is

A. 1

B. 5

C. 10

D. dependent on the configuration of points

Answer: B

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15. In a city, the total income of all people with salary below Rs. 10000 per annum is less than the total income of all people with salary above Rs. 10000 per annum. If the salaries of people in the first group increases by 5% and the salaries of people in the second group decreases by 5% then the average income of all people

A. increases

B. decreases

C. remains the same

D. cannot be determined from the data

Answer: B

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Part Ii Mathematics

1. Let a, b, c, d, e be natural numbers in an arithmetic progression such that a + b + c + d + e is the cube of an integer and b + c + d is square of an integer. The least possible value of the number of digits of c is

A. 2

B. 3

C. 4

D. 5

Answer: B

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2. On each face of a cuboid, the sum of its perimeter and its area is written. Among the six numbers so written, there are three distinct numbers and they are 16, 24 and 31. The volume of the cuboid lies between

A. 7 and 14

B. 14 and 21

C. 21 and 28

D. 28 and 35

Answer: D

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3. Let ABCD be a square and let P be point on segment CD such that DP : PC = 1 : 2. Let Q be a point on segment AP such that $\angle BQP = 90^\circ$. Then the ratio of the area of quadrilateral PQBC to the area of the square ABCD is

A. $\frac{31}{60}$

B. $\frac{37}{60}$

C. $\frac{39}{60}$

D. $\frac{41}{60}$

Answer: D

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4. Suppose the height of a pyramid with a square base is decreased by $p\%$ and the lengths of the sides of its square base are increased by $p\%$ (where $p > 0$). If the volume remains the same, then

A. $50 < p < 55$

B. $55 < p < 60$

C. $60 < p < 65$

D. $65 < p < 70$

Answer: C

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5. There are three kinds of liquids X, Y, Z,. Three jars J_1, J_2, J_3 contain 100 ml of liquids X, Y, Z, respectively. By an operation we mean three steps in the following order:

- stir the liquid in J_1 and transfer 10 ml from J_1 into J_2 ,

- stir the liquid in J_2 and transfer 10 ml from J_2 into J_3 ,

- stir the liquid in J_3 and transfer 10 ml from J_3 into J_1 ,

After performing the operation four times, let x, y, z be the amounts of X, Y, Z, respectively, in J_1. Then

A. $x > y > z$

B. $x > z > y$

C. $y > x > z$

D. $z > x > y$

Answer: B

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Part I Mathematics

1. The sum of non-real roots of the polynomial equation $x^3 + 3x^2 + 3x + 3 = 0$. (i) equals to 0 (ii) lies between 0 and 1 (iii) lies between -1 and 0 (iv) has absolute value bigger than 1

A. equals 0

B. lies between 0 and 1

C. lies between 1 and 0

D. has absolute value bigger than 1

Answer: C

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2. Let n be a positive integer such that $\log_2 \log_2 \log_2 \log_2 \log_2(n) < 0 < \log_2 \log_2 \log_2(n)$

.

Let l be the number of digits in the binary expansion of n. Then the minimum and the maximum possible values of l are

A. 5 and 16

B. 5 and 17

C. 4 and 16

D. 4 and 17

Answer: A

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3. Let ω be a cube root of unity not equal to 1. Then the maximum possible value of $\left|a + bw + cw^2\right|$ where a, b, c $\in$ {+1, -1} is

A. 0

B. 2

C. $\sqrt{3}$

D. $1+\sqrt{3}$

Answer: B

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4. If a, b are positive real numbers such that the lines $ax+9y=5$ and $4x+by=3$ are parallel, then the least possible value of a+ b is

A. 13

B. 12

C. 8

D. 6

Answer: B

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5. Two line segments AB and CD are constrained to move along the x and y axes, respectively, in such a way that the points A, B, C, D are concyclic. If AB = a and CD = b, then the locus of the centre of the circle passing through A, B, C, D in polar coordinates is

A. $r^2 = \frac{a^2 + b^2}{4}$

B. $r^2 \cos 2\theta = \frac{a^2 - b^2}{4}$

C. $r^2 = 4(a^2 + b^2)$

D. $r^2 \cos 2\theta = 4(a^2 - b^2)$

Answer: B

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6. Consider a triangle ABC in the xy -plane with vertices A = (0,0), B = (1,1) and C = (9, 1). If the line x = a divides the triangle into two parts of equal area, then a equals

A. 3

B. 3.5

C. 4

D. 4.5

Answer: A

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7. Let ABC be an acute-angled triangle and let D be the midpoint of BC. If AB = AD, then tan(B)/tan(C) equals

A. $\sqrt{2}$

B. $\sqrt{3}$

C. 2

D. 3

Answer: D

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8. The angles α, β, γ of a triangle satisfy the equations $2\sin\alpha + 3\cos\beta = 3\sqrt{2}$ and $3\sin\beta + 2\cos\alpha = 1$. Then angle γ equals

A. 150°

B. 120°

C. 60°

D. 30°

Answer: D

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9. Let f : R $\rightarrow$ R be a function such that $\lim_{x \rightarrow \infty} f(x) = M > 0$. Then which of the following is false?

A. $\lim_{x \rightarrow \infty} x \sin(1/x) f(x) = M$

B. $\lim_{x \rightarrow \infty} \sin(f(x)) = \sin M$

C. $\lim_{x \rightarrow \infty} x \sin\left(e^{-x}\right) f(x) = M$

D. $\lim_{x \rightarrow \infty} \frac{\sin x}{x} f(x) = 0$

Answer: C

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10. For $x, t \in R$ let $p_t(x) = (\sin t)x^2 - (2\cos t)x + \sin t$ be a family of quadratic polynomials in x with variable coefficients. Let $A(t) = \int_0^1 p_t(x)dx$, Which of the following statements are true?

(I) $A(t) < 0$for all t.

(II) A(t) has infinitely many critical points.

(III) A(t) = 0 for infinitely many t.

(IV) $A'(t) < 0$ for all t.

A.) (I) and (II) only

B. (II) and (III) only

C. (III) and (IV) only

D. (IV) and (I) only

Answer: B

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11. Let

$f(x)=\sqrt{2-x-x^2}$ and $g(x)=\cos x$.

Which of the following statement are true ?

(I) Domain of $f\left((f(x))^2\right)$ = Domain of $f(g(x))$

(II) Domain of $f(g(x)) + g(f(x))$= Domain of $f(g(x))$

(III) Domain of $f(g(x))$= Domain of $f(g(x))$

(IV) Domain of $g\left((f(x))^3\right)$ = Domain of $f(g(x))$

A. Only (I)

B. Only (I) and (II)

C. Only (III) and (IV)

D. Only (I) and (IV)

Answer: B

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12. For real x with $-10 \leq x \leq 10$ define $f(x) = \int_{-10}^{x} 2^{[t]} dt$, where for a real number r we denote by [r] the largest integer less than or equal to r. The number of points of discontinutiy of f in the interval (10,10) is

A. 0

B. 10

C. 18

D. 19

Answer: A

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13. For a real number x let $[x]$ denote the largest integer less than or equal to x and $\{x\} = x - [x]$. The possible integer value of n for which $\int_1^n [x]\{x\}dx$ exceeds 2013 is

A. 63

B. 64

C. 90

D. 91

Answer: D

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14. The area bounded by the curve y = cos x, the line joining $(-\pi/4, \cos(-\pi/4))$ and (0, 2) and the line joining $(\pi/4, \cos(\pi/4))$ and (0, 2) is

A. $\left(\frac{4+\sqrt{2}}{8}\right)\pi-\sqrt{2}$

B. $\left(\frac{4+\sqrt{2}}{8}\right)\pi+\sqrt{2}$

C. $\left(\frac{4+\sqrt{2}}{4}\right)\pi-\sqrt{2}$

D. $\left(\frac{4+\sqrt{2}}{4}\right)\pi+\sqrt{2}$

Answer: A

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15. A box contains coupons labeled 1, 2, 3....n. A coupon is picked at random and the number x is noted. The coupon is put back into the box and a new coupon is picked at random. The new number is y. Then the probability that one of the numbers x, y divides the other is (in the options below [r] denotes the largest integer less than or equal to r)

A. $\frac{1}{2}$

B. $\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

C. $-\frac{1}{n}+\frac{1}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

D. $-\frac{1}{n}+\frac{2}{n^2}\sum_{k=1}^{n}\left[\frac{n}{k}\right]$

Answer: D

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16. Let n $\geq$ 3. A list of numbers $0 \leq x_1 \leq x_2 \leq \ldots \leq x_n$ has mean μ and standard deviatiion σ. A new list of numbers is made as follows : $y_1 = 0, y_2 = x_2, \ldots, y_{n-1} = x_{n-1}, y_n = x_1 + x_n$. The mean and the standard deviation of the new list are $\widehat{\mu}$ and $\widehat{\sigma}$. Which of the following is necessarily true ?

A. $\mu = \widehat{\mu}, \sigma \leq \widehat{\sigma}$

B. $\mu = \widehat{\mu}, \sigma \leq \widehat{\sigma}$

C. $\sigma = \widehat{\sigma}$

D. μ may or may not be equal to $\widehat{\mu}$.

Answer: A

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17. Let $\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}, \overrightarrow{v_4}$ be unit vectors in the xy-plane, one each in the interior of the four quadrants. Which of the following statements is necessarily ture. ?

A. $\overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3} + \overrightarrow{v_4} = 0$

B. There exist i, j with $1 \leq i < j \leq 4$ such

$\overrightarrow{v_i} + \overrightarrow{v_j}$ is in the first quadrant

C. There exits i, j with $1 \leq i < j \leq 4$ such

that $\overrightarrow{v_i} \cdot \overrightarrow{v_j} < 0$

D. There exits i, j with $1 \leq i < j \leq 4$ such

that $\overrightarrow{v_i} \cdot \overrightarrow{v_j} > 0$

Answer: A

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18. The number of integers n with $100 \leq n \leq 999$ and containing at most two distinct digits is

A. 252

B. 280

C. 324

D. 360

Answer: A

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19. For an integer n let Sn = {n + 1, n + 2, ..., n + 18}. Which of the following is true for all n $\geq$ 10?

A. S_n has a multiple of 19

B. S_n has a prime

C. S_n has at least four multiples of 5

D. S_n has at most six primes

Answer: D

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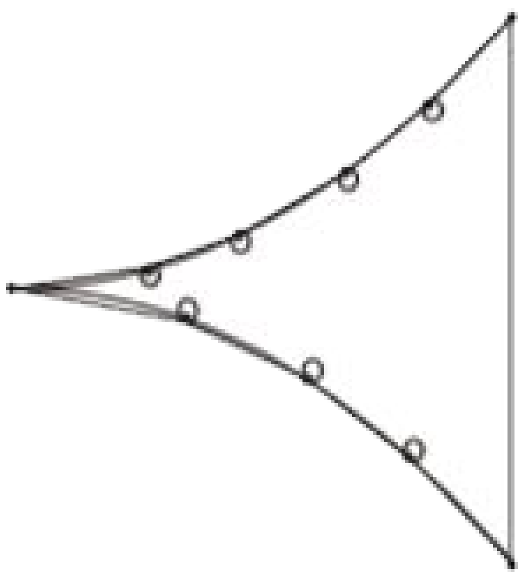
20. Let P be a closed polygon with 10 sides and 10 vertices (assume that the sides do not intersect except at the vertices). Let k be the number of interior angles of P that are greater than 180°. The maximum possible value of k is



A. 3

B. 5

C. 7

D. 9

Answer: C

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Part Ii Mathematics

1. Let f(x) be a non-constant polynomial with real coefficients such that $f\left(\frac{1}{2}\right) = 100$ and f(x) $\leq$ 100 for all real x. Which of the following statements is NOT necessarily true ?

A. The coefficient of the highest degree term in f(x) is negative

B. f(x) has at least two real roots

C. If $x \neq 1/2$ then $f(x) < 100$

D. At least one of the coefficients of f(x) is bigger than 50

Answer: C

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2. Let a, b, c, d be real numbers such that

$$\sum_{k=1}^{n} \left(ak^3 + bk^2 + ck + d\right) = n^4$$

or every natural number n. Then | a | + | b | + |

C | + | d | is equal to

A. 15

B. 16

C. 31

D. 32

Answer: A

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3. The vertices of the base of an isosceles triangle lie on a parabola $y^2 = 4x$ and the base is a part of the line $y = 2x - 4$. If the third vertex of the triangle lies on the x-axis, its coordinates are

A. $\left(\frac{5}{2}, 0\right)$

B. $\left(\frac{7}{2}, 0\right)$

C. $\left(\frac{9}{2}, 0\right)$

D. $\left(\frac{11}{2}, 0\right)$

Answer: C

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4. In a triangle ABC, let G denote its centroid and let M, N be points in the interiors of the segments AB, AC, respectively, such that M, G, N are collinear. If r denotes the ratio of the area of triangle AMN to the area of ABC then

A. $r = 1/2$

B. $r > 1/2$

C. $4/9 \leq r < 1/2$

D. $4/9 < r$

Answer: C

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5. Let XY be the diameter of a semicircle with centre O. Let A be a variable point on the semicircle and B another point on the semicircle such that AB is parallel to XY. The value of $\angle BOY$ for which the inradius of triangle AOB is maximum, is

A. $\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

B. $\sin^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{5}$

Answer: A

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6. Let $f(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$. The number of real roots of f(x) = 0 is

A. 0

B. 1

C. 2

D. 4

Answer: A

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7. Suppose that the earth is a sphere of radius 6400 kilometers. The height from the earths surface from where exactly a fourth of the earths surface is visible, is

A. $3200 km$

B. $3200\sqrt{2}$ km

C. $3200\sqrt{3}$ km

D. 6400 km

Answer: D

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8. Let n be a positive integer. For a real number x, let [x] denote the largest integer not exceeding x and {x} = $x - [x]$. Then $\int_1^{n+1} \frac{(\{x\})^{[x]}}{[x]} dx$ is equal to

A. $\log_e(n)$

B. $\frac{1}{n+1}$

C. $\frac{n}{n+1}$

D. $1 + \frac{1}{2} + \ldots.. + \frac{1}{n}$

Answer: C

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9. A box contains coupons labelled 1,2,...,100. Five coupons are picked at random one after another without replacement. Let the numbers on the coupons be $x_1, x_2, \ldots x_5$. What is the probability that $x_1 > x_2 > x_3$ and $x_3 < x_4 < x_5$?

A. $1/120$

B. $1/60$

C. $1/20$

D. $1/10$

Answer: A

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10. In a tournament with five teams, each team plays against every other team exctly once. Each game is won by one of the playing teams and the winning team scores one point, while the losing team scores zero. Which of the following is NOT necessarily true?

A. There are at least two teams which have at most two points each

B. There are at least two teams which have at least two points each.

C. There are at most three teams which have at least threee points each

D. There are at most four teams which have at most two points each

Answer: D

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