MATHS

BOOKS - KVPY PREVIOUS YEAR

QUESTION PAPER 2020

Part I Mathematics

1. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the equation $[x^2] = x + 1$ has

A. two solutions

B. one solution

C. no solution

D. more than two solutions

Answer: C

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2. Let $p_1(x) = x^3 - 2020x^2 + b_1x + c_1$ and $p_2(x) = x^3 - 2021x^2 + b_2x + c^2$ be polynomials having two common roots α and β. Suppose there exist polynomials $q_1(x)$ and $q_2(x)$ such that $p_1(x)q_1(x) + p_2(x)q_2(x) = x^2 - 3x + 2.$ Then the correct identity is

A. $p_1(3) + p_2(1) + 4028 = 0$

B. $p_1(3) + p_2(1) + 4026 = 0$

C. $p_1(2) + p_2(1) + 4028 = 0$

D. $p_1(1) + p_2(2) + 4028 = 0$

Answer: A

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3. Suppose p,q,r are positive rational numbers such that $\sqrt{p}+\sqrt{q}+\sqrt{r}$ are irrational. Then

A. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

B. $\sqrt{pq}\sqrt{pr}, \sqrt{qr}$ are rational, but $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are irrational

C. $\sqrt{p}, \sqrt{q}, \sqrt{r}$ are rational

D. $\sqrt{pq}\sqrt{pr}\sqrt{qr}$ are irrational

Answer: C

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4. Let A, B, C be three points on a circle of radius 1 such that $\angle ACB = \frac{\pi}{4}$. Then the length of the side AB is

A. $\sqrt{3}$

B. $\frac{4}{3}$

C. $\frac{3}{\sqrt{2}}$

D. $\sqrt{2}$

Answer: D

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5. Let x and y be two positive real numbers such that x + y = 1. Then the minimum value of $\frac{1}{x} + \frac{1}{y}$ is-

A. 2

B. $\frac{5}{2}$

C. 3

D. 4

Answer: D

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6. Let ABCD be a quadrilateral such that there exists a point E inside the quadrilateral satisfying AE = BE = CE = DE. Suppose $\angle DAB, \angle ABC, \angle BCD$ is an arithmetic progression. Then the median of the set $(\angle DAB, \angle ABC, \angle BCD)$ is :-

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

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7. The number of ordered pairs (x, y) of positive integers satisfying $2x + 3y = 5^{xy}$ is

A. 1

B. 2

C. 5

D. infinite

Answer: A

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8. If all the natural numbers from 1 to 2021 are written as 12345.....20202021, then find the 2021st term

A. 0

B. 1

C. 6

D. 9

Answer: B

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9. In a triangle ABC, a point D is chosen on BC such that BD : DC = 2 : 5. Let P be a point on the circumcircleABC such that $\angle PDB = \angle BAC$. Then PD : PC is :-

A. $\sqrt{2}, \sqrt{5}$

B. $2:5$

C. $2:7$

D. $\sqrt{2}:\sqrt{7}$

Answer: D

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10. Let [x] be the greatest integer less than or equal to x, for a real number x. Then the following sum

$$\left[\frac{2^{2020}+1}{2^{2018}+1}\right]+\left[\frac{3^{2020}+1}{3^{2018}+1}\right]+\left[\frac{4^{2020}+1}{4^{2018}+1}\right]+\left[\frac{5^{2020}+1}{5^{2018}+1}\right]+\left[\frac{6^{2020}+1}{6^{2018}+1}\right]$$

is-

A. 80

B. 85

C. 90

D. 95

Answer: C

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11. Let r be the remainder when 2021^{2020} is divided by 2020^{2}.

Then r lies between

A. 0 and 5

B. 10 and 15

C. 20 and 100

D. 107 and 120

Answer: A

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12. In a triangle ABC, the altitude AD and the median AE divide $\angle A$ into three equal parts. If BC=28, then the nearest integer to AB+ AC is

A. 38

B. 37

C. 36

D. 33

Answer: A

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13. The number of permutations of the letters a_1, a_2, a_3, a_4, a_5 in which the first letter a_1 does not occupy the first position (from the left) and the second letter a_2 does not occupy the second position (from the left) is

A. 96

B. 78

C. 60

D. 42

Answer: B

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14. In a book self if m books have black cover and n books have blue cover and all books are different, then the number of ways black books can be arranged side by side are

A. $m!n!$

B. $m!(n+1)!$

C. $(n+1)!$

D. $(m+n)!$

Answer: B

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15. Let's say abcde is a 5 digit number which when multiplied by 9 new number formed is edcba then sum a+b+c+d+e

A. 18

B. 27

C. 36

D. 45

Answer: B

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Part Ii Mathematics

1. The maximum concentration of harmuful chemicals is expected to be found in organisms

A. at the bottom of a food chain

B. at the middle of a food chain

C. at the top of a food chain

D. at any level in a food chain

Answer: C

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2. The genome of SARS - Co V2 is composed of

A. double stranded DNA.

B. double stranded RNA.

C. single stranded DNA.

D. single stranded RNA.

Answer: D

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3. Let A denote the set of all 4-digit natural numbers with no digit being 0. Let $B \subset A$ consist of all numbers x such that no permutation of the digits of x gives a number that is divisible by 4. Then the probability of drawing a number from B with all even digits is

A. $\frac{625}{1641}$

B. $\frac{16}{641}$

C. $\frac{16}{1641}$

D. $\frac{1000}{1641}$

Answer: C

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4. Let ABC be a triangle such that AB = 4, BC = 5 and CA = 6. Choose points D,E,F on AB, BC, CA respectively, such that AD = 2, BE = 3, CF=4. Then $\frac{\text{area}\Delta DEF}{\text{area}\Delta ABC}$ is

A. $\frac{1}{4}$

B. $\frac{3}{15}$

C. $\frac{4}{15}$

D. $\frac{7}{30}$

Answer: C

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5. The number of ordered pairs (x, y) of integers satisfying $x^3 + y^3 = 65$ is

A. 0

B. 2

C. 4

D. 6

Answer: B

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6. A bottle in the shape of a right-circular cone with height h contains some water. When its base is placed on a flat surface, the height of the vertex from the water level is a units. When it is kept upside down, the height of the base from the water level is $\frac{a}{4}$ units. Then the ratio $\frac{h}{a}$ is

A. $\frac{1+\sqrt{85}}{4}$

B. $\frac{1+\sqrt{85}}{8}$

C. $\frac{1+\sqrt{65}}{4}$

D. $\frac{1+\sqrt{65}}{8}$

Answer: B

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7. Consider the following two statements :

I. if n is a composite number, then n divides (n – 1)!

II. There are infinitely many natural numbers n such that n^3+2n^2+n divides n!.

Then

A. I and II are true

B. I and II are false

C. I is true and II is false

D. I is false and II is true

Answer: D

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Part I Mathematics

1. Consider the following statements :

$I.\ \lim_{n \to \infty} \frac{2^n + (-2)^n}{2^n}$ dos not exist

$II.\ \lim_{n \to \infty} \frac{3^n + (-3)^n}{2^n}$ does not exist then

A. I is true and II is fals

B. I is false and II is true

C. I and II are true

D. neither I nor II is true

Answer: A

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2. Consider a regular 10-gon with its vertices on the unit circle. With one vertex fixed, draw straight lines to the other 9 vertices. Call them $L_1, L_2, \ldots, L_9$ and denote their lengths by $l_1, l_2, \ldots, l_9$ respectively. Then the product $l_1, l_2, ..., l_9$ is

A. 10

B. $10\sqrt{3}$

C. $\frac{50}{\sqrt{3}}$

D. 20

Answer: A

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3. The value of the integral

$$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+e^x} dx$$

is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi^2}{2}$

Answer: B

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4. Let $\mathbb{R}$ be the set of all real numbers and $f(x) = \sin^{10} x \left(\cos^8 x + \cos^4 x + \cos^4 x + \cos^2 x + 1\right)$ for x $\in \mathbb{R}$. Let $S = \{\lambda \in \mathbb{R} |$ there exits a point $c \in (0, 2)$ with $f'(c) = \lambda f(c)\}$. then

A. $S = \mathbb{R}$

B. $S = \{0\}$

C. $S = [0, 2\pi]$

D. S is a finite set having more than one element

Answer: A

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5. A person standing on the top of a building of height $60\sqrt{3}$ feel observed the top of a tower to lie at an elevation of 45°. That person descended to the bottom of the building and found that the top of the same tower is now at an angle of elevation of 60°. The height of the tower (in feet) is

A. 30

B. $30(\sqrt{3}+1)$

C. $90(\sqrt{3}+1)$

D. $150(\sqrt{3}+1)$

Answer: C

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6. Assume that $3.313 \leq \pi \leq 3.15$. The integer closest to the value of $\sin^{-1}(\sin 1 \cos 4 + \cos 1 \sin 4)$. Where 1 and 4 appearing in sin and cos are given in radians, is

A. -1

B. 1

C. 3

D. 5

Answer: A

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7. The maximum value of the function $f(x) = e^x + x\ln$ x on the interval $1 \leq x \leq 2$ is

A. $e^2 + In2 + 1$

B. $e^2 + 2In2$

C. $e^{\pi/2} + \frac{\pi}{2}In\frac{\pi}{2}$

D. $e^{3/2} + \frac{3}{2}In\frac{3}{2}$

Answer: B

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8. Let A be a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ 1 & 1 \end{bmatrix}$, where a, b are integers and $-50 \le b \le 50$. The number of such matrices A such that A^{-1}, the inverse of A, exists and A^{-1} contains only integer entries is

A. 101

B. 200

C. 202

D. 101^2

Answer: C

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9. Let $A = (a_{ij})_{1 \le I, j \le 3}$ be a 3×3 invertible matrix where each a_{ij} is a real number. Denote the inverse of the matrix A by A^{-1}. If $\Sigma_{j=1}^{3} a_{ij} = 1$ for $1 \le i \le 3$, then

A. sum of the diagonal entries of A is 1

B. sum of each row of A^{-1} is 1

C. sum of each row and each column of $A^{-}1$ is 1

D. sum of the diagonal entries of A^{-1} is 1

Answer: B

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10. Let x, y be real numbers such that $x > 2y > 0$ and

$2\log(x - 2y) = \log x + \log y.$

Then the possible values (s) of $\frac{x}{y}$

A. is 1 only

B. are 1 and 4

C. is 4 only

D. is 8 only

Answer: C

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11. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (b < a)$. Be an ellipse with major axis AB and minor axis CD. Let F_1 and F_2 be its two foci, with A, F_1, F_2 B in that order on the segment AB. Suppose $\angle F_1CB = 90^\circ$. The eccentricity of the ellipse is

A. $\frac{\sqrt{3}-1}{2}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{5}-1}{2}$

D. $\frac{1}{\sqrt{5}}$

Answer: C

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12. Let A denote the set of all real numbers x such that $x^3 - [x]^3 = (x - [x])^3$, where [x] is the greatest integer less than or equal to x. Then

A. A is a discrete set of at least two points

B. A contains an interval, but is not an interval

C. A is an interval, but a proper subset of $(-\infty, \infty)$

D. $A = (-\infty, \infty)$

Answer: B

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13. S= $\lim_{n \to \infty} \sum_{k=0}^{n} \frac{1}{\sqrt{n^2 + k^2}}$

A. does not exist

B. exists and lies in the interval (0,1)

C. exists and lies in the interval [1, 2)

D. exists and lies in the interval [2, ∞)

Answer: C

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14. Let $\mathbb{R}$ be the set of all real numbers and $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. Suppose $|f(x)-f(y)|\geq|x-y|$ for all real numbers x and y. Then

A. f is one-one, but need not be onto

B. f is onto, but need not be one-one

C. f need not be either one-one or onto

D. f is one-one and onto

Answer: B

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15. Let

$$f(x) = \begin{cases} \frac{x}{\sin x}, & x \in (0,1) \\ 1, & x = 0 \end{cases}$$

Consider the integral

$$I_n = \sqrt{n} \int_0^{1/n} f(x) e^{-nx} dx.$$

Then $\lim_{n \to \infty} I_n$

A. does not exist

B. exists and is 0

C. exists and is 1

D. exists and is $1 - e^{-1}$

Answer: B

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16. The value of the integral

$$\int_1^3 \left((x-2)^4 \sin^3(x-2) + (x-2)^{2019} + 1\right) dx$$

is

A. 0

B. 2

C. 4

D. 5

Answer: B

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17. In a regular 15-sided polygon with all its diagonals drawn, a diagonal is chosen at random. The probability that it is either a shortest diagonal nor a longest diagonal is

A. $\frac{2}{3}$

B. $\frac{5}{6}$

C. $\frac{8}{9}$

D. $\frac{9}{10}$

Answer: A

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18. Let $M = 2^{30} - 2^{15} + 1$, and M^2 be expressed in base 2. The number of 1's in this base 2 representation of M^2 is

A. 29

B. 30

C. 59

D. 60

Answer: B

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19. Let ABC be a triangle such that AB = 15 and AC = 9. The bisector of $\angle BAC$ meets BC in D. If $\angle ACB = 2\angle ABC$, then BD is

A. 8

B. 9

C. 10

D. 12

Answer: C

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20. The figur in the complex plane given by

$$10z\bar{z} - 3(z^2 + \bar{z}^2) + 4i(z^2 - \bar{z}^2) = 0$$

is

A. a straight line

B. a circle

C. a parabola

D. an ellipse

Answer: A

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Part Ii Mathematics

1. Let $a = \sum_{n=101}^{200} 2^n \sum_{k=101}^{n} \frac{1}{K!}$ and $b = \sum_{n=101}^{200} \frac{2^{201} - 2^n}{n!}$

then $\frac{a}{b}$ is

A. 1

B. $\frac{3}{2}$

C. 2

D. $\frac{5}{2}$

Answer: A

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2. Let a, b, c be non-zero real roots of the equation $x^3 + ax^2 + bx + c = 0$. Then

A. there are infinitely many such triples a, b, c

B. there is exactly one such triple a, b, c

C. there are exactly two such triples a, b, c

D. there are exactly three such triples a, b, c

Answer: C

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3. Let $f(x) = \sin x + (x^3 - 3x^2 + 4x - 2)\cos x$ for x in $(0, 1)$.

Consider the following statements

I. f has a zero in (0, 1)

II. f is monotone in (0, 1)

Then

A. I and II are true

B. I is true and II are false

C. I is false and II are true

D. I and II are false

Answer: A

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4. Let A be a set consisting of 10 elements. The number of non-empty relations from A to A that are reflexive but not symmetric is

A. $2^{89} - 1$

B. $2^{89} - 2^{45}$

C. $2^{45} - 1$

D. $2^{90} - 2^{45}$

Answer: D

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5. In a triangle ABC, the angle bisector BD of $\angle B$ intersects AC in D. Suppose $BC = 2, CD = 1$ and $BD = \frac{3}{\sqrt{2}}$. The perimeter of the triangle ABC is

A. $\frac{17}{2}$

B. $\frac{15}{2}$

C. $\frac{17}{4}$

D. $\frac{15}{4}$

Answer: B

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6. Len N be set of natural numbers. For $n \in N$ defiine

$$I_n = \int_0^{\pi} \frac{x \sin^{2n}(x)}{\sin^{2n}(x) + \cos^{2n}(x)} dx.$$

Then for $m, n \in N$

A. $I_m < I_n$ for all $m < n$

B. $I_m > I_n$ for all $m < n$

C. $I_m = I_n$ for all $m \neq n$

D. $I_m < I_n$ for some $m \leq n$ and $I_m > I_n$ for some $m < n$

Answer: C

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7. for $\theta \in [0, \pi]$, let $f(\theta) = \sin(\cos\theta)$ and $g(\theta) = \cos(\sin\theta)$.

Let a =

$\max_{0 \leq \theta \leq \pi} f(\theta), b = \min_{0 \leq \theta \leq \pi} f(\theta), c = \max_{0 \leq \theta \leq \pi} g(\theta)$ and $d = \min_{0 \leq \theta \leq \pi} g(\theta)$

. the correct inequalities satisfied by ` a,b,c,d are

A. $b < d < c < a$

B. $d < b < a < c$

C. $b < d < a < c$

D. $b < a < d < c$

Answer: C

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8. Six consecutive sides of an equiangular octagon are 6, 9, 8, 7, 10, 5 in that order. The integer nearest to the sum of the remaining two sides is

A. 17

B. 18

C. 19

D. 20

Answer: B

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9. The value of the integral

$$\int_1^{\sqrt{2}+1}\left(\frac{x^2-1}{x^2+1}\right)\frac{1}{\sqrt{1+x^4}}dx \text{ is}$$

A. $\frac{\pi}{6\sqrt{2}}$

B. $\frac{\pi}{12\sqrt{2}}$

C. $\frac{\pi}{8\sqrt{2}}$

D. $\frac{\pi}{4\sqrt{2}}$

Answer: B

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10. Let $a = BC$, $b = CA$, $c = AB$ be side lengths of a triangle ABC. And m be the length of the median through A. If $a = 8$, $b-c = 2$, $m = 6$, then the nearest integer to b is

A. 7

B. 8

C. 9

D. 10

Answer: B

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