MATHS

BOOKS - KVPY PREVIOUS YEAR

SOLVED PAPER 2018

Example

1. Suppose $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is real matrix with nonzero entries, ad-bc=0 , and A^2=A. Then a+d equals

A. 1

B. 2

C. 3

D. 4

Answer:

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2. On any given are of positive length on the unit circle |z|=1 in the complex plane,

A. there need not be any root of unity

B. there lies exactly one root of unity

C. there are more than one but finitely many roots of unity

D. there are infinitely many roots of unity

Answer:

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3. For $0 < \theta < \frac{\pi}{2}$, four tangents are drawn at the four points $(\pm 3\cos\theta, \pm 2\sin\theta)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If $A(\theta)$ denotes the area of the quadrilateral formed by these four tangents, the minimum value of $A(\theta)$ is

A. 21

B. 24

C. 27

D. 30

Answer:

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4. Let S $=\{x \in R: \cos(x)+\cos(\sqrt{2}x)<2\}$. Then

A. $S=\phi$

B. S is a non-empty finite set

C. S is an infinite proper subset of R-{0}

D. S=R-{0}

Answer:

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5. On a rectangular hyperbola $x^2 - y^2 = a^2, a > 0$, three points A,B,C are taken as follows : A = (-a,0): B and C are placed symmetrically with respect to the x-axis on the branch of the hyperbola not containing A suppose that the triangle ABC is equilateral. If the side-length of the triangle ABC is ka,then k lies in the interval

A. (0,2)

B. (2,4)

C. (4,6)

D. (6,8)

Answer:

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6. The number of real solution x of the equation

$\cos^2(x\sin(2x)) + \frac{1}{1+x^2} = \cos^2 x + \sec^2 x$ is

A. 0

B. 1

C. 2

D. infinite

Answer:

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7. Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$, be an ellipse with foci F_1 and F_2. Let AO be its semi-minor axis. Where O is the centre of the ellipse. The lines AF_1 and AF_2, when extended, , cut the ellipse again at point B and C respectively. Suppose that the triangle ABC is equilateral. Then the eccentricity of the ellipse is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer:

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8. Let $a = \cos 1^\circ$ and $b = \sin 1^\circ$. We say that a real number is algebraic if is a root of a polynomial with integer coefficients. Then

A. a is algebraic but b is not algebraic

B. b is algebraic but a is not algebraic

C. both a and b are algebraic

D. neither a nor b is algebraic

Answer:

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9. A rectangle with its sides parallel to the x-axis and y-axis is inscibed in the region bounded by the curves $y = x^2 - 4$ and $2y = 4 - x^2$. The maximum possible area of such a rectangle is closest to the integer

A. 10

B. 9

C. 8

D. 7

Answer:

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10. Let $f(x) = x|\sin x|, x \in R.$ Then

A. f is differentiable for all x, except at $x = n\pi$, n=1,2,3,......

B. f is differentiable for all x, except at $x = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots\ldots$

C. f is differentiable for all x, except at $x = n\pi$, n=0,1,2,3,......

D. f is differentiable for all x, except at $x = n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots\ldots$

Answer:

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11. Let $f:[-1, 1] \rightarrow R$ be a function defined by $f(x) = \left\{x^2 \left|\cos\left(\frac{\pi}{x}\right)\right| \text{for} x \neq 0, \text{ for } \ x = 0\right.$, The set of points where f is not differentiable is

A. $\{x \in [-1, 1] : x \neq 0\}$

B. $\left\{x \in [-1, 1] : x = 0 \text{ or } x = \frac{2}{2n+1}, n \in Z\right\}$

C. $\left\{x \in [-1, 1] : x = \frac{2}{2n+1}, n \in Z\right\}$

D. [-1,1]

Answer:

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12. The value of the integral $\int_0^{\pi} (1 - |\sin 8x|)dx$ is

A. 0

B. $\pi - 1$

C. $\pi - 2$

D. $\pi - 3$

Answer:

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13. Let in x denote the logarithm of x with respect to the base e. Let $S \subset R$ be the set all points where the function In $\left(x^2 - 1\right)$ is well-defined. Then the number of function $f: S \rightarrow R$ that are differentiable, satisfy

$f'(x) - \ \text{In}(x^2 - 1)$ for all $x \in S$ and f(2)=0, is

A. 0

B. 1

C. 2

D. infinite

Answer:

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14. Let S be the set of real numbers p such that there is no nonzero continuous function $f: R \to R$ satisfying $\int_0^x f(t)dt = pf(x)$ for all $x \in R$. Then S is

A. the empty set

B. the set of all rational numbers

C. the set of all irrational numbers

D. the whole set R.

Answer:

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15. The porbability of men getting a certain disease is $\frac{1}{2}$ and that of women getting the same disease is $\frac{1}{5}$. The blood test that identifies the disease gives the correct result with probability $\frac{4}{5}$. Suppose a person is chosen at randon from a group of 30 males and 20 females, and the blood test of the person is found to be positive. What is the probability that the chosen person is a man ?

A. $\frac{75}{107}$

B. $\frac{3}{5}$

C. $\frac{12}{19}$

D. `3/10

Answer:

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16. The number of function $f:[0,1] \to [0,1]$ satisfying $|f(x)-f(x)|=|x-y|$ for all x, y in $[0,$

A. exactly 1

B. exactly 2

C. more than 2, but finite

D. infinite

Answer:

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17. Suppose A is a 3×3 matrix consisting of integer entries that are chosen at random from the set $\{-1000, 999, \ldots., 999, 1000\}$. Let P be the probability that either $A^2 = -I$ or A is diagonal, where I is the 3×3 identity matrix. Then

A. $P < \frac{1}{10^{18}}$

B. $P = \frac{1}{10^{18}}$

C. $\frac{5^2}{10^{18}} \leq P \leq \frac{5^3}{10^{18}}$

D. $P \leq \frac{5^4}{10^{18}}$

Answer:

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18. Let $x^2 = 4ky, k > 0$ be a parabola with vertex A. Let BC be its latus rectum. An ellipse with center on BC touches the parabola at A, and cuts BC at point D and E such that BD=DE=EC (B,D,E,C in that order). The eccentricity of the ellipse is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{5}}{3}$

D. $\frac{\sqrt{3}}{2}$

Answer:

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19. Let $f:[0'1] \to [-1, 1]$ and $g:[-1, 1] \to [0, 2]$ be two functions such that g is injective and $g^{\circ} f:[0, 1] \to [0, 2]$ is surjective. Then

A. f must be injective but need not be surjective

B. f must be surjective but need not be injective

C. f must be bijective

D. f must be a constant function

Answer:

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20. Let R be a rectangle , C be a circle, and T be a triangle in the plane. The maximum number of points common to the perimeter of R,C, and T is

A. 3

B. 4

C. 5

D. 6

Answer:

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21. The number of different possible values for the sum x+y+z, where x,y,z are real numbers such that $x^4 + 4y^2 + 16z^4 + 64 = 32xyz$ is

A. 1

B. 2

C. 4

D. 8

Answer:

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22. Let Γ be a circle with diameter AB and centre O.Let l be the tangent to Γ at B.For each point M onΓ different from A,consider the tangent t at M and let it interest l at P.Draw a line parallel to AB through P intersecting OM at Q.The locus of Q as M varies over $\Gamma$is

A. an arc of a circle

B. a parabola

C. an arc of an ellipse

D. a branch of a hyperbola

Answer:

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23. The number of solution x of the equation $\sin(x+x^2)-\sin(x^2)=\sin x$ in the interval [2,3] is

A. 0

B. 1

C. 2

D. 3

Answer:

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24. The number of polynomials $p: R \rightarrow R$ satisfying $p(0) = 0, p(x) > x^2$ for all $x \neq 0$, and $p''(0) = \frac{1}{2}$ is

A. 0

B. 1

C. more than 1,but finite

D. infinite

Answer:

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25. Consider the set A_n of point (x,y) such that $0 \le x \le n, 0 \le y \le n$ where n,x,y are integers. Let S_n be the set of all lines passing through at least two distinct points from A_n. Suppose we choose a line l at random from S_n. Let P_n be the probability that l is tangent to the circle $x^2 + y^2 = n^2\left(1 + \left(1 - \frac{1}{\sqrt{n}}\right)^2\right)$. Then the limit $\lim_{n \to \infty} P_n$ is

A. 0

B. 1

C. $\frac{1}{\pi}$

D. $\frac{1}{\sqrt{2}}$

Answer:

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26. The maximum possible area bounded by the parabola $y = x^2 + x + 10$ and a chord of the parabola of length 1 is

A. $\frac{1}{12}$

B. $\frac{1}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer:

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27. Suppose z is any root of $11z^8 + 20iz^7 + 10iz - 22 = 0$ where $i = \sqrt{-1}$.Then,$S = |z|^2 + |z| + 1$ satisfies

A. $S \leq 3$

B. $3 < S < 7$

C. $7 \leq S < 13$

D. $S \geq 13$

Answer:

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