MATHS

BOOKS - KVPY PREVIOUS YEAR

SOLVED PAPER 2019

Example

1. The number of four-letter words that can be formed, with letters a,b,c such that all three letters occur is

A. 30

B. 36

C. 81

D. 256

Answer:

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2. Let $A = \left\{ \theta \in R : \left(\frac{1}{3}\sin(\theta) + \frac{2}{3}\cos(\theta)\right)^2 = \frac{1}{3}\sin^2(\theta) + \frac{2}{3}\cos^2(\theta) \right\}$ Then

A. $A \cap [0, \pi]$ is an empty set

B. $A \cap [0, \pi]$ has exactly one point

C. $A \cap [0, \pi]$ has exactly two points

D. $A \cap [0, \pi]$ has more than two points

Answer:

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3. The area of the region bounded by the lines x=1, x=2 and the curves $x(y - e^x) = \sin x$ and $2xy = 2\sin x + x^3$ is

A. $e^2 - e - \frac{1}{6}$

B. $e^2 - e - \frac{7}{6}$

C. $e^2 - e + \frac{1}{6}$

D. $e^2 - e + \frac{7}{6}$

Answer:

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4. Let AB be a line segment with midpoint C, and D be the midpoint of AC. Let C_1 be the circle with diameter AB and C_2

be the circle with diameter AC. Let E be a point on C_1 such that EC is perpendicular to AB. Let f be a point on C_2such that DF is perpendicular to AB. Then, the value of $\sin\angle FEC$ is

A. $\frac{1}{\sqrt{10}}$

B. $\frac{2}{\sqrt{10}}$

C. $\frac{1}{\sqrt{13}}$

D. $\frac{2}{\sqrt{13}}$

Answer:

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5. The number of integers x satisfying $-3x^4 + \det\begin{bmatrix}1 & x & x^2\\ 1 & x^2 & x^4\\ 1 & x^3 & x^6\end{bmatrix} = 0$ is equal to

A. 1

B. 2

C. 5

D. 8

Answer:

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6. Let P be a non-zero polynomial such that $P(1+x) = P(1-x)$ for all real x, and P(1)=0. Let m be the largest integer such that $(x-1)^m$ divides P(x) for all such P(x). Then m equals

A. 1

B. 2

C. 3

D. 4

Answer:

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7. Let $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & when x \neq 0 \\ 1 & when x = 0 \end{cases}$ and $A = \{x \in R: f(x) = 1\}$. Then A has

A. exactly one element

B. exactly two elements

C. exactly three elements

D. infinitely many elements

Answer:

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8. Let S be a subset of the plane defined by: $S = \{(x, y) : |x| + 2|y| = 1\}$ Then the radius of the smallest circle with centre at the origin and having non-empty intersection with S is

A. $\frac{1}{5}$

B. $\frac{1}{\sqrt{5}}$

C. $\frac{1}{2}$

D. $\frac{2}{\sqrt{5}}$

Answer:

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9. The number of solutions of the equation sin(9x)+sin(3x)=0 in the closed interval $[0, 2\pi]$ is

A. 7

B. 13

C. 19

D. 25

Answer:

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10. Among all the parallelograms whose diagonals are 10 and 4, the one having maximum area has its perimeter lying in the interval

A. [19.20]

B. [20,21]

C. [21,22]

D. [22,23]

Answer:

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11. The number of ordered pairs (a,b) of positive integers such that $\frac{2a-1}{b}$ and $\frac{2b-1}{a}$ are both integers is

A. 1

B. 2

C. 3

D. more than 3

Answer:

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12. Let $z = x + iy$ and $w = u + iv$ be two complex numbers, such that $|z| = |w| = 1$ and $z^2 + w^2 = 1$. Then, the number of ordered pairs (z, w) is equal to (where, $x, y, u, v \in R$ and $i^2 = -1$)

A. 0

B. 4

C. 8

D. infinite

Answer:

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13. Let $\sigma_1, \sigma_2, \sigma_3$ be planes passing through the origin. Assume that σ_1 is perpendicular to the vector (1,1,1), σ_2 is perpendicular to a vector (a,b,c) and σ_3 is perpendicular to the vector (a^2, b^2, c^2). What are all the positive values of a,b and c so that $\sigma_1 \cap \sigma_2 \cap \sigma_3$ is a single point?

A. Any positive value of a,b, and c other than 1

B. Any positive values of a,b and c where either $a \neq b, b \neq c$ or $a \neq c$

C. Any three distinct positive values of a,b and c

D. There exist no such positive real numbers a,b,and c

Answer:

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14. Ravi and Rashmi are each holding 2 red cards and 2 black cards (all four red and all four black cards are identical). Ravi picks a card at random from Rashmi, and then Rashmi picks a card at random from Ravi. This process is repeated a second time. Let p be the probability that both have all 4 cards of the same colour. Then p satisfies

A. $p \leq 5\%$

B. $5\% < p \leq 10\%$

C. $10\% < p \leq 15\%$

D. $15\% < p$

Answer:

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15. Let A_1, A_2 and A_3 be the region on R^2 defined by

$A_1 = \{(x, y): x \geq 0, y \geq 0, 2x + 2y - x^2 - y^2 > 1 > x + y\}$

,$A_2 = \{(x, y): x \geq 0, y \geq 0, x + y > 1 > x^2 + y^2\}$,

$A_3 = \{(x, y): x \geq 0, y \geq 0, x + y > 1 > x^3 + y^3\}$.

Denote by $|A_1|$,$|A_2|$, and $|A_3|$ respectively. Then

A. $|A_1| > |A_2| > |A_3|$

B. $|A_1| > |A_3| > |A_2|$

C. $|A_1| = |A_2| > |A_3|$

D. $|A_1| = |A_3| > |A_2|$

Answer:

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16. Let $f: R \to R$ be a continuous function such that $f(x^2) = f(x^3)$ for all $x \in R$. Consider the following statements. I. f is an odd function. II. f is an even function. III. f is differentiable everywhere Then

A. I is true and III is false

B. II is true and III is false

C. both I and III are true

D. both II and III are true

Answer:

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17. Suppose a continuous function $f:[0,\infty)\to R$ satisfies $f(x)=2\int_0^x tf(t)dt+1$for all $x\geq 0$ Then f(1) equals

A. e

B. e^2

C. e^4

D. e^6

Answer:

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18. Let $a > 0, a \neq 0$. Then the set S of all positive real numbers b satisfying $(1 + a^2)(1 + b^2) = 4ab$ is

A. an empty set

B. a singleton set

C. a finite set containing more than one element

D. $(0, \infty)$

Answer:

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19. Let $f: R \to R$ be function defined by $f(x) = \begin{cases} \frac{\sin(x^2)}{2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Then, at x=0, f is

A. not continuous

B. continuous but not differentiable

C. differentiable and the derivative is not continuous

D. differentiable and the derivative is continuous.

Answer:

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20. The points C and D on a semicircle with AB as diameter are such that AC=1, CD=2, and DB=3. Then the length of AB lies in the interval

A. [4,4.1]

B. [4.1,4.2]

C. [4.2,4.3]

D. $[4.3, \infty]$

Answer:

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21. Let $f(x) = x^6 - 2x^5 + x^3 + x^2 - x - 1$ and $g(x) = x^4 - x^3 - x^2 - 1$ be two polynomials. Let a,b,c and d be the roots of g(x)=0. Then the value of f(a)+f(b)+f(c)+f(d) is

A. -5

B. 0

C. 4

D. 5

Answer:

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22. The number of solutions to $\sin\left(\pi \sin^2(\theta)\right) + \sin\left(\pi \cos^2(\theta)\right) = 2\cos\left(\frac{\pi}{2}\cos(\theta)\right)$ satisfying $0 \leq \theta \leq 2\pi$ is

A. 1

B. 2

C. 4

D. 7

Answer:

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23. Let $J = \int_0^1 \frac{x}{1+x^8} dx.$ Consider the following assertions: $I.$ J>1/4$II.$ J

A. only I is true

B. only II is true

C. both I and II are true

D. neither I nor II is true

Answer:

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24. Let $f:(-1,1)\to R$ be a differentiable function satisfying $(f'(x))^4=16(f(x))^2$ for all $x\in(-1,1)$, f(0)=0. The number of such functions is

A. 2

B. 3

C. 4

D. more than 4

Answer:

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25. Let A be the set of vectors $\overrightarrow{a} = (a_1, a_2, a_3)$ satisfying

$$\left(\sum_{i=1}^{3} \frac{a_i}{2^i}\right)^2 = \sum_{i=1}^{3} \frac{a_i^2}{2^i}$$ Then

A. A is empty

B. A contains exactly one element

C. A has 6 elements

D. A has infinitely many elements

Answer:

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26. Let $f:[0, 1] \rightarrow [0, 1]$ be a continuous function such that $x^2 + (f(x))^2 \leq 1$ for all $x \in [0, 1]$ and $\int_0^1 f(x)dx = \frac{\pi}{4}$.

Then $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{f(x)}{1 - x^2} dx$ equals

A. $\frac{\pi}{12}$

B. $\frac{\pi}{15}$

C. $\frac{\sqrt{2} - 1}{2}\pi$

D. $\frac{\pi}{10}$

Answer:

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