MATHS

BOOKS - BITSAT GUIDE

BINOMIAL THEOREM AND MATHEMATICAL INDUCTION

Practice Exercise

1. In $\left(33 + \dfrac{1}{33}\right)^{n}$ if the ratio of 7th term from the beginning to the 7th term from the end is 1/6, then find the value of n.

A. 7

B. 8

C. 9

D. 10

Answer: C

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2. In the expansion of $\left[\dfrac{1}{a} + a^{\log_{10} a}\right]^5$, if the value of the third term is 1000, then the value of a is

A. 10

B. 100

C. 1000

D. 99

Answer: B

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3. If $z = \left(\dfrac{\sqrt{3}}{2} + \dfrac{i}{2} \right)^5 + \left(\dfrac{\sqrt{3}}{2} - \dfrac{i}{2} \right)^5$, then

A. Re(z)=0

B. Im (z)=0

C. Re (z) $> 0, Im(z) > 0$

D. Re (z) > 0, In (z) < 0

Answer: B

4. The expression

$$\left[x + (x^3 - 1)^{\frac{1}{2}}\right]^5 + \left[x - (x^3 - 1)^{\frac{1}{2}}\right]^5 \text{ is a polynomial of}$$

degree

A. 5

B. 7

C. 9

D. 6

Answer: B

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5. Find the coefficient of x^4 in the expansion of

$\left(x/2 - 3/x^2\right)^{10}$.

A. $\dfrac{305}{256}$

B. $\dfrac{405}{256}$

C. $\dfrac{504}{256}$

D. 0

Answer: B

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6. The expression $7^{2n} + 2^{3n-3}.3^{n-1}$, where $n \in N$, is divisible by

A. 7

B. 25

C. 2

D. 5

Answer: B

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7. If 7th term in the binomial expansion of $\left(\dfrac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x \right)^9$, $x > 0$ is equal to 729, then x can be

A. e^2

B. e

C. $\dfrac{e}{2}$

D. $2e$

Answer: B

8. If the coefficients of the rth, (r +1)th and (r + 2)th terms in the binomial expansion of (1 + y)^(m)` are in A.P., then m and r satisfy the equation

A. $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

B. $m^2 - m(4r + 1) + 4r^2 - 2 = 0$

C. $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

D. $m^2 - m(4r - 1) + 4r^2 - 2 = 0$

Answer: B

9. Find the middle term in the expansion of $(1 + x)^{2n}$

A. $\dfrac{1.3.5\ldots(2n - 1)}{n!} \cdot 2^n \cdot x^n$

B. $\dfrac{1.2.3.4\ldots(n + 1)}{(n + 1)!}$

C. $\dfrac{1.2.3.4\ldots n}{n!}$

D. None of these

Answer: A

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10. Find the middle term in the expansion $\left(\dfrac{2}{3}x^2 - \dfrac{3}{2x}\right)^{20}$.

A. $^{20}C_{10}x^{11}$

B. $^{20}C_9x^{10}$

C. $^{20}C_{10}x^{10}$

D. $^{20}C_9x^9$

Answer: C

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11. If the last term of $\left(2^{1/3} - \dfrac{1}{\sqrt{2}}\right)^n$ is $\left(\dfrac{1}{3.9^{1/3}}\right)^{\log_3 8}$,

then the 5th term from the beginning is

A. 210

B. 310

C. 200

D. 300

Answer: A

12. In the expansion of $\left(5^{1/6} + 2^{1/8}\right)^{100}$, the number of irrational terms are

    A. 93

    B. 95

    C. 97

    D. 99

Answer: C

13. Find the value of the greatest term in the expansion of

$$\sqrt{3}\left(1 + \frac{1}{\sqrt{3}}\right)^{20}.$$

A. $\binom{20}{7} \dfrac{1}{27}$

B. $\binom{20}{6} \dfrac{1}{81}$

C. $\dfrac{1}{9}\binom{20}{9}$

D. $\binom{20}{8} \dfrac{1}{3}$

Answer: A

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14. The greatest term in the expansion of $(3 + 5x)^{15}$, when

x=1/5, is

A. $^{15}C_3$

B. $^{15}C_4 \cdot 3^{11}$

C. $^{15}C_{10} \cdot 3^{10}$

D. $^{15}C_3 \cdot 3^{13}$

Answer: B

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15. Find the term independent of x in the expansion of $\left(3x - \dfrac{2}{x^2}\right)^{15}$.

A. $-3003\left(3^{10}\right)\left(2^5\right)$

B. $-3003\left(3^{10}\right)\left(2^4\right)$

C. $3003\left(3^{10}\right)\left(2^5\right)$

D. None of these

Answer: A

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16. If p is a real number and the middle term in the expansion of $\left(\dfrac{p}{2} + 2\right)^8$ is 1120, then find the value of p.

A. ± 3

B. ± 1

C. ± 2

D. ± 4

Answer: C

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17. The constant term in the expansion of $\left(1 + x + \dfrac{2}{x}\right)^6$ is

A. 479

B. 517

C. 569

D. 581

Answer: D

18. Find the term in $\left(\dfrac{a}{\sqrt{b}}3 + \sqrt{\dfrac{b}{a3}}\right)^{21}$ which has the same power of a and b.

A. 9

B. 10

C. 8

D. 6

Answer: A

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19. If the coefficient of x^7 and x^8 in $\left(2 + \dfrac{x}{3}\right)^n$ are equal, then n is

A. 56

B. 55

C. 45

D. 15

Answer: B

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20. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:7:42, then find the value of n.

A. 51

B. 53

C. 55

D. 57

Answer: C

21. If the sum of all the coefficients in the expansion of $\left(1 - 3x + 10x^2\right)^n$ is α and the sum of all the coefficients in the expansion of $\left(1 + x^2\right)^n$ is β, then which of the following is correct ?

A. $\alpha = 3\beta$

B. $\alpha = \beta^3$

C. $\beta = \alpha^3$

D. $\alpha = 2\beta$

Answer: B

22. If $C_0, C_1, C_2, \ldots, C_n$ denote the binomial coefficients

in the expansion of $(1+x)^n$, then

$$C_0^2 + 2C_1^2 + 3C_2^2 + \ldots + (n+1)C_n^2 =$$

A. $(2n+1)^{2n}C_n$

B. $(2n-1)^{2n}C_n$

C. $\left(\dfrac{n}{2} + 1\right)^{2n}C_n$

D. $\left(\dfrac{n}{2} + 1\right)^{2n-1}C_n$

Answer: C

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23. If a_1, a_2, a_3, a_4 be the coefficient of four consecutive

terms in the expansion of $(1+x)^n$, then prove that:

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}.$$

A. $\dfrac{2a_2}{a_2 + a_3}$

B. $\dfrac{2a_2}{a_3}$

C. $\dfrac{a_2}{a_2 + a_3}$

D. $\dfrac{a_3}{a_2}$

Answer: A

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24. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(X^2 + \dfrac{2}{x}\right)^{15}$ is

A. $7 : 16$

B. $7:64$

C. $1:4$

D. $1:32$

Answer: D

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25. The coefficient of the term independent of x in the expansion of $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}} \right)^{10}$

A. 4

B. 120

C. 210

D. 310

Answer: C

26. The coefficient of x^7 in the expansion of $\left(1 - x - x^2 + x^3\right)^6$ is :

A. -132

B. -144

C. 132

D. 144

Answer: B

27. The value of $^{50}C_4 + \sum\limits_{r=1}^{6} {}^{56-r}C_3$, is

A. $^{56}C_4$

B. $^{56}C_3$

C. $^{55}C_3$

D. $^{55}C_4$

Answer: A

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28. If $\dbinom{n}{r+1} = 56$, $\dbinom{n}{r} = 28$ and $\dbinom{n}{r-1} = 8$, then $n + r$ is equal to

A. 8

B. 10

C. 12

D. 9

Answer: B

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29. If $\left(1 + x - 2x^2\right)^6 = 1 + a_1 x + a_2 x^{12} + + a_{12} x^{12}$,

then find the value of $a_2 + a_4 + a_6 + + a_{12}$.

A. 32

B. 63

C. 64

D. 31

Answer: D

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30. The least positive integer n such that $\binom{n-1}{3} + \binom{n-1}{4} > \binom{n}{3}$ is

A. 6

B. 7

C. 8

D. 9

Answer: C

31. If $C_r = \begin{pmatrix} 10 \\ r \end{pmatrix}$, then $\Sigma_{r=1}^{10} C_{r-1} C_r$ is equal to

A. $\begin{pmatrix} 20 \\ 9 \end{pmatrix}$

B. $\begin{pmatrix} 20 \\ 10 \end{pmatrix}$

C. $\begin{pmatrix} 20 \\ 13 \end{pmatrix}$

D. $\begin{pmatrix} 20 \\ 8 \end{pmatrix}$

Answer: A

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32. In $n \in N$, then $121^n - 25^n + 1900^n - (-4)^n$ is divisible by

A. 1904

B. 2000

C. 2002

D. 2006

Answer: B

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33. The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by

9 is:

A. 0

B. 2

C. 7

D. 8

Answer: B

34. If $x = 7 + 4\sqrt{30}^{2n} = [x] + f$, then the value of $x(1 - f)$ is

A. 2^{2n}

B. 3^{2n}

C. 1^{2n}

D. 4

Answer: C

35. The expression $n^3 + 3n^2 + 5n + 3$, $n \in N$ is divisible by

A. 3

B. 4

C. 5

D. 6

Answer: A

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36. The sum of the series

$.^{20}C_0 - .^{20}C_1 + .^{20}C_2 - .^{20}C_3 + ... - . + .^{20}C_{10}$ is -

A. $- .^{20}C_{10}$

B. $\frac{1}{2} \, ^{20}C_{10}$

C. 0

D. $^{20}C_{10}$

Answer: B

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37. The number of terms in the expansion of

$(a + b + c)^n$, where $n \in N$.

A. $\dfrac{(n+1)(n+2)}{2}$

B. $n + 1$

C. $n + 2$

D. $(n + 1)n$

Answer: A

38. The vaule of $\displaystyle\sum_{r=0}^{n-1}\left(\dfrac{C_r}{{}^nC_r + {}^nC_{r+1}}\right)$ is equal to

A. $\dfrac{n}{2}$

B. $\dfrac{n+1}{2}$

C. $\dfrac{n-1}{2}$

D. 2n

Answer: A

39. The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}} + 3^{\frac{1}{5}} \right)^{10}$ is :

A. 25

B. 32

C. 9

D. 41

Answer: D

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40. If a and b are the coefficients of x^r and x^{n-r}, respectively in the expansion of $(1 + x)^n$, then

A. a=b

B. $a + b = n^2$

C. $a = nb$

D. $a + b = 2^{n/2}$

Answer: A

41. If n is a positive integer, then $\left(\sqrt{3} + 1\right)^{2n} - \left(\sqrt{3} - 1\right)^{2n}$

is

A. an irrational number

B. an odd positive integer

C. an even positive integer

D. a rational number other than positive integers

Answer: A

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42. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.\ 7} + \frac{1}{7.\ 11} + \frac{1}{11.\ 15} + \ + \frac{1}{(4n-1)(4n+3)} = \frac{n}{3(4n+3)}$$

A. $\dfrac{n}{3(4n+3)}$

B. $\dfrac{n}{5(5n+3)}$

C. $\dfrac{n}{3(7n+3)}$

D. 0

Answer: A

43. In the expansion of $\left(x^3 - \dfrac{1}{x^2} \right)^{15}$, the constant term, is

A. $^{15}C_9$

B. 0

C. $^{10}C_9$

D. $- \,^{15}C_9$

Answer: D

44. If $s_n = \sum\limits_{r=0}^{n} \dfrac{1}{.^n C_r}$ and $t_n = \sum\limits_{r=0}^{n} \dfrac{r}{.^n C_r}$, then $\dfrac{t_n}{s_n}$ is equal to

A. $\dfrac{n}{2}$

B. $\dfrac{n}{2} - 1$

C. $n - 1$

D. $\dfrac{2n - 1}{2}$

Answer: A

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45. The number of terms in the expansion of

$\left(1 + 5\sqrt{2}x\right)^{19} + \left(1 - 5\sqrt{2}x\right)^{19}$ is

A. 10

B. 7

C. 13

D. 4

Answer: A

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1. $2^{3n} - 7n - 1$ is divisible by

A. 64

B. 36

C. 49

D. 25

Answer: C

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2. If n is a positive integer, then $n^3 + 2n$ is divisible by

A. 2

B. 6

C. 15

D. 3

Answer: D

3. The sum of the coefficients in the expansion of $(5x - 4y)^n$, where n is a positive integer, is

    A. 0

    B. n

    C. 1

    D. -1

Answer: C

4. Let $a = \sum\limits_{n=0}^{\infty} \frac{x^{3n}}{(3n)}!$, $b = \sum\limits_{n=1}^{\infty} \frac{x^{3n-2}}{3n-2}!$ and

$C = \sum\limits_{n=1}^{\infty} \frac{x^{3n-1}}{3n-1}!$ and w be a complex cube root of unity

Statement 1: a+b+c

$= e^x$, $a + bw + cw^2 = e^{wx}$ and $a + bw^2 + cw = e^{w^2}$

Statement 2: $a^3 + b^3 + C^3 - 3abc = 1$

A. 1

B. 0

C. -1

D. -2

Answer: A

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5. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 17

Answer: C

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6. The coefficient of x^n in the expansion of $\log_e\left(\dfrac{1}{1 + x + x^2 + x^3}\right)$, when n is odd, is

A. $-\dfrac{2}{n}$

B. $-\dfrac{1}{n}$

C. $\dfrac{1}{n}$

D. $\dfrac{2}{n}$

Answer: B

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7. Find the value of

$$\dfrac{^{\wedge}(8)C_0}{6} -^8 C_1 +^8 C_2 \times 6 -^8 C_3 \times 6^2 + \dots \dots +^8 C_8 6^7$$

A. 0

B. 6^7

C. 6^8

D. $\dfrac{5^8}{6}$

Answer: D

8. If the coefficient of second, third and fourth terms in the expansion of $(1 + x)^n$ are in AP, then n is equal to

A. 7

B. 4

C. 5

D. 6

Answer: A

9. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then

$C_0 + 3C_1 + 5C_2 + \ldots + (2n+1)C_n =$

A. 3^n

B. 2^n

C. 1

D. 0

Answer: D

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10. The value of $(\sqrt{5}+1)^5 - (\sqrt{5}-1)^5$ is

A. 252

B. 352

C. 452

D. 552

Answer: B

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11. The coefficient of x^5 in the expansion of $\left(2 - x + 3x^2\right)^6$ is

A. -4692

B. 4692

C. 2346

D. -5052

Answer: D

12. If $C_1, C_2, C_3, \ldots C_n$ denote the coefficients in the binomial expansion of $(1+x)^n$, then the value of $^nC_1 + 2.\,^nC_2 + 3.\,^nC_3 + \ldots + n.\,^nC_n$ is

A. $(n-1)2^n$

B. $n.2^{n-1}$

C. $n.2^{n+1}$

D. None of these

Answer: B

13. Larger of $99^{50} + 100^{50}$ and 101^{50} is ....

A. 101^{50}

B. $99^{50} + 100^{50}$

C. Both are equal

D. None of these

Answer: A

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14. $\dfrac{1}{n!} + \dfrac{1}{2!(n-2)!} + \dfrac{1}{4!(n-4)!} + \ldots$ is equal to

A. $\dfrac{2^{n-1}}{n!}$

B. $\dfrac{2^n}{(n+1)!}$

C. $\dfrac{2^n}{n!}$

D. $\dfrac{2^{n-2}}{(n-1)!}$

Answer: A

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15. When 2^{301} is divided by 5, the least positive remainder is

A. 4

B. 8

C. 2

D. 6

Answer: C

16. The middle term in the expansion of $\left(x - \dfrac{1}{x} \right)^{18}$ is

A. $^{18}C_9$

B. $- \, ^{18}C_9$

C. $^{18}C_{10}$

D. $- \, ^{18}C_{10}$

Answer: B