MATHS

BOOKS - BITSAT GUIDE

EXPONENTIAL AND LOGARITHMIC SERIES

Practice Exercise

1. The sum of the series

$$1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\left(1+2+2^2+2^3\right)+\text{...is}$$

A. e^2

B. $e^2 + e$

C. $e^2 - e$

D. $e^2 - e - 1$

Answer: C

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2. The sum of the series $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \ldots$ is

A. e^{-1}

B. $2e^{-1}$

C. e^{-2}

D. $2e^{-2}$

Answer: A

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3. The sum of the series $1 + \frac{3}{2!} + \frac{7}{3!} + \frac{15}{4!} + \ldots$ is

A. $e(e+1$

B. $e(1-e)$

C. $e(e-1)$

D. 3e

Answer: C

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4. The sum of the series $\frac{5}{1\cdot 2\cdot 3}+\frac{7}{3\cdot 4\cdot 5}+\frac{9}{5\cdot 6\cdot 7}+\ldots$ is

A. $\log\frac{8}{e}$

B. $\log\frac{e}{8}$

C. $\log mn$

D. $\log\frac{m}{n}$

Answer: A

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5. The sum of the series is

$$9 + \frac{16}{2!} + \frac{27}{3!} + \frac{42}{4!} + ... \infty$$

A. 5e

B. 7e

C. 9e

D. $11e - 6$

Answer: D

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6. The sum of the series $\frac{1}{1.2}+\frac{1.3}{1\cdot 2\cdot 3\cdot 4}+\frac{1\cdot 3\cdot 5}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}+\ldots$ is

A. $e-1$

B. $e^{1/2}-1$

C. $e^{1/2}+e$

D. None of these

Answer: B

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7. If $\frac{e^{5x}+e^{x}}{e^{3x}}$ is expand in a series of ascending powers of x and n is and odd natural number then the coefficent of x^n is

A. $\frac{2^n}{n!}$

B. $\frac{2^{n+1}}{(2n!)}$

C. $\frac{2^{2n}}{(2n)!}$

D. 0

Answer: D

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8. The coefficient of x^n in the expansion of $\frac{a+bx+cx^2}{e^x}$ is

A. $\frac{(-1)^n}{n!}\left[cn^2-(b+c)n+a\right]$

B. $\frac{(-1)^n}{n!}\left[cn^2+(b+c)n+a\right]$

C. $\frac{(-1)^n}{n!}\left[cn^2+(b+c)n-a\right]$

D. None of the above

Answer: A

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9. The sum of the infinite series $\frac{2^2}{2!} + \frac{2^4}{4!} + \frac{2^6}{6!} + \ldots$ Is equal to

A. $\frac{e^2+1}{2e}$

B. $\frac{e^4+1}{2e^2}$

C. $\frac{\left(e^2-1\right)^2}{2e^2}$

D. $\frac{\left(e^2+1\right)^2}{2e^2}$

Answer: C

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10. The sum of the infinite series $1+\frac{1}{2!}+\frac{1\cdot 3}{4!}+\frac{1\cdot 3\cdot 5}{6!}+\ldots.$ is

A. e

B. e^2

C. $\sqrt{e}$

D. $\frac{1}{e}$

Answer: C

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11. $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+$ ..is equal to

A. $e^{1/2}$

B. e^{-1}

C. e

D. $e^{-1/3}$

Answer: B

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12. Prove that

$$\frac{1^3}{1!}+\frac{2^3}{2!}+\frac{3^3}{3!}+\frac{4^3}{4!}+\ldots\ldots=5e$$

A. 5e

B. 4e

C. 3e

D. 2e

Answer: A

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13. Prove that,

$$1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots.=\frac{3e}{2}$$

A. e

B. 2e

C. $\frac{3e}{2}$

D. $\frac{4e}{5}$

Answer: C

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14. $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots\ldots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\ldots\ldots}=$

A. $e+1$

B. $\frac{e-1}{e+1}$

C. $e-1$

D. 0

Answer: B

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15. The coefficent of x^n in the series

$$1+\frac{a+bx}{1!}+\frac{(a+bx)^2}{2!}+\frac{(a+bx)^3}{3!}+\ldots\text{is}$$

A. $\frac{(ab)^n}{n!}$

B. $e^b \cdot \frac{a^n}{n!}$

C. $e^a \cdot \frac{b^n}{n!}$

D. $e^{a+b} \cdot \frac{a^n}{n!}$

Answer: C

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16. The sum of the series $\sum_{n=1}^{\infty} \frac{2n}{(2n+1)!}$ is

A. e

B. e^{-1}

C. 2e

D. None

Answer: B

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17. The coefficient of x^k in the expansion of $\frac{1-2x-x^2}{e^{-x}}$ is

A. $\frac{1-k-k^2}{k!}$

B. $\frac{k^2+1}{k!}$

C. $\frac{1-k}{k!}$

D. $\frac{1}{k!}$

Answer: A

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18. If $\frac{1}{e^{3x}}\left(e^{x}+e^{5x}\right)=a^{0}+a_{1}x+a_{2}x^{2}+\ldots,$ then $2a_{1}+2^{3}a_{3}+2^{5}a_{5}+\ldots$ is equal to

A. e

B. e^{-1}

C. 1

D. 0

Answer: D

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19. If n is not a multiple of 3 then the coefficient of x^n in the expansio of $\log_e\left(1+x+x^2\right)$ is

A. $\frac{1}{n}$

B. $\frac{2}{n}$

C. $-\frac{1}{n}$

D. $-\frac{2}{n}$

Answer: A

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20. If α, β are the roots of the equation $x^2 - px + q = 0$, then the value of $(\alpha + \beta)x - \left(\frac{\alpha^2 + \beta^2}{2}\right)x^2 + \left(\frac{\alpha^3 + \beta^3}{3}\right)x^3 - \ldots.$ is

A. $\log\left(1 - px + qx^2\right)$

B. $\log\left(1 + px - qx^2\right)$

C. $\log\left(1 + px + qx^2\right)$

D. None of these

Answer: A

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21. The sum of the series $\frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \frac{4}{5}x^5 + \ldots$ is

A. $\frac{x}{1+x} + \log(1+x)$

B. $\frac{x}{1-x} + \log(1-x)$

C. $\frac{-x}{1+x} + \log(1+x)$

D. None of these

Answer: B

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22.

$$\frac{1}{2}\left(\frac{1}{2}+\frac{1}{3}\right)-\frac{1}{4}\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{6}\frac{1}{2^3}+\frac{1}{3^3}+\ldots\infty$$

is equal to

A. $\log_e 2$

B. $\log_e 3$

C. $2\log_e 2$

D. $\frac{1}{2}\log_e 2$

Answer: D

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23. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is

A. 3

B. 1

C. 2

D. 0

Answer: B

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24. The least value of the expression $2\log_{10} x - \log_x 0.01$ for $x > 1$ is .....

A. 2

B. 3

C. 5

D. 4

Answer: D

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25. If $S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2n)!}$ these S is equal to

A. $x + x^{-1}$

B. $x - x^{-1}$

C. $\frac{1}{2}\left(x + x^{-1}\right)$

D. 0

Answer: C

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26. If x, y, z are three consecutive positive integers, then

$$\frac{1}{2}\log_e x + \frac{1}{2}\log_e z + \frac{1}{2xz+1} + \frac{1}{3}\left(\frac{1}{2xz+1}\right)^3 + \ldots$$

is equal to

A. $\log_e x$

B. $\log_e y$

C. $\log_e z$

D. 0

Answer: B

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27. The coefficient of n^{-r} in the expansion of $\log_{10}\left(\frac{n}{n-1}\right)$ is

A. $\frac{1}{r \log_e 10}$

B. $-\frac{1}{r\log_e 10}$

C. $-\frac{1}{r!\log_e 10}$

D. None of these

Answer: A

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28. $\log_e.\ \frac{1+3x}{1-2x}$ is equal to

A. $-5-\frac{5x^2}{2}-\frac{35x^3}{3}-\ldots$

B. $-5x+\frac{5x^2}{2}-\frac{35x^3}{3}+\ldots$

C. $5x-\frac{5x^2}{2}+\frac{35x^3}{3}-\ldots$

D. $5x + \frac{5x^2}{2} + \frac{35x^3}{3} + \ldots$

Answer: C

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29. The sum of the infinite series

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{3^2}+\frac{1}{4^2}\right)+\frac{1}{6}\left(\frac{1}{3^3}+\frac{1}{4^3}\right)-$$

...is equal to

A. $\frac{1}{2}\log 2$

B. $\log\frac{3}{5}$

C. $\log\frac{5}{3}$

D. $\frac{1}{2}\log\frac{5}{3}$

Answer: D

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30. The sum of the series $1+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{4^2}+\frac{1}{7}\cdot\frac{1}{4^3}+\ldots$ is

A. $\log_e 1$

B. $\log_e 2$

C. $\log_e 3$

D. $\log_e 4$

Answer: C

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31. The sum of the series $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{3\cdot 4\cdot 5}+\frac{1}{5\cdot 6\cdot 7}+\ldots$ is

A. $\log_e 2-\frac{1}{2}$

B. $\log_e 2$

C. $\frac{\log_e 2}{2}-\frac{1}{2}$

D. $\log_e 2+1$

Answer: A

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32. The sum of the series $\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 - \ldots$ is

A. $1 - \log_e 2$

B. $1 + \log_e 2$

C. $\log_e 2$

D. $\log_e 3$

Answer: C

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33. The sum of the infinite series $\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{1}{3}\right)^4 + \frac{1}{5}\left(\frac{1}{3}\right)^6 + \ldots$ is

A. $\frac{1}{4}\log_e 2$

B. $\frac{1}{2}\log_e 2$

C. $\frac{1}{6}\log_e 2$

D. $\frac{1}{4}\log_e \frac{3}{2}$

Answer: C

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34. $\frac{x-y}{x}+\frac{1}{2}\left(\frac{x-y}{x}\right)^2+\frac{1}{3}\left(\frac{x-y}{x}\right)^3+\ldots.$

is equal to

A. $\log_e(x-y)$

B. $\log_e(x+y)$

C. $\log_e\left(\frac{x}{y}\right)$

D. $\log_e xy$

Answer: C

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35. $\log_e 3 - \frac{\log_e 9}{2^2} + \frac{\log_e 27}{3^2} - \frac{\log_e 81}{4^2} + \ldots$ is equal to

A. $(\log_e 3)(\log_e 2)$

B. $\log_e 3$

C. $\log_e 2$

D. $\frac{\log_e 5}{\log_e 3}$

Answer: A

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Bitsat Archives

1. If $\frac{e^x}{1-x} = B_0 + B_1 x + B_2 x^2 + \ldots + B_n x^n + \ldots,$

then the value of $B_n - B_{n-1}$ is

A. 1

B. 1/n

C. $\frac{1}{n!}$

D. None of these

Answer: A

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2. The sum of the series $\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots$ is

A. e^2

B. $\log_e 2 + 1$

C. $\log_e 3 - 2$

D. $1 - \log_e 2$

Answer: D

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3. Let $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)}!$, $b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{3n-2}!$ and $C = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{3n-1}!$ and w be a complex cube root of unity

Statement 1: a+b+c $= e^x, a + bw + cw^2 = e^{wx}$ and $a + bw^2 + cw = e^{w^2}$

Statement 2: $a^3 + b^3 + C^3 - 3abc = 1$

A. 1

B. 0

C. -1

D. -2

Answer: A

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4. The coefficient of in the expansion of $\log_e\left(\frac{1}{1+x+x^2+x^3}\right)$, when n is odd, is

A. $-\frac{2}{n}$

B. $-\frac{1}{n}$

C. $1/n$

D. None of these

Answer: B

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5. $7\log\left(\frac{16}{15}\right)+5\log\left(\frac{25}{24}\right)+3\log\left(\frac{81}{80}\right)$ is equal to

A. $\log 2$

B. 3

C. 5

D. 7

Answer: A

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6. The value of $\frac{2}{1!}+\frac{2+4}{2!}+\frac{2+4+6}{3!}+$ ... is

A. e

B. 2e

C. 3e

D. None of these

Answer: C

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7. The value of $\frac{1}{3}+\frac{1}{3\cdot 3^3}+\frac{1}{5\cdot 3^5}+\frac{1}{7\cdot 3^7}+\ldots$ is

A. $\frac{1}{2}\log_e 2$

B. $\log_e 2$

C. $\log_e 3$

D. None of these

Answer: A

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8. If $a = \log_2 3, b = \log_2 5$ and $c = \log_7 2,$ then $\log_{140} 63$ in terms of a, b, c is

A. $\frac{2ac + 1}{2c + abc + 1}$

B. $\frac{2ac + 1}{2a + c + a}$

C. $\frac{2ac+1}{2x+ab+a}$

D. None of the above

Answer: D

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