MATHS

BOOKS - V PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Question Bank

1. Check the continuity of the function $f(x) = 2x + 3$ at x=1

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2. Examine whether the function $f(x) = x^2$ is continuous at x=0

3. Discuss the continuity of the function f is given by $f(x) = |x|$ at $x = 0$

4. Show that the function $f(x) = \begin{cases} x^3 + 3 & \text{if} \ \ x \neq 0 \\ 1 & \text{if} \ \ x = 0 \end{cases}$ is not continuous at x=0

5. Check the point where the constant function $f(x) = k$ is continuous.

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6. Prove that the identity function on real numbers given by $f(x) = x$ is continuous at every real number

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7. Discuss the continuity of the function f is given by $f(x) = |x|$ at $x = 0$

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8. Discuss the continuity of the function

$$f(x) = x^3 + x^2 - 1$$

9. Discuss the continuity of the function

$$f(x) = \frac{1}{x}, \; x \neq 0$$

10. Check the continuity of the function

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x < 0 \\ -x + 2 & \text{if} \quad x > 0 \end{cases}$$

11. Discuss the continuity of the function given by

$$f(x) = \begin{cases} x & \text{if} & x \geq 0 \\ x^2 & \text{if} & x < 0 \end{cases}$$

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12. Show that every polynomial function is continuous

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13. Prove that the greatest integer function $f: R \to R$ given by $f(x) = [x]$ is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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14. Prove that every rational function is continuous.

15. Discuss the continuity of sine function.

16. Show that the function defined by $f(x) = \sin(x^2)$ is a continuous function.

17. Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$?

18. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$

19. Examine the following functions for continuity $f(x) = \dfrac{1}{x - 5}$

20. Prove that the function $f(x) = x^n$ is continuous at x=n, where n is a positive integer.

21. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at x=0

22. Find all points of discontinuity of f where f is defined by $f(x) = \begin{cases} 2x + 3 & x \leq 2 \\ 2x - 3 & x > 2 \end{cases}$

23. Find all points of discontinuity of f, where f is defined

by $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$

24. Evalute

$\lim\limits_{x \to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

25. Find all points of discontinuity of f, where f is defined

by $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$

26. Examine the continuity of the function $f(x) = \begin{cases} x + 1 & x \geq 1 \\ x^2 + 1 & x < 1 \end{cases}$

27. Examine the continuity of the function $f(x) = \begin{cases} x^3 - 3 & \text{if} & x \leq 2 \\ x^2 + 1 & \text{if} & x > 2 \end{cases}$.

28. Examine whether the function defined by

$$f(x) = \begin{cases} x + 5 & x \le 1 \\ x - 5 & x > 1 \end{cases}$$ is continuous or not .

29. Discuss the continuity of the function

$$f(x) = \begin{cases} 2x & \text{if} & x < 0 \\ 0 & \text{if} & 0 \le x \le 1 \\ 4x & \text{if} & x > 1 \end{cases}.$$

30. Find the derivative of the function given by

$$f(x) = \sin(x^2)$$

31. Differentiate the function $f(x) = \sin\left[\cos x^2\right]$ with respect to x.

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32. Differentiate the functions with respect to x:

$\sin\left(x^2 + 5\right)$

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33. Differentiate the functions with respect to x:

$\cos(\sin x)$

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34. Differentiate the functions with respect to x:

$\sin(ax + b)$

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35. Differentiate $\sec\left(\tan\left(\sqrt{x}\right)\right)$ w.r.t.x

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36. Differentiate $\dfrac{\sin(ax + b)}{\cos(cx + d)}$ w.r.t.x

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37. Differentiate $\cos\left(x^3\right)\sin^2\left(x^5\right)$ w.r.t.x

38. Differentiate the functions with respect to x:

$$2\sqrt{\cot x^2}$$

39. Differentiate $\cos\left(\sqrt{x}\right)$ w.r.t.x

40. Prove that the function f is given by $f(x) = |x - 1|$, $x \in R$ is not differentiable at $x = 1$

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41. Find $\dfrac{dy}{dx}$ if $x - y = \pi$

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42. Find $\dfrac{dy}{dx}$ if $y + \sin y = \cos x$

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43. Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exist.

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44. Find the derivative of f given by $f(x) = \tan^{-1} x$ assuming it exists.

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45. Find $\left(\dfrac{dy}{dx}\right)$ in the following $2x + 3y = \sin x$

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46. Find $\left(\dfrac{dy}{dx}\right)$ in the following $2x + 3y = \sin y$

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47. Find $\left(\dfrac{dy}{dx}\right)$ in the following $ax + by^2 = \cos y$

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48. Find $\left(\dfrac{dy}{dx}\right)$ in the following $xy + y^2 = \tan x + y$

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49. Find $\dfrac{dy}{dx}$ of the following

$x^2 + xy + y = 100$

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50. Find $\dfrac{dy}{dx}$ of the following

$x^3 + x^2y + xy^2 + y^3 = 81$

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51. Find $\left(\dfrac{dy}{dx}\right)$ in the following. $\sin^2 y + \cos xy = \pi$

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52. Find $\left(\dfrac{dy}{dx} \right)$ in the following $\sin^2 x + \cos^2 y = 1$

53. Find $\dfrac{dy}{dx}$ of the following

$y = \sin^{-1} \left(\dfrac{2x}{1 + x^2} \right)$

54. Find $\left(\dfrac{dy}{dx} \right)$ in the following $y = \tan^{-1} \left(\dfrac{3x - x^3}{1 - 3x^2} \right)$

55. Find $\left(\dfrac{dy}{dx}\right)$ in the following $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

56. Find $\dfrac{dy}{dx}$ of the following

$y = \sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

57. Find $\left(\dfrac{dy}{dx}\right)$ in the following. $y = \cos^{-1}\left(\dfrac{2x}{1+x^2}\right)$

58. Find $\left(\dfrac{dy}{dx}\right)$ in the following

$$y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

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59. Find $\dfrac{dy}{dx}$ of the following

$$y = \sec^{-1}\left(\dfrac{1}{2x^2 - 1}\right)$$

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60. Differentiate the following w.r.t. X:

(i) e^{-x}

(ii) $\sin(\log(x)), \; x > 0$

(iii) $\cos^{-1}(e^x)$

(iv) $e^{\cos x}$

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61. Differentiate the following w.r.t.x $\dfrac{e^x}{\sin x}$

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62. Differentiate the following w.r.t.x $e^{\sin^{-1} x}$

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63. Differentiate the following w.r.t x. e^{x^3}

64. Differentiate the following w.r.t. x

$$\sin\left(\tan^{-1} e^{-x}\right)$$

65. Differentiate the following w.r.t.x $\log(\cos e^x)$

66. Differentiate the following w.r.t. x

$$e^x + e^{x^2} + \ldots\ldots\ldots + e^{x^5}$$

67. Differentiate the following w.r.t.x $\sqrt{e^{\sqrt{x}}}, x > 0$

68. Differentiate the following w.r.t.x $\log(\log x), x > 1$

69. Differentiate the following w.r.t. x

$\dfrac{\cos x}{\log x}, x > 0$

70. Differentiate the following w.r.t. x

$\cos(\log x + e^x), \ x > 0$

71. Differentiate $\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t.x.

72. Differentiate a^x w.r.t. x, where a is a positive constant.

73. Differentiate $x^{\sin x}$ w.r.t. x

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74. Differentiate the functions $\cos x \cos 2x \cos 3x$

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75. Differentiate $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect to x.

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76. Differentiate the function $(\log x)^{\cos x}$

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77. Differentiate the functions $x^x - 2^{\sin x}$

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78. Differentiate the function $(x+3)^2 . (x+4)^3 . (x+5)^4$

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79. Differentiate the function $\left(x + \dfrac{1}{x}\right)^x + x^{1+\frac{1}{x}}$

80. Differentiate the function $(\log x)^x + x^{\log x}$

81. Differentiate the function $(\sin x)^x + \sin^{-1}\sqrt{x}$

82. Differentiate the function $x^{\sin x} + (\sin x)^{\cos x}$

83. Differentiate the function $x^{x \cos x} + \dfrac{x^2 + 1}{x^2 - 1}$

84. Differentiate the function $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

85. Find $\left(\dfrac{dy}{dx}\right)$ of the function $x^y + y^x = 1$

86. Find $\left(\dfrac{dy}{dx}\right)$ of the function $y^x = x^y$

87. Find $\left(\dfrac{dy}{dx}\right)$ of the function $(\cos x)^y = (\cos y)^x$

88. Find $\left(\dfrac{dy}{dx}\right)$ of the function $xy = e^{x-y}$

89. Find the derivative of the functions given by

$f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$ and hence

find $f'(1)$

90. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ by using product rule

91. Find $\left(\dfrac{dy}{dx}\right)$, if $x = a\cos\theta$, $y = a\sin\theta$

92. Find $\left(\dfrac{dy}{dx}\right)$, if $x = at^2$, $y = 2at$

93. Find $\dfrac{dy}{dx}$

if $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

94. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx}\right)$

$x = 2at^2, y = at^4$

95. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx}\right)$

$x = a\cos\theta, y = b\cos\theta$

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96. Find $\dfrac{dy}{dx}$, $x = \sin t, y = \cos 2t$

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97. Find $\dfrac{dy}{dx}$, $x = 4t, y = \dfrac{4}{t}$

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98. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx} \right)$

if $x = \cos\theta - \cos 2\theta,\ y = \sin\theta - \sin 2\theta$

99. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx} \right)$

if $x = a(\theta - \sin\theta),\ y = a(1 + \cos\theta)$

100. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx} \right)$

if $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}, y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$

101. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx}\right)$ if $x = a\left(\cos t + \log\tan\left(\dfrac{t}{2}\right)\right), y = a\sin t$

102. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx}\right)$ if $x = a\sec\theta, y = b\tan\theta$

103. If x and y are connected parametrically by the equation without eliminating the parameter, find $\left(\dfrac{dy}{dx}\right)$ if $x = a(\cos\theta + \theta\sin\theta)$ $y = a(\sin\theta - \theta\cos\theta)$

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104. If $x = a^{\sin^{-1}t}$, $y = a^{\cos^{-1}t}$, $a > 0$, show that $\dfrac{dy}{dx} = -\dfrac{y}{x}$

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105. Find $\dfrac{d^2y}{dx^2}$, if $y = x^3 + \tan x$

106. If $y = A \sin x + B \cos x$, then prove that $\dfrac{d^2 y}{dx^2} + y = 0$

107. If $y = 3e^{2x} + 2e^{3x}$, prove that $\dfrac{d^2 y}{dx^2} - 5\left(\dfrac{dy}{dx}\right) + 6y = 0$

108. If $y = \sin^{-1} x$, then show that $\left(1 - x^2\right) \dfrac{d^2 y}{dx^2} - x \dfrac{dy}{dx} = 0$

109. Find the second order derivative of the following functions $x^2 + 3x + 2$

110. Find the second order derivative of the following functions x^{20}

111. Find the second order derivative of the following functions xcosx

112. Find the second order derivative of the following functions $\log x$

113. Find the second order derivatives of the function $x^3 \log x$

114. Find the second order derivatives of the function $e^x \sin 5x$

115. Find the second order derivative of the function

$$e^{6x} \cos 3x$$

116. Find the second order derivative of the function

$$\tan^{-1} x$$

117. Find the second order derivative of the function

$$\log(\log x)$$

118. Find the second order derivative of the function $\sin(\log x)$

119. If $y = 5\cos x - 3\sin x$, prove that $\dfrac{d^2y}{dx^2} + y = 0$

120. If $y = \cos^{-1} x$ find $\dfrac{d^2y}{dx^2}$ in terms of y alone.

121. If $y = 3\cos(\log x) + 4\sin(\log x)$ Show that $x^2 y_2 + x y_1 + y = 0$

122. If $y = ae^{mx} + be^{nx}$, show that

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

123. If $y = 500e^{7x} + 600e^{-7x}$. Show that $\dfrac{d^2y}{dx^2} = 49y$

124. If $e^y(x + 1) = 1$, show that $\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2$

125. If $y = \left(\tan^{-1} x\right)^2$, show that

$$\left(x^2 + 1\right)^2 y_2 + 2x\left(x^2 + 1\right)y_1 = 2.$$

126. Verify Rolle's theorem for the function

$f(x) = x^2 + 2$ in the interval [-2,2]

127. Verify Mean Value Theorem for the function $f(x) = x^2$ in the interval $[2, 4]$

128. Verify Rolle's Theorem for the function $f(x) = x^2 + 2x - 8 , x \in [-4, 2]$

129. If $f : [-5, 5] \to R$ is a differentiable function and if f'(x) does not vanish any where, then prove that $f(-5) \neq f(5)$

130. Verify mean value theorem for the function $f(x) = x^2 - 4x - 3$ in the interval [1,4] .

131. Verify Lagrange 's mean value theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a,b], where a=1 and b=3. Find all $c \in (1, 3)$ at which f'(c)=0.

132. Differentiate w.r.t. x, the following functions:

(i) $\sqrt{3x + 2} + \dfrac{1}{\sqrt{2x^2 + 4}}$

(ii) $e^{\sec^2 x} + 2\cos^{-1} x$

(iii) $\log_7(\log x)$

133. Differentiate the following w.r.t. x

(i) $\cos^{-1}(\sin x)$

(if) $\tan^{-1}\left(\dfrac{\sin x}{1 + \cos x}\right)$

(iii) $\sin^{-1}\left(\dfrac{2^{x+1}}{1 + 4^x}\right)$

134. Find $f'(x)$ if $f(x) = (\sin x)^{\sin x}$ for all $0 < x < \pi$

135. For a positive constant a find $\left(\dfrac{dy}{dx}\right)$, where $y = a^{t + \frac{1}{t}}$, and $x = \left(t + \dfrac{1}{t}\right)^{a}$

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136. Differentiate $\sin^{2} x$ w.r.t $e^{\cos x}$

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137. Differentiate w.r.t x the function $(3x^{2} - 9x + 5)^{9}$

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138. Differentiate w.r.t x the function $\sin^3 x + \cos^6 x$

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139. Differentiate w.r.t x the function $(5x)^{3\cos 2x}$

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140. Differentiate w.r.t x the function $\sin^{-1}(x\sqrt{x}), 0 \le x \le 1$

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141. Differentiate w.r.t x the function $\dfrac{\cos^{-1}\left(\frac{x}{2}\right)}{\sqrt{2x+7}}$

142. Differentiate w.r.t x the functions $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

143. Differentiate w.r.t x the function $(\log x)^{\log x}$, $x > 1$

144. Differentiate w.r.t x the function

$\cos(a\cos x + b\sin x)$, for some constant a and b

145. Differentiate w.r.t x the function

$(\sin x - \cos x)^{(\sin x - \cos x)}$

146. Differentiate the following w.r.t.x $x^x + x^a + a^x + a^a$

for some fixed $a > 0$ and $x > 0$

147. Differentiate w.r.t x the function $x^{x^2-3} + (x-3)^{x^2}$,

for $x > 3$

148.   Find

$\dfrac{dy}{dx}$,   if $y = 12(1 - \cos t), x = 10(t - \sin t). \dfrac{\pi}{2} < t < \dfrac{\pi}{2}$

149. Find $\left(\dfrac{dy}{dx}\right)$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}$,

$-1 \le x \le 1$

150. if $x\sqrt{1+y} + y\sqrt{1+x} = 0$ prove that $\left(\dfrac{dy}{dx}\right) = -\dfrac{1}{(1+x)^2}$

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151. if $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$ prove that $\dfrac{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ is constant independent of a and b

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152. If $\cos y = x\cos(a + y)$ prove that $\left(\dfrac{dy}{dx}\right) = \dfrac{\cos^2(a + y)}{\sin a}$

153. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, find $\dfrac{d^2 y}{dx^2}$

154. Using mathematical induction prove that $\dfrac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n.

155. Using the fact that $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and the

technique differentiation. Obtain the sum formula for cosines.

156. "All continuous function are not differentiable." Justify your answer with an example

157. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$ prove that $\left(\dfrac{dy}{dx}\right) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

158. If $y = e^{a \cos^{-1} x}$, then show that

$$\left(1 - x^2\right) y_2 - x y_1 - a^2 y = 0$$

159. Let $y = \left(x + \sqrt{1 + x^2}\right)^m$

i) Find $\left(\dfrac{dy}{dx}\right)$

ii) Show that $\left(1 + x^2\right) \dfrac{d^2 y}{dx^2} + x \left(\dfrac{dy}{dx}\right) - m^2 y = 0.$

160. Let $y = \tan^{-1}\left(\dfrac{1 - \cos x}{\sin x}\right)$

i) Express $1 - \cos x$ and $\sin x$ in terms of $\sin\left(\dfrac{x}{2}\right)$ and $\cos\left(\dfrac{x}{2}\right)$

ii) Hence find $\left(\dfrac{dy}{dx}\right)$.

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161. If $xy = ax^2 + \dfrac{b}{x}$, prove that $x^2 y_2 + 2\left(x\left(\dfrac{dy}{dx}\right) - y\right) = 0$

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162. Establish that $g(x) = 1 - x + |x|$ is continious at origin.

163. a) Find $\left(\dfrac{dy}{dx}\right)$ if

i) $y = \sin(x^3 + 7)$

ii) $x = a(t - \sin t), y = a(1 - \cos t)$

b) If $y = a\cos(\log x) + b\sin(\log x)$

Prove that $x^2 \dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right) + y = 0$

164. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if} \quad x < 0 \\ x + 1 & \text{if} \quad x \geq 0 \end{cases}$$

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165. Find the point of discontinuities of the function.

$$\frac{x + 1}{12x^2 + 7x - 10}$$

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166. Consider $f(x) = \begin{cases} 3x - 8 & x \leq 5 \\ 2k & x > 5 \end{cases}$. Find the value

of k if f(x) is continuous at x=5`.

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167. Find the point of discontinuity, if any of the following functions

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if} \quad x \neq 1 \\ 2 & \text{if} \quad x = 1 \end{cases}$$

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168. Find the value of m, if any, so that the function

$$h(x) = \begin{cases} m(x^2 - 2x) & \text{if} \quad x < 0 \\ \cos x & \text{if} \quad x \geq 0 \end{cases}$$ is continuous

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