



MATHS

BOOKS - V PUBLICATION

RELATIONS AND FUNCTIONS

Question Bank

1. Let P={1,2,3} , Q={a, b} Find P x Q and Q x P

2. A={p, q, r}, B={m}

Write 'A x B' and all subs of 'A x B'.



3. Let A be the set of all students of a boys school. Show that the relation R in A given by $R=\{(a, b): a is$ sister of b} is the empty relation and $R'=\{(a, b): the$ difference between heights of a and b is less than 3 meters} is the universal relation.



4. Let T be the set of all triangles in a plane with R as relation in T given by $R = \{(T_1, T_2): T_1 \cong T_2\}$. Show that R is an equivalence relation.

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5. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1(\text{ is perpendicular to L})_2\}$. Show that R is symmetric but neither reflexive nor transitive.



6. Show that the relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$

is reflexive but neither symmetric nor transitive.

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7. Show that the relation R on the set Z of integers,

given by $R=\{(a,\ b)\!:\!2$ divides $a-b\}$, is an

equivalence relation.



8. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) \colon both a$

and *b* are either odd or even}. Show that *R* is an equivalence relation. Further, show that all the elements of the subset $\{1, 3, 5, 7\}$ are related to each other and all the elements of the subset $\{2, 4, 6\}$ are related to each other, but no element of the subset $\{1, 3, 5, 7\}$ is related to any element of the subset $\{2, 4, 6\}$.

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9. Determine whether each of the following relations is

reflexive, symmetric and transitive.

Relation R in the set

A = {1,2,,3,...,13,14} defined as R= {(x,y) : 3x - y = 0}

10. Show that the relation R in the set R of real numbers, defined at $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.



11. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive,

symmetric or transitive.



12. Show that the relation R on R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.



13. Check whether the relation R on R defined by $R = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

14. Show that the relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.



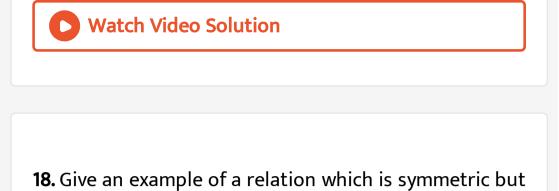
15. Show that the relation R on the set A of all the books in a library of a college given by $R = \{(x, y) : x \text{ and } y \text{ have the same number of pages}\}$, is an equivalence relation.



16. Show that the relation R on the set $A = \{1, 2, 3, 4, 5\}$, given by $R = \{(a, b) : |a - b| \text{ is even }\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But, no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.



17. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$,given by R = {(a,b) : |a-b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 in each case.



neither reflexive nor transitive.



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19. Show that the relation R. in the set A of all points in a place given by

R = {(P,Q): the distance of the points P from the point Q from the origin }, is an equivalence relation. Further show that the set of all points related to a point p
eq (0,0) is the circle passing through P with origin as

centre.



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20. Show that the relation R. defined in the set A of all triangle as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2, \text{ is an equivalence relation. Consider three right angled triangles <math>T_1$ with sides 3,4,5, T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangle among T_1, T_2 and T_3 are related ?



21. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2): P_1 \text{ and } P_2 \text{ have the same number of sides }, \text{is an equivalence relation.What is the set of all elements in A related to the right angled triangle T with sides 3,4 and 5?$

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22. Let L be the set of all line in XY place and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$ Show that R is an equivalence relation. Find the set of all line related to the line y=2x+4

23. Let R be the relation in the set {1,2,3,4,} given by R = {(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)}.Choose the correct answer. a) R is reflexive and symmetric but not transitive b) R is reflexive and transitive but not symmetric c) R is symmetric and transitive but not reflexive d) R is an equivalence relation

A. R' isreflexive and transitive but not transitive

B. R' is reflexive and transitive but not symmetric.

C. R' is symmetric and transitive but. not reflexive.

D. R' is an equivalence relation.

Answer: B



24. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$

choose the correct answer

A. (2,4) in R'

B. (3,8) in R'

C. (6,8) in R'

D. (8,7) in R'

Answer: C



25. Let A be the set of all 50 students of class X in a school. Let f: A o Nbe function defined by f(x) = roll numbers of the student x. Show that

f is one-one but not onto



26. The function $f\!:\!N o N$ given by f(x)=2x

27. Prove that the function $f\!:\!R o R$ given by f(x)=2x is one-one and onto.

28. Show that the function f:N o N, given by f(1)=f(2)=1 and f(x)=x-1, for every x>2, is

onto but not one - one

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29. Check the injective and surjective of the following

functions : $f\!:\!R o R$ given by $f(x)=x^2$



30. Show that $f\colon N o N$ given by

$$f(x) = egin{cases} (x+1) & ext{if} \ xisodd \ (x-1) & ext{if} \ xiseven \end{cases}$$

is both one-one and onto.



31. Check the injectivity and surjectivity of the following

functions

 $f{:}N
ightarrow N$ defined by $f(x)=x^3$



32. Prove that the greatest integer function $f: R \to R$ given by f(x) = [x] is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.



33. Show that the modulus function $f \colon R \to R$ given

by f(x) = |x|, is neither one-one nor onto.

34. Show that the signum function $f \colon R o R$ given by

$$f(x) = egin{cases} 1 & ext{if} \ x > 0 \ 0 & ext{if} \ x = 0 & ext{is} \ ext{neither} \ ext{one-one} \ ext{nor} \ -1 & ext{if} \ x < 0 \end{cases}$$

onto.



35. Let A = {1, 2, 3}, B { 4, 5, 6} and let f = {(1,4),(2,5)(3,6)}

be a function from A to B. Show that f is one-one.



36. In each of the following cases, states whether the function is one-one,

onto or bijective. Justify your answer. $f\!:\!R o R$

defined by f(x) = 3 - 4x



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37. Let A and B be sets. Show that

 $f{:}\,A imes B o B imes A$ such that

f(a,b)=(b,a) is a bijective function

38. Let f: N to N be defined by

$$f(n) = \left\{egin{array}{c} \left(rac{n+1}{2}
ight) ext{ if } nisodd \ rac{n}{2} ext{ if } niseven \end{array}
ight.$$
 for all $n \in N$

State whether the function f is bijective. Justify yur

answer.

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39. Let $A = R - \{3\}$ and $B = R - \{1\}$ Consider the

function $f\!:\!A o B$ defined by $f(x)=rac{x-2}{x-3}$

Is f one-one and onto? Justify your answer.

40. Let $f\!:\!R o R$ be defined as f(x) = x^4 Choose the

correct answer

A. 'f' is one-one onto

B. 'f' is many-one onto

C. f' is one-one but not onto

D. 'f' is neither one-onenor onto.

Answer: D



41. Let $f \colon R \to R$ be defined as f(x) = 3x Choose the

correct answer

A. f' is one-one onto

B. f' is many-one onto

C. 'f' is one-one but not onto

D. f' is neither one-one nor onto

Answer: A



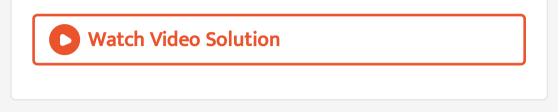
42. Let $f: \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g: \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined as f(2) = 3, f(3) = 4, f(4) = f(5) = 5 and g(3) = g(4) = 7 and g(5) = g(9) = 11. Find gof.

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43. Find gof and fog, if f: R o R and g: R o R are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that gof
eq fog.

44. Show that if $f \colon A o B$ and $g \colon B o C$ are one-one,

then gof: A
ightarrow C is also one-one.



45. Show that if $f \colon A \to B$ and $g \colon B \to C$ are onto,

then gof: A
ightarrow C is also onto.



46. Are 'f' and 'g' both necessarily onto, if gof, is onto?



47. Let $f: \{1, 2, 3, \} \rightarrow \{a, b, c\}$ be one-one, and onto function given by f(1) = a, f(2) = b and f(3) = c. Shów that there exists a function $g: \{a, b, c\} \rightarrow \{1, 2, 3, \}$ such that $gof = I_x$ and $fog = , I_y$ where, $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$

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48. Consider $f: R \to R$ given f(x) = 4x + 3. Show that f

is invertible. Find the inverse of f.

49. Consider of $f \colon N o N$ and $h \colon N o R$ defined as

f(x) = 2x, g(y) = 3y + 4

and h(z) = sin z, \forall , x, y and z in N. Show that ho(gof) =

(hog)of.



50. Let S = {1, 2, 3}. Determine whether the function $f: S \rightarrow S$ defined as below have inverses. Find f^{-1} , if it exists (a) f = {(1,1), (2,2), (3,3)} (b) f = {(1,2), (2,1), (3,1)}

 $(c) f = \{(1,3), (3,2), (2,1)\}$



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51. Let f: {1, 3, 4} \rightarrow {1, 2, 5} and
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g : {1, 2, 5} \rightarrow (1, 3} be given by
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f = {(1,2), (3,5), (4,1)} and
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g = {(1,3), (2,3), (5,1)}

Write down g o f

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52. Let f, g, and h be functions from R to R. Show that

(f+g)oh = foh + goh

53. Find f o g and g o f

f(x) = |x| and g(x) = |5x - 2|

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54. If f(x) =
$$rac{4x+3}{6x-4}, x
eq rac{2}{3}$$
, show that (fof)(x) = x, for all $x
eq rac{2}{3}$.

What is the inverse of f?

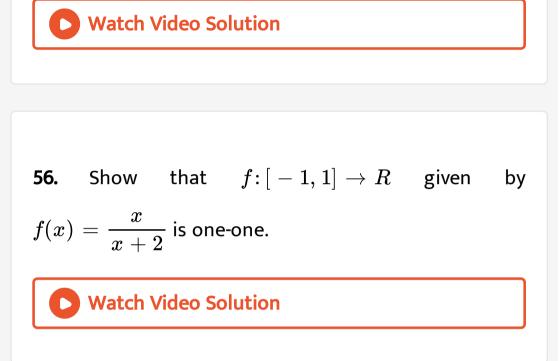
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55. State with reason whether the following functions

have inverse

f: {1,2,3,4} to {10} with

 $f = \{(1,10),(2,10),(3,10),(4,10)\}$



57. Consider $f: R \to R$ given f(x) = 4x + 3. Show that f

is invertible. Find the inverse of f.

58. Consider $f\colon R^+ o [-5,\infty)$ given by $f(x)=9x^2+6x-5.$ Show that f is invertible with $f^{-1}(y)=rac{\sqrt{y+6}-1}{3}$

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59. Let $f \colon X \to Y$ be invertible, show that f has unique

inverse



60. Consider f : {1, 2, 3} \rightarrow {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. find f^{-1} and show that $(f^{-1})^{-1}$ = f



61. Let $f: X \to Y$ be invertible, show that the inverse of $f^{-1} = f$, i.e., $(f^{-1})^{-1} = f$. Watch Video Solution

62. If $f\!:\!R o R$ be given by f(x) = $\left(3-x^3
ight)^{rac{1}{3}}$, then (f o

f) (x) is

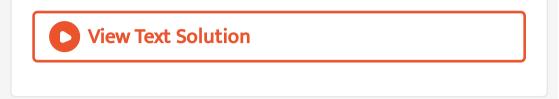
63. f: $R - \left\{\frac{-4}{3}\right\} \to R$ be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \to R - \left\{\frac{-4}{3}\right\}$ given by

Answer: B



64. Can you define a function from 'RxR rarr R' such as

'f_3: RxR rarr, R' defined by 'f_3(a, b)=a/b ?'



65. Can you define a function from 'R xx R rarr R' such

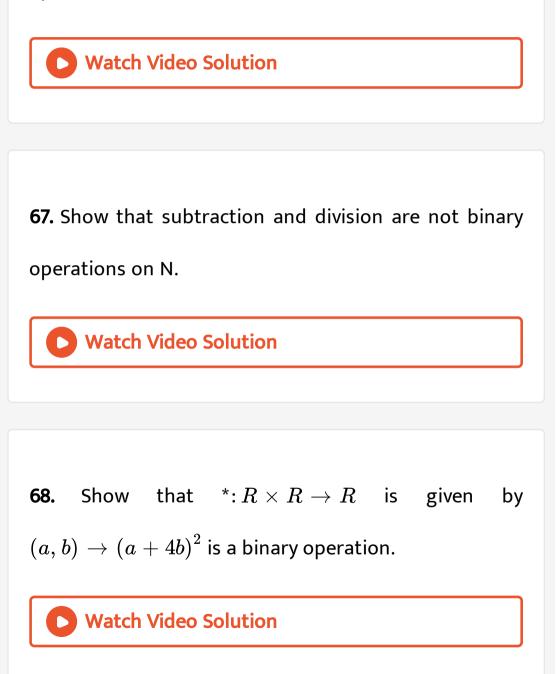
as 'f_4: R xx R rarr R' defined by 'f_4(a, b)=sqrt (a b)'



66. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary

operation on R. Further, show that division is a binary

operation on the set R of nonzero real numbers.



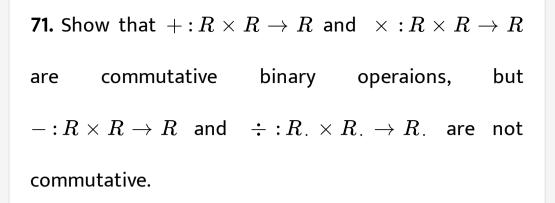
69. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \to P$ given by $(A, B) \to A \cup B$ and $\cap : P \times P \to P$ given by $(A, B) \to A \cap B$ are binary operations on the set P.



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70. Show that the $\vee : R \times R \to R$ given by $(a, b) \to \max \{a, b\}$ and the $*: R \times R \to R$ given by $(a, b) \to \min \{a, b\}$ are binary operations.







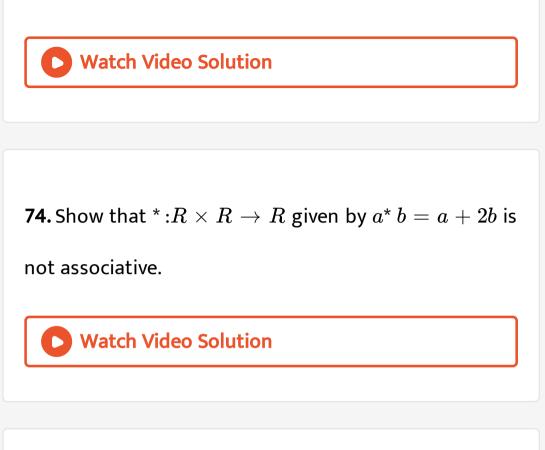
72. Show that *: R imes R o R defined by a * b = a + 2b

is not commutative.



73. Show that addition and multiplication are associativé binary operation on R. But subtraction is

not associative on R. Division is not associative on R.



75. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations. $-: R \times R \to R$ and $\div : R. \times R \to R$

76. Show that -a is not the inverse of $a \in N$ for the addition operation '+' on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation 'X' on N for $a \neq 1$



77. Determine whether or not each of the definitions of
* given below gives a binary operation. In the event
that * is not a binary operation, give justification for
this

on Z^+ , define st by a st b = |a-b|

78. For each binary operation * defined below, determine whether * is commutative or associative on Z, define a * b = a - b

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79. Consider the binary operation on a set $\{1, 2, 3, 4, 5\}$. defined by $a \wedge b = \min \{a, b\}$. Write the operation table of the operation.



80. Consider the binary operation * on the set A= {1,2,3,4,5} given by the following multiplication table Compute (2 * 3) * 4 and 2 * (3 * 4)

*	ľ	2	3	4	5
1	1	1	1	i	1
2	1	2	1	2	1
3	1	1	3	1	1
4	. 1	2	1	4	1
5	1	.1	1	•1	5

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81. Let * ' be the binary operation on N given by a * 'b

= L.c.m. of a and b. Find 5 * 7, 20 * 16



82. Consider an operation * defined on the set

$$A=\{1,2,4,8\}$$
 by $a*b=LCM$ of a and b.

Show that * is a binary operation.



- 83. Let * be a binary operation on N defined by
- a * b = HCF of a and b
- Is * commutative?



84. Let * be a binary operation on the set Q of

rational numbers as follows

a * b = a - b.

Check whether * is commutative and associative



85. Let

A = N imes N and $\, * \,$

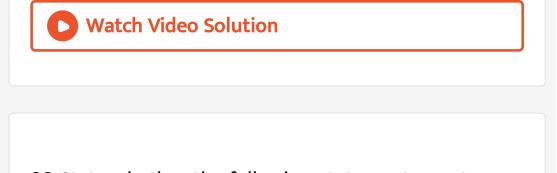
be a binary operation on A defined by (a,b)*(c,d) = (a +

c,b+d)

Prove that

*

is associative



86. State whether the following statements are true or false. Justify

For an arbitary binary operation * on N, $a*a=a orall a \in N$

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87. Consider a binary operation * on N defined as $a * b = a^3 + b^3$. Choose the correct answer. a) both associative and commutative b)commutative but not

associative c) associative but not commutative d)

neither commutative nor associative

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88. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.



89. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by R_1 ={(x, y): x-y is divisible by 3} and R_2 be another relation on X given by

 $R_2 = \{(x,y) \colon \{x,y\} \subset \{1,4,7\}$ or $\ \{x,y\} \subset \{2,5,8\}$ or . $\{x,y\} \subset \{3,6,9\}\}$. Show that $R_1 = R_2$



90. Let $f \colon X o Y$, be a function. Define a relation R in X given by: $R = \{(a, b) \colon f(a) = f(b)\}$ Examine if R is

an equivalence relation.

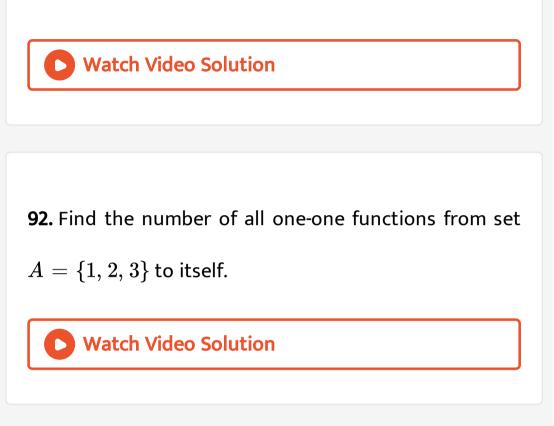
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91. Determine which of the following binary operation

on the set R are associative and which are

communtative.

a * b = 1



93. Consider the identity function, $I_N\colon N o N$ defined as $I_n(x)=x\,orall x\in N.$ Show that although I_N is onto but $I_N+I_N\colon N o N$ defined as.

$$(I_N+I_N)(x)=I_N(x)+I_N(x)=x+x=2x$$
 is not

onto

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94. Consider a function
$$f:\left[0,rac{\pi}{2}
ight] o R$$
 given by f(x) = sin x and $g:\left[0,rac{\pi}{2}
ight] o R$

given by g(x) = cos x. Show that f and g are one-one, but

f + g is not one-one.



95. Let $f \colon R o R$ be defined as f(x) = 10x+7

Find the function $g \colon R o R$ such that gof = fog = I_R



96. Let $f \colon W o W$ be defined as f(n) = n-1

if n is odd and

f(n) = n+1 if n is even.

Show that f is invertible. Find the inverse of f. Here W is

the set of all whole numbers.



97. If $f\!:\!R o R$ is defined by

 $f(x)=x^2-3x+2$, find f(f(x))



98. Show that the function $f \colon R o R$ given by $f(x) = x^3$ is injective.

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99. Given a non empty set X, consider P(X) which is the set of all subsets of X. Define the relation R in P(X) as follows. For subsets A, B in P(X), ARB if and only of $A \subset B$. Is R an equivalence relation on P(X). Justify your answer.

100. Find the number of all onto functions from the set

 $\{1, 2, 3. \ldots n\}$ to itself.

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101. Let
$$S = \{a, b, c\}$$
 and $T = \{1, 2, 3\}$, Find F^{-1} of

the following functions F from S to T if it exist.

i)
$$F = \{(a,3), (b,2), (c,1)\}$$

ii)
$$F = \{(a,2), (b,1), (c,1)\}$$

102. Let $A=\{-1,0,1,2,\}B=\{-4,-2,0,2\}$ and $f,g\colon A o B$ be functions defined by $f(x)=x^2-x,x\in A$ and $g(x)=2\Big|x-rac{1}{2}\Big|-1,x\in A$

.Are f and g equal?. Justify your answer.

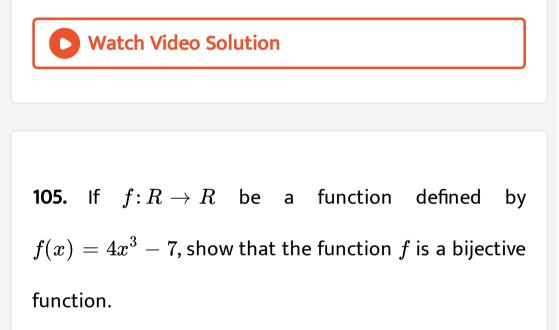


103. Number of binary operations on the set $\{a, b\}$ are

(A) 10 (B) 16 (C)20 (D) 8



104. Show that the relation R on the set $N \times N$ defined by (a, b)R(c, d) iff ad(b + c) = bc(a + d) is an equivalence relation.





106. Consider the binary operation *: Q o Q where Q is the set of rational numbers as defined as a*b = a + b - abFind 2*3

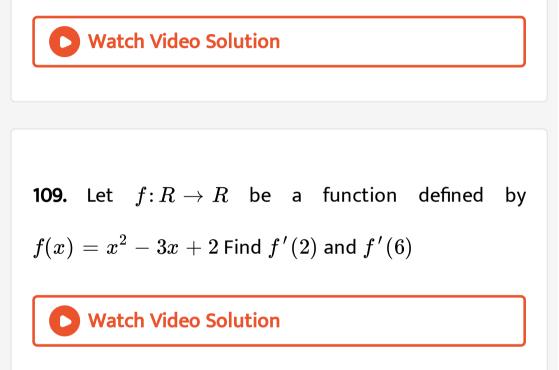
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107. Prove that the function $f \colon Q o Q$ given by f(x) = 2x - 3 for all $x \in Q$ (the set of all rational

numbers) is a bijection.

108. Prove that the function $f\colon N o N$, defined by

 $f(x) = x^2 + x + 1$ is one-one but not onto.



110. If f: R
ightarrow R be defined by f(x) = 5x - 3, then prove

that f is one-one and onto and find a formula for $f^{\,-1}$

111. Let $f,g\colon R o R$ be defined by $f(x)=x^2+1$ and $g(x)=\sin x.$ Find fog and gof .

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112. If $f, g \colon R o R$ are defined by $f(x) = x^2 + 3x + 1$,

find

g(x)=2x-3,

(i) fog(ii) fof(iii) (gof)(2) (iv) (gog)(-5)

113. Let * be a binary operation on N, the set of all

natural numbers defined by

 $a^{\star}b$ = a^{b} , $a, b \in N$. Is (N,*) associative or commutative?

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114. Let A = N imes N and let * be a binary operation on

A defined by

 $(a, b)^*(c, d)$ =(ac, bd) show that

(i) (A,*) is associative

(ii)(A,*) is commutative

