MATHS

BOOKS - V PUBLICATION

RELATIONS AND FUNCTIONS

Question Bank

1. Let P={1,2,3} , Q={a, b} Find P x Q and Q x P

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2. A={p, q, r}, B={m}

Wriţe 'A x B' and all subs of 'A x B'.

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3. Let A be the set of all students of a boys school. Show that the relation R in A given by R={(a, b): a is sister of b} is the empty relation and R'={(a, b): the difference between heights of a and b is less than 3 meters} is the universal relation.

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4. Let T be the set of all triangles in a plane with R as relation in T given by $\mathrm{R} = \{(\mathrm{T}_1, \ \mathrm{T}_2) : T_1 \cong \mathrm{T}_2\}$. Show that R is an equivalence relation.

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5. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1(\text{ i s p e r p e n d i c u l a r t o L})_2\}$. Show that R is symmetric but neither reflexive nor transitive.

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6. Show that the relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is reflexive but neither symmetric nor transitive.

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7. Show that the relation R on the set Z of integers, given by $R = \{(a, b) : 2$ divides $a - b\}$, is an equivalence relation.

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8. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b) :$ both a

and b are either odd or even}. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

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9. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set

A = {1,2,,3,...,13,14} defined as R= {(x,y) : 3x -y = 0}

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10. Show that the relation R in the set R of real numbers, defined at $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

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11. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$as $R = \{(a, b) : b = a + 1\}$is reflexive, symmetric or transitive.

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12. Show that the relation R on R defined as $R = \{(a,\ b) : a \leq b\}$, is reflexive and transitive but not symmetric.

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13. Check whether the relation R on R defined by $R = \{(a,\ b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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14. Show that the relation R on the set $A = \{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

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15. Show that the relation R on the set A of all the books in a library of a college given by $R = \{(x, y) : x$ and y have the same number of pages$\}$, is an equivalence relation.

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16. Show that the relation R on the set $A = \{1, 2, 3, 4, 5\}$, given by $R = \{(a, b) : |a - b|$ is even }, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But, no element of {1, 3, 5} is related to any element of {2, 4}.

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17. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$,given by

R = {(a,b) : |a-b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 in each case.

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18. Give an example of a relation which is symmetric but neither reflexive nor transitive.

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19. Show that the relation R. in the set A of all points in a place given by

R = {(P,Q): the distance of the points P from the point Q from the origin }, is an equivalence relation. Further show that the set of all points related to a point

$p \neq (0, 0)$is the circle passing through P with origin as centre.

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20. Show that the relation R. defined in the set A of all triangle as $R = \{(T_1, T_2) : T_1$is similar to T_2,is an equivalence relation. Consider three right angled triangles $T_1$with sides 3,4,5,T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangle among T_1, T_2 andT_3 are related ?

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21. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have the same number of sides },is an equivalence relation.What is the set of all elements in A related to the right angled triangle T with sides 3,4 and 5?

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22. Let L be the set of all line in XY place and R be the relation in L defined as$R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$Show that R is an equivalence relation. Find the set of all line related to the line y=2x+4

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23. Let R be the relation in the set {1,2,3,4,} given by R = {(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)}.Choose the correct answer. a) R is reflexive and symmetric but not transitive b) R is reflexive and transitive but not symmetric c) R is symmetric and transitive but not reflexive d) R is an equivalence relation

A. R' isreflexive and transitive but not transitive

B. R' is reflexive and transitive but not symmetric.

C. R' is symmetric and.transitive but. not reflexive.

D. R' is an equivalence relation.

Answer: B

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24. Let R be the relation in the set N given by

$R = \{(a, b) : a = b - 2, b > 6\}$

choose the correct answer

A. (2,4) in R'

B. (3,8) in R'

C. (6,8) in R'

D. (8,7) in R'

Answer: C

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25. Let A be the set of all 50 students of class X in a school. Let $f: A \rightarrow N$

be function defined by f(x) = roll numbers of the student x. Show that

f is one-one but not onto

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26. The function $f: N \rightarrow N$ given by $f(x) = 2x$

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27. Prove that the function $f: R \rightarrow R$ given by $f(x) = 2x$ is one-one and onto.

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28. Show that the function $f: N \rightarrow N$, given by $f(1) = f(2) = 1$ and $f(x) = x - 1$, for every $x > 2$, is onto but not one - one

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29. Check the injective and surjective of the following functions : $f: R \rightarrow R$ given by $f(x) = x^2$

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30. Show that $f: N \rightarrow N$ given by

$$f(x) = \begin{cases} (x+1) & \text{if} \quad x is odd \\ (x-1) & \text{if} \quad x is even \end{cases}$$

is both one-one and onto.

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31. Check the injectivity and surjectivity of the following functions

$f: N \rightarrow N$ defined by $f(x) = x^3$

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32. Prove that the greatest integer function $f: R \rightarrow R$ given by $f(x) = [x]$ is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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33. Show that the modulus function $f: R \rightarrow R$ given by $f(x) = |x|$, is neither one-one nor onto.

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34. Show that the signum function $f: R \rightarrow R$ given by

$$f(x) = \begin{cases} 1 & \text{if } \ x > 0 \\ 0 & \text{if } \ x = 0 \\ -1 & \text{if } \ x < 0 \end{cases}$$ is neither one-one nor onto.

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35. Let A = {1, 2, 3}, B { 4, 5, 6} and let f = {(1,4),(2,5)(3,6)} be a function from A to B. Show that f is one-one.

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36. In each of the following cases, states whether the function is one-one,

onto or bijective. Justify your answer. $f: R \to R$ defined by f(x) =3 - 4x

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37. Let A and B be sets. Show that

$f: A \times B \to B \times A$ such that

$f(a, b) = (b, a)$ is a bijective function

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38. Let f: N to N be defined by

$$f(n) = \begin{cases} \left(\frac{n+1}{2}\right) & \text{if} \quad nisodd \\ \frac{n}{2} & \text{if} \quad niseven \end{cases} \text{ for all } n \in N$$

State whether the function f is bijective. Justify yur answer.

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39. Let $A = R - \{3\}$ and $B = R - \{1\}$ Consider the function $f: A \rightarrow B$ defined by $f(x) = \dfrac{x-2}{x-3}$

Is f one-one and onto? Justify your answer.

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40. Let $f: R \rightarrow R$ be defined as f(x) = x^4 Choose the correct answer

A. 'f' is one-one onto

B. 'f' is many-one onto

C. f' is one-one but not onto

D. 'f' is neither one-onenor onto.

Answer: D

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41. Let $f: R \to R$ be defined as f(x) = 3x Choose the correct answer

A. f' is one-one onto

B. f' is many-one onto

C. 'f' is one-one but not onto

D. f' is neither one-one nor onto

Answer: A

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42. Let $f:\{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$ and $g:\{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$ be functions defined as $f(2) = 3, f(3) = 4,$ $f(4) = f(5) = 5$ and $g(3) = g(4) = 7$ and $g(5) = g(9) = 11$. Find gof.

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43. Find gof and fog, if $f:R \rightarrow R$ and $g:R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $gof \neq fog$.

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44. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are one-one, then $gof: A \rightarrow C$ is also one-one.

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45. Show that if $f: A \rightarrow B$ and $g: B \rightarrow C$ are onto, then $gof: A \rightarrow C$ is also onto.

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46. Are 'f' and 'g' both necessarily onto, if gof, is onto?

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47. Let $f: \{1, 2, 3, \} \to \{a, b, c\}$ be one-one, and onto function given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Shów that there exists a function $g: \{a, b, c\} \to \{1, 2, 3. \}$ such that $gof = I_x$ and $fog = , I_y$ where, $X = \{1, 2, 3\}$ and $Y = \{a, b, c\}$

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48. Consider $f: R \to R$ given f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

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49. Consider of $f: N \to N$ and $h: N \to R$ defined as

f(x) = 2x, g(y) = 3y + 4

and h(z) = sin z, $\forall$, x, y and z in N. Show that ho(gof) =

(hog)of.

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50. Let S = {1, 2, 3}. Determine whether the function

$f: S \to S$ defined as below have inverses. Find f^{-1}, if

it exists

(a) f = {(1,1), (2,2), (3,3)}

(b) f = {(1,2), (2,1), (3,1)}

(c) f = {(1,3), (3,2), (2,1)}

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51. Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and

g : {1, 2, 5} $\rightarrow$ (1, 3} be given by

f = {(1,2), (3,5), (4,1)} and

g = {(1,3), (2,3), (5,1)}

Write down g o f

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52. Let f, g, and h be functions from R to R. Show that

$(f+g)oh = foh + goh$

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53. Find f o g and g o f

f(x) = |x| and g(x) = |5x -2|

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54. If f(x) = $\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that (fof)(x) = x, for all $x \neq \frac{2}{3}$.

What is the inverse of f ?

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55. State with reason whether the following functions have inverse f : {1,2,3,4} to {10} with

f = {(1,10),(2,10),(3,10),(4,10)}

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56. Show that $f: [-1, 1] \rightarrow R$ given by $f(x) = \frac{x}{x+2}$ is one-one.

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57. Consider $f: R \rightarrow R$ given f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

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58. Consider $f: R^{+} \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}$

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59. Let $f: X \rightarrow Y$ be invertible, show that f has unique inverse

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60. Consider f : {1, 2, 3} $\rightarrow$ {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. find f^{-1} and show that $\left(f^{-1}\right)^{-1}$ = f

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61. Let $f: X \to Y$ be invertible, show that the inverse of f^{-1} = f,

i.e.,$\left(f^{-1}\right)^{-1} = f.$

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62. If $f: R \to R$ be given by f(x) = $\left(3 - x^3\right)^{\frac{1}{3}}$, then (f o f) (x) is

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63. f : $R-\left\{\frac{-4}{3}\right\} \rightarrow R$ be a function defined as $f(x)=\frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \rightarrow R-\left\{\frac{-4}{3}\right\}$ given by

A. g(y)=(3 y)/(3-4 y)'

B. 'g(y)=(4 y)/(4-3 y)'

C. g(y)=(4 y)/(3-3 y)'

D. g(y)=(3 y)/(4-3 y)'

Answer: B

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64. Can you define a function from 'RxR rarr R' such as 'f_3: RxR rarr, R' defined by 'f_3(a, b)=a/b ?'

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65. Can yóu define a function from 'R xx R rarr R' such as 'f_4: R xx R rarr R' defined by 'f_4(a, b)=sqrt (a b)'

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66. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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67. Show that subtraction and division are not binary operations on N.

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68. Show that $*: R \times R \rightarrow R$ is given by $(a, b) \rightarrow (a + 4b)^2$ is a binary operation.

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69. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \to P$ given by $(A, B) \to A \cup B$ and $\cap : P \times P \to P$ given by $(A, B) \to A \cap B$ are binary operations on the set P.

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70. Show that the $\vee : R \times R \to R$ given by $(a, b) \to \max\{a, b\}$ and the $*: R \times R \to R$ given by $(a, b) \to \min\{a, b\}$ are binary operations.

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71. Show that $+ : R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operaions, but $- : R \times R \to R$ and $\div : R_{.} \times R_{.} \to R_{.}$ are not commutative.

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72. Show that *: $R \times R \to R$ defined by a * b $= a + 2b$ is not commutative.

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73. Show that addition and multiplication are associativé binary operation on R. But subtraction is not associative on R. Division is not associative on R.

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74. Show that $* : R \times R \to R$ given by $a^* b = a + 2b$ is not associative.

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75. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations. $- : R \times R \to R$ and $\div : R_. \times R_. \to R_.$

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76. Show that $-a$ is not the inverse of $a \in N$ for the addition operation '+' on N and $\frac{1}{a}$ is not the inverse of $a \in N$ for multiplication operation 'X' on N for $a \neq 1$

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77. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on Z^+, define $*$ by $a * b = |a - b|$

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78. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative on Z, define $a * b = a - b$

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79. Consider the binary operation on a set $\{1, 2, 3, 4, 5\}$. defined by $a \wedge b = \min\{a, b\}$. Write the operation table of the operation.

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80. Consider the binary operation $*$ on the set A= {1,2,3,4,5} given by the following multiplication table

Compute $(2 * 3) * 4$ and $2 * (3 * 4)$

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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81. Let $*$ ' be the binary operation on N given by $a * 'b$ = L.c.m. of a and b. Find $5 * 7, 20 * 16$

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82. Consider an operation $*$ defined on the set

$A = \{1, 2, 4, 8\}$ by $a * b = LCM$ of a and b.

Show that $*$ is a binary operation.

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83. Let $*$ be a binary operation on N defined by

$a * b = HCF$ of a and b

Is $*$ commutative?

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84. Let $*$ be a binary operation on the set Q of rational numbers as follows

$a * b = a - b.$

Check whether * is commutative and associative

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85. Let

$A = N \times N$ and $*$

be a binary operation on A defined by (a,b)*(c,d) = (a + c,b + d)

Prove that

$*$

is associative

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86. State whether the following statements are true or false. Justify

For an arbitary binary operation $*$ on N, $a * a = a \forall a \in N$

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87. Consider a binary operation $*$ on N defined as $a * b = a^3 + b^3$. Choose the correct answer. a) both associative and commutative b)commutative but not associative c) associative but not commutative d) neither commutative nor associative

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88. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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89. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R_1 be a relation in X given by R_1={(x, y): x-y is divisible by 3} and R_2 be another relation on X given by

$R_2 = \{(x, y) : \{x, y\} \subset \{1, 4, 7\}$or $\{x, y\} \subset \{2, 5, 8\}$

or . $\{x, y\} \subset \{3, 6, 9\}\}$. Show that $R_1 = R_2$

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90. Let $f : X \to Y$, be a function. Define a relation R in X given by: $R = \{(a, b) : f(a) = f(b)\}$ Examine if R is an equivalence relation.

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91. Determine which of the following binary operation on the set R are associative and which are communtative.

$a * b = 1$

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92. Find the number of all one-one functions from set $A = \{1, 2, 3\}$ to itself.

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93. Consider the identity function, $I_N : N \rightarrow N$ defined as $I_n(x) = x \forall x \in N$. Show that although I_N is onto but $I_N + I_N : N \rightarrow N$ defined as.

$(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto

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94. Consider a function $f : \left[0, \frac{\pi}{2}\right] \rightarrow R$ given by f(x) = sin x and $g : \left[0, \frac{\pi}{2}\right] \rightarrow R$ given by g(x) = cos x. Show that f and g are one-one, but f + g is not one-one.

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95. Let $f : R \rightarrow R$ be defined as f(x) = 10x+7

Find the function $g : R \rightarrow R$ such that gof = fog = I_R

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96. Let $f : W \rightarrow W$ be defined as f(n) = n-1

if n is odd and

f(n) = n+1 if n is even.

Show that f is invertible. Find the inverse of f. Here W is the set of all whole numbers.

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97. If $f : R \rightarrow R$ is defined by

$f(x) = x^2 - 3x + 2$, find f(f(x))

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98. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective.

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99. Given a non empty set X, consider $P(X)$ which is the set of all subsets of X . Define the relation R in $P(X)$ as follows. For subsets A, B in $P(X)$, ARB if and only of $A \subset B$. Is R an equivalence relation on $P(X)$. Justify your answer.

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100. Find the number of all onto functions from the set $\{1, 2, 3. \dots ..n\}$to itself.

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101. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$, Find F^{-1} of the following functions F from S to T if it exist.

i) $F = \{(a, 3), (b, 2), (c, 1)\}$

ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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102. Let $A = \{-1, 0, 1, 2, \} B = \{-4, -2, 0, 2\}$ and $f, g : A \to B$ be functions defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$ .Are f and g equal?. Justify your answer.

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103. Number of binary operations on the set $\{a, b\}$ are

(A) 10 (B) 16 (C)20 (D) 8

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104. Show that the relation R on the set $N \times N$ defined by $(a, b)R(c, d)$ iff $ad(b+c) = bc(a+d)$ is an equivalence relation.

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105. If $f: R \to R$ be a function defined by $f(x) = 4x^3 - 7$, show that the function f is a bijective function.

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106. Consider the binary operation $* : Q \rightarrow Q$ where Q is the set of rational numbers as defined as $a * b = a + b - ab$

Find $2 * 3$

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107. Prove that the function $f: Q \rightarrow Q$ given by $f(x) = 2x - 3$ for all $x \in Q$ (the set of all rational numbers) is a bijection.

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108. Prove that the function $f: N \rightarrow N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

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109. Let $f: R \rightarrow R$ be a function defined by $f(x) = x^2 - 3x + 2$ Find $f'(2)$ and $f'(6)$

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110. If $f: R \rightarrow R$ be defined by f(x) = 5x - 3, then prove that f is one-one and onto and find a formula for f^{-1}

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111. Let $f, g: R \rightarrow R$ be defined by $f(x) = x^2 + 1$and $g(x) = \sin x$. Find fog and gof .

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112. If $f, g: R \rightarrow R$ are defined by $f(x) = x^2 + 3x + 1,$ $g(x) = 2x - 3,$ find $(i) fog (ii) fof (iii) (gof)(2) (iv) (gog)(-5)$

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113. Let * be a binary operation on N, the set of all natural numbers defined by

$a^* b$=a^b, $a, b \in N$. Is (N,*) associative or commutative?

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114. Let $A = N \times N$ and let * be a binary operation on A defined by

(a, b)*(c, d)=(ac, bd) show that

(i) $(A$,*) is associative

(ii)$(A$,*) is commutative

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