

MATHS

BOOKS - V PUBLICATION

THREE DIMENSIONAL GEOMETRY

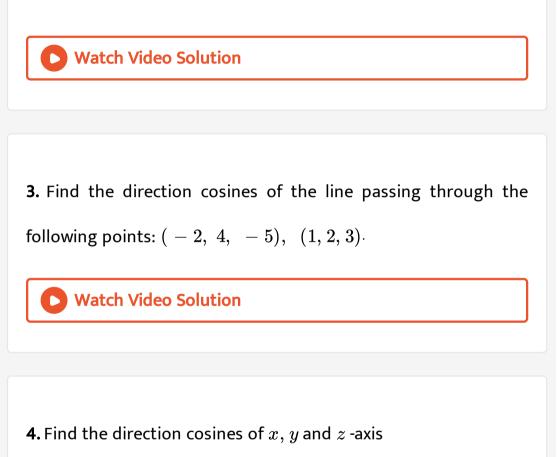
Question Bank

1. If a line makes angles. $90^\circ,\,60^\circ,\,30^\circ$ with the x, y and z axes

respectively,

find its direction cosines

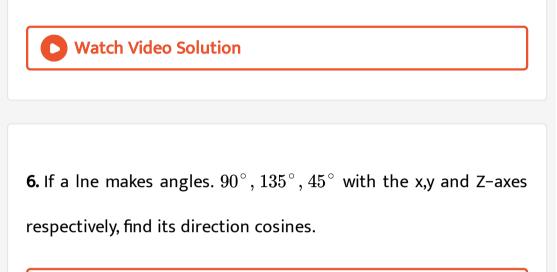
2. If a line has direction ratios '2,-1,-2', determine direction cosines.





5. Show that the points 'A(2,3,-4), B(1,-2, 3)' and 'C(3,8,-11)' are

collinear



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7. Find the direction cosines of a line which makes equal angles

with the coordinate axes.



8. If a line has the direction ratios '-18,(12) ,-4', then what are its

directión cosines?

Watch Video Solution 9. Show that the points (2, 3, 4), (-1, -2, 1), (5, 8, 7) are collinear. Watch Video Solution 10. Find the direction cosines of the sides of the triangle whose

vertices are '(3,5,-4),(-1,1,2)' and '(-5,-5,-2)'



11. Find the vector and Cartesian equations of the line through the point (5, 2, -4) and which is pralel to the vector $3\hat{i} + 2\hat{j} - 8k$.

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12. Find the vector equation for the line passing through the points (1, 0, 2) and (3, 4, 6).

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13. The Cartesian equation of a line is $\displaystyle rac{x+3}{2} = \displaystyle rac{y-5}{4} = \displaystyle rac{z+6}{2}$

. Find the vector equation for the line.

14. Find the angle between the pair of lines given by

$$ar{r}=3\hat{i}+2\hat{j}-4\hat{k}+\lambda\Big(\hat{i}+2\hat{j}+2\hat{k}\Big)$$
 and $\overrightarrow{r}=5\hat{i}-2\hat{j}+\mu\Big(3\hat{i}+2\hat{j}+6\hat{k}\Big)$



15. find the angle between the pair of lines
$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$$
 and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

16. Find the shortest distance between the lines l_1 and l_2 $\bar{r} = \hat{i} + \hat{j} + \lambda \left(2\hat{i} - \hat{j} + k\right)$ and $\bar{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

17. Find the distance between the lines l_1 and l_2 , given by

$$ar{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda \Big(2\hat{i} + 3\hat{j} + 6\hat{k} \Big)$$
 and $ar{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu \Big(2\hat{i} + 3\hat{j} + 6\hat{k} \Big)$



18. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}; \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

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19. Show that the line through the points (1, -1, 2) and (3, 4,-2) is

perpendicular to the through the points (0, 3,2) and (3, 5, 6).



20. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (-1, -2, 1), (1, 2, 5).



21. The equation of the line which passes through the point (1,2,3) and parallel to the vector 3i+2j-2k is



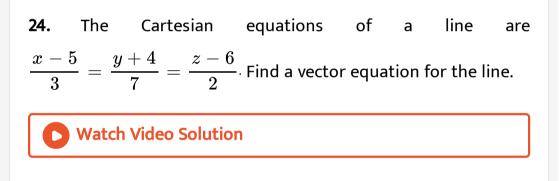
22. Find the equation of the line in vector and in cartesian form that passes through the point with position vector '2 hati-hatj+4 hatk' and is in the direction 'hati+2 hatj-hatk'.



23. Find the Cartesian equation of the line with passes through

the point (-2, 4, -5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{3} = \frac{z+8}{6}.$

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25. Find the Cartesian equation of the line passing through origin and (5,-2,3)



26. Find the vector and the cartesian equations of the lines that

passes through

the points (3, -2, -5) and (3, -2, 6)



27. Find the angle between the following pairs of lines:

i)
$$\overrightarrow{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda\Big(3\hat{i}+2\hat{j}+6\hat{k}\Big)$$
 and

$$\overrightarrow{r}=7\hat{i}-6\hat{k}+\mu\Bigl(\hat{i}+2\hat{j}+2\hat{k}\Bigr)$$

(ii)
$$\overrightarrow{r}=3\hat{i}+\hat{j}-2\hat{k}+\lambda\Big(\hat{i}-\hat{j}-2\hat{k}\Big)$$
 and

$$\overrightarrow{r}=2\hat{i}-\hat{j}-56\hat{k}+\mu\Bigl(3\hat{i}-5\hat{j}-4\hat{k}\Bigr)$$
 .

28. Find the angles between the lines

$$rac{x-2}{2} = rac{y-1}{5} = rac{z+3}{-3}$$
 and $rac{x+2}{-1} = rac{y-4}{8} = rac{z-5}{4}$

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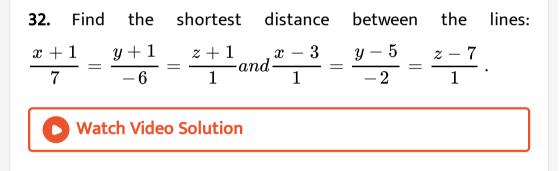
29. Find the values of p so that the lines
$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$$
 and
$$\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$
 are at right angles.

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30. Show that the lines
$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$$
 and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

31. Find the shortest distance between the skew lines $\vec{r} = (\vec{i} + 2\vec{i})$

 $j + k + \lambda (i - j + k)$ and $r = (2i - j - k) + \eta (2i + j + 2k)$



33. Find the shortest distance between the lines whose vector equations are $\overrightarrow{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\overrightarrow{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

34. Find the shortest distance between the lines whose vector equations $ext{are} o r = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $o r = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$



35. Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is

 $2\hat{i}-3\hat{j}+4\hat{k}$ Also, find its Cartesian form.

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36. Find the direction cosines of the unit vector perpendicular to the plane $\rightarrow r6\hat{i} - 3\hat{j} - 2\hat{k} + 1 = 0$ passing through the origin.

37. Find the distance of the plane 2x - 3y + 4z - 6 = 0 from the origin.

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38. Find the coordinates of the foot of the perpendicular drawn

from the origin to the plane 2x - 3y + 4z - 6 = 0.



39. Find the vector and cartesian equations of the plane which passes through the point (5, 2, -4) and perpendicular to the line with direction ratios 2, 3, -1



40. Find the vector equations of the plane passing through the

points R(2, 5, -3), S(-2, -3, 5) and T(5, 3, -3).



41. Find the equation of the plane with intercepts 2, 3 and 4 on

the x, y and z-axis respectively.

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42. Find the vector equation of the Plane Passing through the intersection of the planes $\bar{r}.(i + j + k) = 6$ and $\bar{r}.(2i + 3j + 4k) = -5$ and through the point (1,1,1).

43. Show that the lines

$$rac{x+3}{-3} = rac{y-1}{1} = rac{z-5}{5}$$
 and $rac{x+1}{-1} = rac{y-2}{2} = rac{z-5}{5}$ are

coplanar.

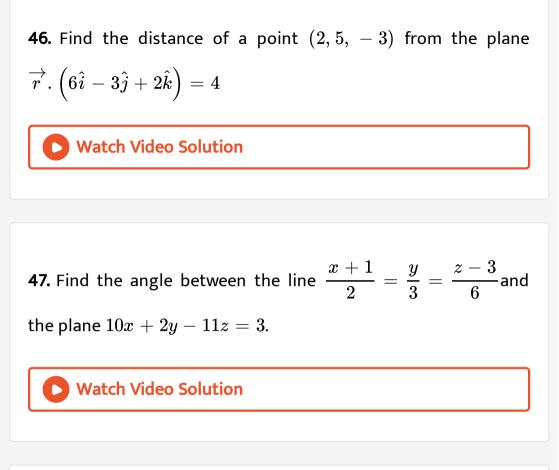


44. Find the angle between the two planes 2x + y - 2z = 5 and

3x - 6y - 2z = 7 using vector method.

45. Find the angle between the two planes 3x - 6y + 2z = 7

and 2x + 2y - 2z = 5



48. Determine the direction cosines of the normal to the plane and the distance from the origin.

z = 2.

49. Find the vector equation of a plane which is at a distance of 7

units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}.$



50. Find the cartesian equation fo the following planes. $\overrightarrow{r}.\left(\hat{i}+\hat{j}-\hat{k}
ight)=2$

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51. Find the vector and cartesian equations of the planes

a) that passes through the point $(1,\,0,\,-2)$ and the normal to the plane is $\hat{i}+\hat{j}-\hat{k}$

b) that passes through the point (1,4,6) and the normal vector to the plane is $\hat{i}-2\hat{j}+\hat{k}$

52. Find the equation of the planes that pass through the points..

(1, 1, -1), (6, 4, -5), (-4, -2, 3)

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53. Find the intercepts cut off by the plane 2x + y - z = 5



54. Find the equation of the plane with intercept 3 on the y-axils and parallel to ZOX plane.

55. Find the equation of the plane through the intersection of the planes 3x - y + 2z = 4 and x + y + z = 2 and the point (2, 2, 1).

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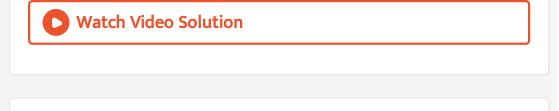
56. Find the vector equation of the plane passing through the

intersection of the palnes
$$ec{r}.\left(2\hat{i}+2\hat{j}-\hat{3}k
ight)=7, ec{r}.\left(2\hat{i}+5\hat{j}+3\hat{k}
ight)=9$$
 and through

the point (2, 1, 3)

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57. Find the vector equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5 which is perpendicular to the plane x - y + z = 0.



58. Find the angle between the planes whose vector equations

are

$$\overrightarrow{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}
ight)=5$$
 and $\overrightarrow{r}.\left(3\hat{i}-3\hat{j}+5\hat{k}
ight)=3$

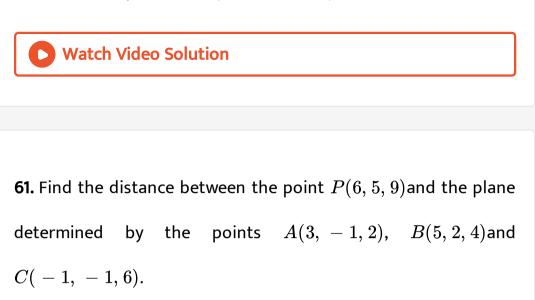
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59. A line makes angles \angle , β , γ and δ with the diagonals of a cube. Show that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$.

60. Find the equation of the plane that contains the point (1, -1,

2) and is perpendicualr

to each of the planes 2x+3y-3z=5 and x+2y-3z=8



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62. Show that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar.

63. Find the coordinates of the point where the line through the

points A (3, 4, 1) and B (5, 1, 6) crosses the XY-plane.

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64. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points(3, 5, -1), (4, 3, -1).



65. If l_1 , m_1 , n_1 and l_2 , m_2 , n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1$, $n_1l_2 - n_2l_1$, $l_1m_2 - l_2m_1$. 66. Find the angle between the lines whose direction ratios are a,

b, c and b - c, c - a, a - b.

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67. Find the equation of a line parallel to x-axis and passing through the origin.

68. If the coordinates of the points

 A, B, C, D be (1, 2, 3), (4, 5, 7), (-4, 3, -6) and (2, 9, 2)

respectively then find the angle between the lines $AB \ and \ CD$.

69. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

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70. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane \overrightarrow{r} . $(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

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71. Find the equation of the plane passing through (a, b, c) and

parallel to the plane $\overrightarrow{r}\hat{i}+\dot{\hat{j}}+\hat{k}=2.$

72. Find the shortest distance between the lines

$$\overrightarrow{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$$
 and
 $\overrightarrow{r} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$

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73. Find the co-ordinates of the point where the line through

(5,1,6) and (3,4,1)

crosses the YZ plane

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74. Find the coordinates of the point where the line through (5,

1, 6) and (3, 4, 1) crosses the ZX-plane.

75. Find the co-ordinates of the point where the line through (3,

-4, -5) and (2, -3, 1)

crosses the plane 2x+y+z=7.

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76. Find the equation of the plane passing through the point (-1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0



77. If the points (1,1,p) and (-3,0,1) be equidistant from the plane \overrightarrow{r} . $\left(3\hat{i}+4\hat{j}-12\hat{k}
ight)+13=0$, then find the value of p



78. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x -axis.

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79. If O be the origin and the co-ordinates of P be (1,2,-3), then find the equation of the plane passing through P and perpendicular to OP.

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80. Find the equation of the plane which contains the line of

intersection of the planes $\overrightarrow{r}.\left(\hat{i}+2\hat{j}+3\hat{k}
ight)-4=0$,

 $\overrightarrow{r}.\left(2\hat{i}+\hat{j}-\hat{k}
ight)+5=0$ and which is perpendicular

to the plane $\overrightarrow{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}
ight)+8=0$

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81. Find the distance of the point (-1, -5, -10) from the point of intersection of the line $\overrightarrow{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane \overrightarrow{r} . $(\hat{i} - \hat{j} + \hat{k}) = 5$

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82. Find the vector equation of the line passing through (1, 2, 3)and parallel to the planes $\overrightarrow{r}.(\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\overrightarrow{r}.(3\hat{i} + \hat{j} + \hat{k}) = 6$

83. Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ Watch Video Solution

84. Prove that if a plane has the intercepts a, b, c and it is at a

distance p units

from the origin, then
$$rac{1}{a^2}+rac{1}{b^2}+rac{1}{c^2}=rac{1}{p^2}$$

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85. Distance between the two planes: '2 x+3 y+4 z=4' and '4 x+6

y+8 z=(12)' is

A. 2 units

B. 4 units

C. 8 units

D. 2sqrt29' units

Answer: D

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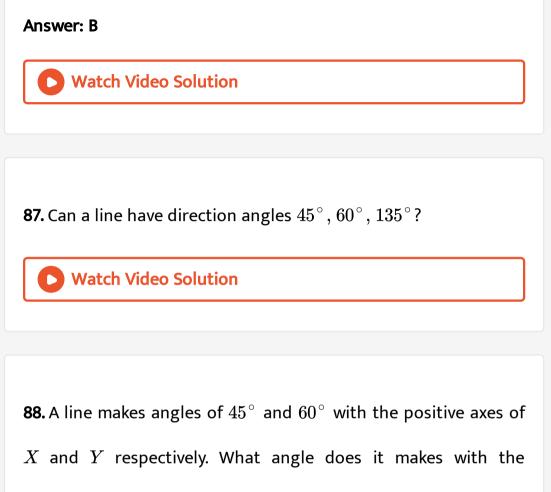
86. The planes '2 x-y+4 z=5' and '5 x-2.5 y+10 z=6' are

A. Perpendicular

B. Parallel

C. Intersect y-axis

D. passes through '(0,0, 5/4)'.

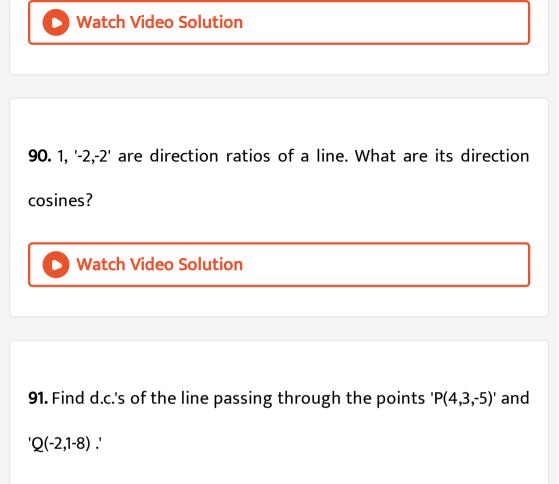


positive axis of \boldsymbol{Z}



89. Find the direction cosine of a line which makes equal angles

with the co-ordinate axes.



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92. A line passes through the points (6, -7, -1) and (2, -3, 1)Find the direction ratios and direction cosines of the line so directed that the angle α is acute.



93. Find the angle between the lines whose diréction ratios are

'2,3,4' and '1,-2,1'

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94. Find the obtuse angle between the lines with direction ratios

'3,-6,2' and '1,-2,-2 .'



95. If points 'P, Q' are '(2,3,-6)' and '(3,-4,5)', find the angle that OP

makes with OQ.



96. If A, B, C, D are the points (3, 4, 5), (4, 6, 3), (-1, 2, 4)and (1, 0, 5), find the angle between CD and AB

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97. Find the coordinates of foot of perpendicular drawn from the point (1, 2, 1) to the line joining the points (1, 4, 6) and (5, 4, 4)

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98. Prove that the line joining the midpoints of the two sides of a

triangle is parallel to the third side and half of its length.

99. The cartesian equations of a line are 6x - 2 = 3y + 1 = 2z - 2.

Find direction of the ratios of the line and write down the cartesian

and vector equations of the line through $(2,\ -1,\ -1)$, which is

parallel to the given line.



100. Find the shortest distance between the lines:

$$ec{r} = 3\hat{i}+8\hat{j}+3\hat{k}+\lambda\Big(3\hat{i}-\hat{j}+\hat{k}\Big)$$
 and $ec{r} = -3\hat{i}-7\hat{j}+6\hat{k}+\mu\Big(-3\hat{i}+2\hat{j}+4\hat{k}\Big)$

101. Find the shortest distance between the lines

$$\overrightarrow{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$
 and
 $\overrightarrow{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

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102. Find the angle between the line $rac{x-2}{3}=rac{y+1}{-1}=rac{z-3}{-2}$

and the plane 3x + 4y + z + 5 = 0

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103. Find the length and foot of the perpendicular from

the point (7, 14, 5) to the plane 2x + 4y - z = 2.

