



MATHS

BOOKS - A N EXCEL PUBLICATION

APPLICATION OF DERIVATIVES

Question Bank

1. Find the rate of change of the area of a circle with respect to its radius r when r = 5

cm.



an edge is 10 cm.



3. A stone is dropped into a quiet lake and waves move in circle at a speed of 4 cm per

second, At the instant, when the radius of the

circular wave is 10 cm, how fast is the enclosed

area increasing?

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4. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate if 2 cm/minute. When x = 10 cm and y = 6 cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.



5. The total cost C (x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$ Find the marginal cost when 3 unit are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.



6. The total revenue is Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue, when x = 5, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant.

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7. A man of height 2 metres walks at uniform speed of 2 km/h away from a lamp post which

is 6 metres high. Find the rate at which the

length of his shadow increasing.



8. Find the rate of change of the area of a

circle with respect to its radius r when r = 3

cm.



9. Find the rate of change of the area of a circle with respect to its radius r when r = 4 cm.



10. The volume of a cube is increasing at the rate of $8cm^3/S$. How fast is the surface area

increasing when the length of an edge is 12cm.



11. The radius of a circle is increasing uniformly

at the rate of 3cm/s. Find the rate

at which the area of the circle is increasing

when the radius is 10 cm.



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12. The edge of a variable cube is increasing at

the rate of 3 cm/s. How fast is the

volume of the cube increasing when the edge

is 10 cm long?

13. A stone is dropped into a quiet lake and waves move in circle at the speed of 5 cm/s. At the instant when the radius of

the circular wave is 8 cm, how

fast is the enclosed area increasing.



14. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of

its circumference?

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15. The length of a rectangle is decreasing at the rate of 5 cm/mi and the width is increasing at the rate of 4cm/min.When length is 8 cm and width is 6 cm, find the rate of change of its area.

16. Suppose that a spherical balloon is inflated and it has volume 'v' and radius 'r' at time 't'. If the balloon is inflated by pumping 900c.c. of gas per second. Find the rate at which the radius of the balloon is increasing when the radious is 15 cm.

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17. A balloon, which always remains spherical has a variable radius. Find the rate at

which volume is increasing with the radius

when the later is 10 cm.



18. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the decreasing when the foot of the ladder is 4m away from the wall. 19. A particle moves along the curve $6y = x^3 + 2$ Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-co-ordinate.

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20. The radius of an air bubble is increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?



21. A balloon, which always remains spherical,

has a variable diameter $rac{3}{2}(2x+1).$

Find the rate of change of its volume w.r.t. x.



22. Sand is pouring from a pipe. The falling sand forms a Cone on the ground in such a way that the height of the Cone is always one-

sixth of the radius of the base.

if the sand is pouring at the rate of $12cm^3/s$

,Find the rate of change of height of the Cone.

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23. The total cost C (x) in Rupees, associatedwith the production of x units of an item isgiven by

 $C(x) = 0.007 x^3 - 0.03 x^2 + 30 x + 4000$ Find

the marginal cost when 3 unit are produced.



24. The total revenue is Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 5$. Find the marginal revenue when x = 7

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25. Choose the correct answer The rate of change of the area of a circle w.r.t its radius r at r = 6 cm is.

A. 10π

 $\mathsf{B}.\,12\pi$

 $C.8\pi$

D. 11π

Answer: B

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26. Choose the correct answer The total revenue in Rupees received from the sale of x units of a product is given by



 $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly

increasing.

28. Find the intervals in which the function $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly decreasing.



29. Show that the function given by f(x) = 3x + 17 is strictly increasing on R.

30. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R. Watch Video Solution



 $f(x) = \sin x$ is strictly increasing on $\left(0, rac{\pi}{2}
ight)$

32. Show that the function given by $f(x) = \sin x$ is strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$ **Watch Video Solution**

33. Show that the function given by $f(x) = \sin x$ is neither increasing nor decreasing in $(0, \pi)$

34. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is strictly increasing



35. Find the intervals in which the function f

given $f(x) = 2x^2 - 3x$ is

Strictly Decreasing.



36. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is Strictly Increasing.

37. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is

Strictly Decreasing.

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38. Find the intervals in which the function $f(x) = x^2 + 2x - 5$ strictly increasing or decreasing.

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39. Find the intervals in which the following

functions are strictly increasing

or decreasing: $10 - 6x - 2x^2$

40. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ strictly

increasing or decreasing.



41. Find the intervals in which the following

functions are strictly increasing

or decreasing: $6-9x-x^2$

42. Find the intervals in which the function $f(x) = (x+1)^3(x-3)^3$ strictly increasing or decreasing.



domain.

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44. Find the value of x for which $y = [x(x-2)]^2$ is an increasing function. Watch Video Solution

45. Prove that
$$y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$$
 is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

46. Prove that the logarithmic function is strictly increasing on $(0, \infty)$ **Watch Video Solution**

47. Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on ? (-1, 1)

48. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$? cos x

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49. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$? cos2x

50. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$? cos3x

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51. Prove that tan x is strictly increasing on

$$\left(0, \frac{\pi}{2}\right)$$

52. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing ?

A. (0,1)
B.
$$\left(\frac{\pi}{2}, \pi\right)$$

C. $\left(0, \frac{\pi}{2}\right)$

D. None of these

Answer: D



53. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$

is strictly increasing on (1,2)







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56. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$ and strictly increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.

57. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ in increasing in R.

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58. The interval in which $y = x^2 e^{-x}$ is increasing is a) (0, 2) b) (-2, 0) c) $(2, \infty)$ d) $(-\infty, \infty)$

- A. $(\,-\infty,\infty)$
- B. (-2, 0)



D. (0,2)

Answer: D



59. Find the equation of the tangent and normal at the point (1,2) on the parabola $y^2=4x.$

60. Find the point at which the tangent to the

curve
$$y=\sqrt{4x-3}-1$$
has its slops $\displaystyle rac{2}{3}$.



61. Find the equation of all lines having slope 2

and being tangent to the curve
$$y + rac{2}{x-3} = 0$$
62. Find points on the curve $\displaystyle rac{x^2}{4} + \displaystyle rac{y^2}{25} = 1$ at

which the tangents are parallel

to x-axis

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63. Find points on the curve $\frac{x}{x}$

$$rac{x^2}{4}+rac{y^2}{25}=1$$
 at

which the tangents are

parallel to y-axis

64. Find the equation of tangent to the curve given by $x = a \sin^3 t$, $y = b \cos^3 t$ at a point where $t = \frac{\pi}{2}$.

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65. Find the slope of the tangent to the curve

$$y = 3x^4 - 4x \;\; {
m at} \;\; x = 4$$

66. Find the slope of the tangent to the curve.

$$y=rac{x-1}{x-2}, x
eq 2$$

at x = 10



67. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point

whose x-coordinate is 2.

68. Find the slope of the tangent to the curve

 $y=x^3-3x+2$ at the point

whose x-coordinate is 3.

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69. Find the slope of the normal to the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$ at $\theta = \frac{\pi}{4}$

70. Find the slope of the normal to the curve

$$x=1-a\sin heta$$
 , $y=b\cos^2 heta$
at $heta=rac{\pi}{4}$



71. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$

is parallel to the x-axis.



72. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

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- 73. Find the point on the curve $y = x^3 11x + 5$ at which the tangent is
- y = x 11

74. Find the equation of all lines having slope

-1 that are tangent to the curve $y=rac{1}{x-1}, x
eq 1$

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75. Find the equation of all lines having slope

2 which are tangents to the curve $y=rac{1}{x-3}, x
eq 3.$

76. Find the equation of all lines having slope zero which are tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$ Watch Video Solution



parallel to x-axis





79. Find the equation on tangents and normals to the given curves

$$y=x^4-6x^3+13x^2-10x+5$$
 at $(0,5)$

80. Find the equation of the tangent and normal to the given curve at the indicated points $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (1, 3)

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81. Find the equation of tangents and normals

to the given curves $y=x^3$ at (1,1)

82. Find the equation of the tangent and normal to the given curve at the indicated points $y = x^2$ at (0,0)





84. Find the equation of the tangent line to

the curve $y=x^2-2x+7$

which is parallel to the line 2x - y + 9 = 0

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85. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is perpendicular to the line 5y - 15x = 13.

86. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where x = 2 and x = -2 are parallel.



87. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the

y co-ordinate of the point.



88. For the curve $y = 4x^3 - 2x^5$, find all points at which the tangent passes through the origin.



89. Find the points on the curve

$$x^2 + y^2 - 2x - 3 = 0$$
 at which the tangent

are parallel to x-axis.

90. Find the equation of the normal at the point $\left(am^2, am^3
ight)$ for the curve $ay^2=x^3.$

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91. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line x + 14y + 4 = 0.

92. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point (at^22at) .



93. Prove that the curve $x = y^2$ and xy = k cut at right angles, if $8k^2 = 1$.

94. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) .

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95. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line 4x - 2y + 5 = 0.

96. Choose the correct answer from the bracket The slope of the normal to the curve $y = 2x^2 + 3 \sin x$ at x = 0 is.



$$\mathsf{D.}-\frac{\mathsf{I}}{3}$$

1

Answer: D



97. Choose the correct answer from the bracket The line y = x + 1 is a tangent to the curve $y^2 = 4x$ at the point.

- A. (1,2)
- B. (2,1)
- C. (1, -2)
- D. (-1, 2)

Answer: A



98. Using differentiable, find the approximate value of each of the following upto 3 places of decimal. $\sqrt{25.3}$



99. Using differentiable, find the approximate

value of each of the following upto 3 places of

decimal. $\sqrt{49.5}$



100. Use differentials to find the approximate

value of $\sqrt{0.6}$ up to 3 places of decimals.



101. Use differentials to find the approximate value of $(0.009)^{\frac{1}{3}}$ up to 3 places of decimals.

102. Use differentials to find the approximate value of $(0.999)^{\frac{1}{10}}$ up to 3 places of decimals.

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103. Use differentials to find the approximate value of $(15)^{\frac{1}{4}}$ up to 3 places of decimals.

104. Using differentiable, find the approximate value of each of the following upto 3 places of decimal. $(26)^{1/3}$



105. Using differentiable, find the approximate

value of each of the following upto 3 places of

decimal. $(255)^{1/4}$



106. Using differential, find the approximate value of each of the following upto 3 places of decimal. $(82)^{1/4}$

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107. Find the approximate value of $\sqrt{401}$.

108. Using differential, find the approximate value of each of the following upto 3 places of decimal. $(0.0037)^{1/2}$



109. Use differentials to find the approximate value of $(0.009)^{\frac{1}{3}}$ up to 3 places of decimals.



110. Using differentiable, find the approximate value of each of the following upto 3 places of decimal. $(81.5)^{1/4}$



111. Using differentiable, find the approximate value of each of the following upto 3 places of decimal. $(3.968)^{3/2}$

112. Find the approximate value of f(2.01), where $f(x)=4x^2+5x+2$ **Vatch Video Solution**

113. Find the approximate value of f(5.001) where $f(x) = x^3 - 7x^2 + 15$.

114. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 1%.



115. Find the approximate change in the surface area of a cube of side x meters caused

by decreasing the side by 1%.



116. If the radious of a sphere is measured as 7m with an error of 0.02m then find the approximate error in calculating its volume.

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117. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.



118. If $f(x)= 3x^2 + 15x + 5$, then find the approximate value of f(3.02)

A. 47.66

B. 57.66

C. 67.66

D. 77.66

Answer: D

119. Find the approximate change in volume of

a cube of side x meters caused by an increase

in the side by 3%.

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120. Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$f(x) = x^3 - 3x$$

121. Find the local maximum and local minimum value (if any) of the function $y=x^3-27x+5$

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122. Find the local maximum and local minimum values of the function of f

given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$

minimum values of the function

of f given by $f(x)=2x^3-6x^2+6x+5$

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124. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2,1).



125. Let AP and BQ be two vertical poles at point A and B, respectively. If AP= 16 m, BQ = 22 m and AB=20 m, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.



126. If length of three sides of a trapezium other than base are equal to 10cm. Then find the area of the trapezium when it is maximum.





127. Show that all rectangles with a given

perimeter, the square has the maximum area.



128. Prove that of all rectangles with given

area square has minimum perimeter?

129. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.



130. Find the absolute maximum and absolute

minimum value of f(x)= $x^2 + 2$ in [-2,2].
131. Find the absolute maximum value and minimum value of the function.

$$f(x)=2x^3-15x^2+36x+1, x\in [1,5]$$

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132. Find the absolute maximum value and minimum value of the function.

$$f(x)=12x^{rac{4}{3}}-6x^{rac{1}{3}},x\in [\,-1,1]$$

133. Find the minimum and maximum value, if

any, of the function $f(x) = \left(2x-1
ight)^2 + 3$



134. Find the maximum and minimum values, If any, of the following functions given by $f(x) = 9x^2 + 12x + 2$

135. Find the maximum and minimum values, If

any, of the following functions

given by $f(x) = -(x-1)^2 + 10$



136. Find the maximum and minimum values, If

any, of the following functions

given by $g(x) = x^3 + 1$

137. Find the maximum and minimum values, If

any, of the following functions f(x) = |x+2| - 1



138. Find the maximum and minimum values, If

any, of the following functions g(x) = -|x+1|+3



139. Find the maximum and minimum values, If

any, of the following functions h(x) = sin (2x)+5

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140. Find the maximum and minimum values, If

any, of the following functions $f(x) = |\sin 4x + 3|$



141. Find the maximum and minimum values, If any, of the following functions h (x) = x + 1, $x \in (-1, 1)$



142. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum value, as the case may be: $f(x) = x^2$ 143. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum value, as the case may be: $g(x) = x^2 - 3x$

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144. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum value,



145. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum value,

as the case may be:

 $f(x) = \sin x - \cos x, 0 < x < 2\pi$

146. Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$f(x) = x^3 - 6x^2 + 9x + 15$$

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147. Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$g(x)=rac{x}{2}+rac{2}{x},x>0$$

148. Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$g(x)=rac{1}{x^2+2}$$

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149. Find the local maxima and local minima, if

any, of the following functions. Find also the

local maximum and the local minimum value,

as the case may be:
$$f(x)=x\sqrt{1-x}, x>0$$



151. Prove that the following functions do not have maxima or minima $g(x) = \log x$



153. Find the absolute maximum value and minimum value of the following functions.

$$f(x)=x^3, x\in [\,-2,2]$$



154. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals $f(x) = \sin x + \cos x$, x in $[0, \pi]$

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155. Find the absolute maximum value and minimum value of the following functions. $f(x) = 4x - \frac{x^2}{2}, x \in \left[-2, \frac{9}{2}\right]$



156. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals $f(x) = (x-1)^2 + 3, x \in [-3,1]$

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157. Find the maximum profit that a company can make, if the profit function

is given by $p(x) = 41 - 72x - 18x^2$



158. Consider the function $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ Find the absolute extreme values of f(x). $x \in [0,3]$ are respectively



159. at what points in the interval $[0, 2\pi]$, does

the function $\sin 2x$ attain its maximum value?

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160. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals `f(x)=sinx+cosx,x in [0,pi]

161. Find the maximum value of $2x^3-24x+107$ in the interval [1,3]. Find the maximum value of the same function in $[-3,\ -1]$

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162. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0,2]. Find the value of a ?

163. Find two numbers whose sum is 24 and

whose product as large as possible.



164. Find two positive numbers x and y such

that x+y = 60 and xy^3 is maximum.

165. Find two numbers x and y such that their

sum is 35 and the product x^2y^5 is a maximum.

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166. Find two positive numbers whose sum is

16 and the sum of whose Cubes is minimum.



167. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to from the box. What should be the side of the square to be cut off so that the volume of the box is maximum?



168. A rectangle sheet of tin with adjacent sides 45 cm and 24 cm is to be made into a

box without top, by cutting off equal squares from the corners the folding up the flaps. Taking the side of the square cut off as x, express the volume of the box as the function

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of x.

169. Show that all rectangles with a given

perimeter, the square has the maximum area.

170. Of all the cylinders with given surface area, show that the volume is maximum when height is equal to the diameter of the base .



171. Of all the closed cylindrical cans (right circular) of a given volume of 100 cubic. cm. find the dimensions of the can which has the minimum surface area.



172. A wire of length 28 m is cut into two pieces. One of the Pieces is be made into a square and the other in to a circle. What should be the length of the two pieces so that combined area of the square and the circle is minimum using differentiation?

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173. Show that the right circular cone of least curved surface and given volume has an

altitude equal to $\sqrt{2}$ times the radius of the

base.



175. The point on the curve $x^2 = 2y$ which is nearest to the point (0,5) is –

A.
$$\left(2\sqrt{2},4\right)$$

- $\mathsf{B.}\left(2\sqrt{2},0\right)$
- C. (0,0)
- D. (2,2)

Answer: A

176. Choose the correct answer. For all real values of x , the minimum value of $rac{1-x+x^2}{1+x+x^2}$ is a)0 b)1 c)3 d)1/3 A. 0 B.1 C. 3 $\mathsf{D}.\,\frac{1}{3}$ Answer: D

177. The maximum value of
$$[x(x-1)+1]^{1/3} 0 \le x \le 1$$
is a) $\left(rac{1}{2}
ight)^{rac{1}{3}}$ b)1/2 c) 1 d)0

A.
$$\left(\frac{1}{2}\right)^{1/3}$$

B. $\frac{1}{2}$

C. 1

D. 0

Answer: C



178. Show that the function given by
$$f(x) = \frac{\log x}{x}$$
 has maximum at x = e **Watch Video Solution**

179. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of $3cm/\sec$. How fast is the area decreasing when the two equal sides are equal to the base?

180. Find the equation of the tangent and normal at the point (1,2) on the parabola $y^2 = 4x.$



181. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If the building of tank costs Rs. 70/sq metres for the base and Rs. $45/_{square}$

metres. For sides, what is the cost of least

expensive tank?



182. The sum of the perimeter of a circle and square is k , where k is a constant. Prove that the sum of their areas is least when the side of the square is double the radius of the circle.

183. A window is in the form of a rectangle surmounted by a semi circular opening. The total perimeter of the window is 10 m. Find the dimension of the window to admit maximum light through the whole opening.