

MATHS

BOOKS - A N EXCEL PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Question Bank

1. Check the continuity of the function f(x)=2x+3 at

x=1



2. Examine whether the function $f(x)=x^2$ is continuous at x=0



3. Show that the function $f(x)=egin{cases} x^3+3 & ext{if} & x
eq 0 \\ 1 & ext{if} & x=0 \end{cases}$ is not continuous at x=0



4. Discuss the continuity of the function $f(x) = x^3 + x^2 - 1$



5. Discuss the continuity of the function $f(x) = rac{1}{x}, x
eq 0$



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6. Let $f(x)=\left\{\begin{array}{l} \frac{x^2-9}{x-3}:x\neq 3 \\ 5:x=3 \end{array}\right\}$ Is f(x) continuous at x=3?



7. Show that the function defined by $f(x) = \sin(x^2)$ is a continuous function?



8. Prove that the function f(x)=5x-3 is continuous at x=0, at x=-3 and at x=5?



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9. Examine the continuity of the function $f(x)=2x^2-1$ at x=3



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10. Examine the following functions for continuity f(x) = x - 5



11. Examine the following functions for continuity

$$f(x) = \frac{1}{x-5}$$



12. Examine the following functions for continuity

$$f(x)=\frac{x^2-25}{x+5}$$



13. Examine the following functions for continuity

$$f(x) = |x - 5|$$



14. Prove that the function $f(x) = x^n$ is continuous at x=n, where n is a positive integer.



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15. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at x=0



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16. Is the function f defined by $f(x)= \begin{cases} x, & \text{if} \quad x \leq 1 \\ 5, & \text{if} \quad x>1 \end{cases}$ continuous at x=1?



17. Is the function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ continuous at x=2?



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18. Find all points of discontinuity of f where f is defined

by
$$f(x)=\left\{egin{array}{ccc} 2x+3 & x\leq 2 \ 2x-3 & x>2 \end{array}
ight.$$



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19. Find all points of discontinuity of f, where f is defined

$$\mathsf{by}\, f(x) = \left\{ \begin{aligned} |x|+3, & \text{if} \quad x \leq -3 \\ -2x, & \text{if} \quad -3 < x < 3 \\ 6x+2, & \text{if} \quad x \geq 3 \end{aligned} \right.$$

20. Evalute

$$\lim_{x o 0} \, f(x)$$
 , where $f(x) = egin{cases} rac{|x|}{x} & x
eq 0 \ 0 & x = 0 \end{cases}$



21. Find all points of discontinuity of f, where f is defined

by
$$f(x) = \left\{ egin{array}{ll} rac{x}{|x|}, & ext{if} & x < 0 \ -1, & ext{if} & x \geq 0 \end{array}
ight.$$



22. Find all points of discontinuity of f, where f is defined

by
$$f(x) = \left\{ egin{array}{ll} x+1 & ext{if} & x \geq 1 \ x^2+1 & ext{if} & x < 1 \end{array}
ight\}$$



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23. Is the function defined by

$$f(x) = \left\{ egin{array}{ll} x+5, & ext{if} & x \leq 1 \ x-5, & ext{if} & x>1 \end{array}
ight.$$
 a continuous function?



24. Find the relation between 'a' and 'b' if the function fdefined by

$$f(x) = \left\{egin{array}{ll} ax+1 & x \leq 3 \ bx+3 & x>3 \end{array}
ight.$$
 is continuous.

25. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

26. Discuss the continuity of the following functions:



 $f(x) = \sin x + \cos x$



27. Discuss the continuity of the following functions: $f(x) = \sin x - \cos x$



28. Discuss the continuity of the following functions: $f(x) = \sin x \cdot \cos x$



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29. Find the values of k so that function f is continuous at the indicated points $f(x)=egin{cases} kx^2 & ext{if} & x\leq 2atx=2 \ 3 & ext{if} & x>2 \end{cases}$



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30. Find the values of k so that function f is continuous at the indicated points

$$f(x) = \left\{ egin{aligned} kx + 1 & ext{if} & x \leq \pi at x = \pi \ \cos x & ext{if} & x > \pi \end{aligned}
ight.$$



31. Prove that the function defined by



 $f(x) = \cos x^2$

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32. Show that the function $f(x) = |\cos x|$ is a continuous function.



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33. Find all the points of discontinuity of f defined by f(x) = |x| - |x+1|





34. Differentiate $\sin(x^2+5)$ w.r.t.x



35. Differentiate cos(sinx)w.r.t.x





37. Differentiate
$$\dfrac{\sin(ax+b)}{\cos(cx+d)}$$
 w.r.t.x



38. Differentiate $\cos\left(x^3\right)\sin^2\left(x^5\right)$ w.r.t.x



39. Differentiate $2\sqrt{\cot\left(x^2
ight)}$ w.r.t.x



40. Differentiate $\cos\left(\sqrt{x}\right)$ w.r.t.x



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41. Differentiate the following w.r.t.x $2\sin^{-1}x + \tan^{-1}x + 1$



42. Differentiate the following w.r.t.x $x \sin^{-1} x$





44. Differentiate the following w.r.t.x $e^{m an^{-4}x}$



45. Differentiate the following w.r.t.x $\sin^{-1}(x^3)$



46. Differentiate the following w.r.t.x $(\cot^{-1} x)^2$



47. Find $\dfrac{dy}{dx}$ if $x-y=\pi$



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48. Find $\frac{dy}{dx}$ if $y + \sin y = \cos x$



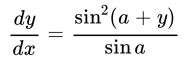
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49. If $\sin y = x \sin(a+y)$, find x in terms of y and hence find (dx)/(dy)



50. If
$$\sin y = x \sin(a+y)$$
, prove that

$$=\frac{\sin^2(a+y)}{\sin^2(a+y)}$$



51. Find $\dfrac{dy}{dx}$ if $2x+3y=\sin x$



52. Find $\frac{dy}{dx}$ if $2x + 3y = \sin y$



53. Find
$$\frac{dy}{dx}$$
 if $ax + by^2 = \cos y$



54. Find
$$\frac{dy}{dx}$$
 if $xy+y^2=\tan x+y$



55. Find
$$\frac{dy}{dx}$$
, if $x^2+y^2+xy=100$



57. Find
$$\dfrac{dy}{dx}$$
 if $\sin^2 y + \cos(xy) = \pi$



58. Find
$$\dfrac{dy}{dx}$$
 of the following $\sin^2 x + \cos^2 y = 1$

59. Find
$$\dfrac{dy}{dx}$$
 of the following $y=\sin^{-1}\!\left(\dfrac{2x}{1+x^2}
ight)$



60. If
$$y= an^{-1}igg(rac{3x-x^2}{1-3x^2}igg), rac{-1}{\sqrt{3}} < x < rac{1}{\sqrt{3}}$$
 Find $rac{dy}{dx}$



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61. Find
$$\dfrac{dy}{dx}$$
 of $y=\cos^{-1}\dfrac{1-x^2}{(1+x^2)}$, 0



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62. Find
$$\dfrac{dy}{dx}$$
 If $y=\sin^{-1}\!\left(\dfrac{1-x^2}{1+x^2}\right)$, $0< x< 1$



63. If
$$y=\cos^{-1}\Bigl(\dfrac{2x}{1+x^2}\Bigr), \ -1 < x < 1,$$
 Find $\dfrac{dy}{dx}$



64. Consider the function
$$f(x)=\sin^{-1}\Bigl(2x\sqrt{1-x^2}\Bigr), \, rac{-1}{\sqrt{2}}\leq x\leq rac{1}{\sqrt{2}}$$

Find f'(x)`.



65. Find
$$\frac{dy}{dx}$$
 of the following $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$



66. Find
$$\frac{dy}{dx}$$
 of $y = e^x \log x$



67. Find
$$\frac{dy}{dx}$$
 of $y=rac{1-\log x}{1+\log x}$



68. Find
$$\dfrac{dy}{dx}$$
 of $y=4^x$



69. Find
$$\dfrac{dy}{dx}$$
 of $y=log_2x$

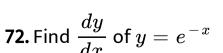


70. Find
$$\frac{dy}{dx}$$
 of y=log_0x`



71. Find
$$rac{dy}{dx}y=2^{\sin^2x}$$







73. Find $\frac{dy}{dx}$ of $y = \sin(\log x)$



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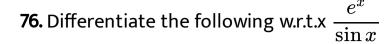
74. Find $\dfrac{dy}{dx}y=\cos^{-1}ig(e^2ig)$



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75. Find $\frac{dy}{dx}$ of $y = e^{\cos x}$







77. Differentiate the following w.r.t.x $e^{\sin^{-1}x}$



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78. Differentiate the following w.r.t.x e^{x^2}



80. Differentiate the following w.r.t.x $\log(\cos e^x)$



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81. Differentiate the following w.r.t.x $e^x + e^{x^2} + ... + e^{x^5}$



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82. Differentiate the following w.r.t.x $\sqrt{e^{\sqrt{x}}}, x>0$



83. Differentiate the following w.r.t.x $\log(\log x), x > 1$



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84. Differentiate the following w.r.t.x $\frac{\cos x}{\log x}$, x>0



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 $\cos(\log x + e^2), x > 0$

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85. Differentiate the

following w.r.t.x

86. Find
$$\dfrac{dy}{dx}$$
 of $y=x^x$



87. Differentiate $y = a^x$



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88. Differentiate $x^{\sin x}$ w.r.t. x



89. Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t.x.



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90. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$



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91. Differentiate cosx.cos2x.cos3x



92. Differentiate $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$ with respect

to x.



93. Differentiate $(\log x)^{\cos x}$ w.r.t.x



94. Differentiate $x^2-2^{\sin x}$



95. Differentiate $(x+3)^2(x+4)^3(x+5)^4$ w.r.t.x



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96. Differentiate $\left(x+rac{1}{x}
ight)^x+x^{\left(1+1/x
ight)}$ w.r.t.x



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97. Differentiate $(\log x)^x + x^{\log x}$ w.r.t.x



98. Find
$$\frac{dy}{dx}$$
 of

$$y = (\sin x)^x + \sin^{-1} \sqrt{x}$$



99. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ w.r.t.x



100. Find $\frac{dy}{dx}$ if $x^y + y^x = 1$



101. Find
$$\dfrac{dy}{dx}$$
 of $y^x = x^y$



102. Find
$$\frac{dy}{dx}$$
 of $(\cos x)^y = (\cos y)^x$



103. Find
$$\frac{dy}{dx}$$
 if $xy = e^{(x-y)}$



104. Find the derivative of the function given by $f(x)=(1+x)\big(1+x^2\big)\big(1+x^4\big)\big(1+x^8\big) \ \ \text{and} \ \ \text{hence}$ find f '(1) ?



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105. Differentiate $\left(x^2-5x+8\right)\left(x^3+7x+9\right)$ by using product rule



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106. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by expanding the product to obtain a single polynomial

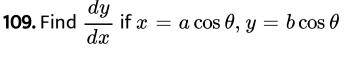
107. Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ by logarithmic differentiation.



$$x=2at^2$$
 , $y=at^4$

108. Find $\frac{dy}{dx}$ of





110. Find $\frac{dy}{dx}$, $x = \sin t$, $y = \cos 2t$



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111. Find $\frac{dy}{dx}, x=4t, y=rac{4}{t}$



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112. Find $\frac{dy}{dx}$ if $x=\cos\theta-\cos2\theta$ and $y=\sin\theta-\sin2\theta$



113. Find
$$\dfrac{dy}{dx}$$
 if $x=a(heta-\sin heta)$ and $y=a(1+\cos heta)$



114. Find
$$\dfrac{dy}{dx}$$
 if $x=a(\cos t + \log \tan (t/2))$ and $y=a\sin t$



115. Find
$$\frac{dy}{dx}$$
, $x = a \sec \theta$, $y = b \tan \theta$



116. Find
$$\frac{dy}{dx}$$
 of

$$x=a(\cos heta+ heta\sin heta)$$
 , $y=a(\sin heta- heta\cos heta)$



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117. If
$$x=\sqrt{a^{\sin^{-1}t}}$$
 , $y=\sqrt{a^{\cos^{-1}t}}$ Show that $\dfrac{dy}{dx}=\dfrac{-y}{x}$



118. Find second order derivative for $y=3x^4+4x^2-x+1$



119. Find second order derivative for y= xsinx



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120. Find second order derivative for f(x)=cos2x



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121. Find second order derivative for $y = \frac{\log x}{r}$



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122. Find second order derivative for x=ct and $y=rac{c}{t}$



123. Find
$$\frac{d^2y}{dx^2}$$
, if $y=x^3+\tan x$



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124. $y = a \cos x + b \sin x$ is the solution of the differential equation

$$rac{d^2y}{dx^2}+y=0$$



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$$rac{d^2y}{dx^2}-5rac{dy}{dx}+6y=0$$

125. If $y = 3e^{2x} + 2e^{3x}$, prove that

126. If
$$y=\sin^{-1}x$$
,then show that $(1-x^2)rac{d^2y}{dx^2}-xrac{dy}{dx}=0$



127. Find the second order derivative of the following functions $x^2 + 3x + 2$



128. Find the second order derivative of the following functions x^{20}



129. Find the second order derivative of the following functions xcosx



130. Find the second order derivative of the following functions $\log x$



131. Find the second order derivative of the following functions $x^3 \log x$



132. Find the second order derivative of the following functions $e^x \sin 5x$



133. Find the second order derivative of the following functions $e^{6x}\cos 3x$



134. Find the second order derivative of the following functions $an^{-1}x$



135. Find the second order derivative of the following functions log(logx)



136. Find the second order derivative of the following functions sin(logx)



138. If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.



139. Let $y=3\cos(\log x)+4\sin(\log x)$

Prove that $x^2y_2+xy_1+y=0$



140. If $y=ae^{mx}+be^{nx}$, show that

$$rac{d^2y}{dx^2}-(m+n)rac{dy}{dx}+mny=0$$

141. If $y = 500e^{7x} + 600e^{-7x}$. Show that $\frac{d^2y}{dx^2} = 49y$



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142. If
$$e^y(x+1)=1$$
 , show that $\dfrac{d^2y}{dx^2}=\left(\dfrac{dy}{dx}
ight)^2$



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143. If $y=\left(\tan^{-1}x\right)^2$, show that

$$\left(x^2+1\right)^2 y_2 + 2x \left(x^2+1\right) y_1 = 2$$
 .



144. Verify Rolle's Theorem for the function

$$f(x)=x^2+2x-8$$
 , $x\in[\,-4,2]$



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145. Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the converse of Rolle's theorem from these examples? $f(x) = [x] \text{forx} \in [5, 9]$



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146. Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the

converse of Rolle's theorem from these examples? f(x)=[x]for $x \in [-2, 2]$



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147. Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the converse of Rolle's theorem from these examples? $f(x) = x^2 - 1$ for $\mathbf{x} \in [1, 2]$



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148. If $f\!:\![\,-5,5] o R$ is a differentiable function and if f'(x) does not vanish any where, then prove that $f(-5) \neq f(5)$

149. Verify mean value theorem for the function $f(x)=x^2-4x-3$ in the interval [1,4] .



150. Verify Lagrange 's mean value theorem, if $f(x)=x^3-5x^2-3x$ in the interval [a,b], where a=1 and b=3. Find all $c\in(1,3)$ at which f'(c)=0.





152. Differentiate the following w.r.t x $\sin^3 x + \cos^6 x$



153. Differentiate the following w.r.t x $(5x)^{3\cos 2x}$



154. Differentiate the following w.r.t x $(\log x)^{\log x}, x>1$



155. Differentiate the following w.r.t.x $x^x + x^a + a^x + a^a$ for some fixed a>0 and x>0

157. If x= a(cos t +t sin t) and y=a(sin t - t cos t), find $\frac{d^2y}{dx^2}$

Find



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 $\frac{dy}{dx}, \quad ext{if} \quad y = 12(1-\cos t), \, x = 10(t-\sin t). \, \frac{\pi}{2} < t < \frac{\pi}{2}$

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158. If $y=e^{a\cos^{-1}x}$, then show that

$$(1-x^2)y_2-xy_1-a^2y=0$$

