



## MATHS

### BOOKS - A N EXCEL PUBLICATION

### CONTINUITY AND DIFFERENTIABILITY

#### Question Bank

1. Check the continuity of the function  $f(x) = 2x + 3$  at  $x=1$



Watch Video Solution

2. Examine whether the function  $f(x) = x^2$  is continuous at  $x=0$

 [Watch Video Solution](#)

3. Show that the function  $f(x) = \begin{cases} x^3 + 3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$  is not continuous at  $x=0$

 [Watch Video Solution](#)

4. Discuss the continuity of the function  $f(x) = x^3 + x^2 - 1$

 [Watch Video Solution](#)

5. Discuss the continuity of the function

$$f(x) = \frac{1}{x}, x \neq 0$$

 [Watch Video Solution](#)

6. Let  $f(x) = \left\{ \begin{array}{l} \frac{x^2-9}{x-3} : x \neq 3 \\ 5 : x = 3 \end{array} \right\}$  Is  $f(x)$  continuous at  $x=3$ ?

 [Watch Video Solution](#)

7. Show that the function defined by  $f(x) = \sin(x^2)$  is a continuous function?

 [Watch Video Solution](#)

8. Prove that the function  $f(x) = 5x - 3$  is continuous at  $x = 0$ , at  $x = -3$  and at  $x = 5$ ?



Watch Video Solution

9. Examine the continuity of the function  $f(x) = 2x^2 - 1$  at  $x = 3$



Watch Video Solution

10. Examine the following functions for continuity  
 $f(x) = x - 5$



Watch Video Solution

11. Examine the following functions for continuity

$$f(x) = \frac{1}{x - 5}$$

 [Watch Video Solution](#)

12. Examine the following functions for continuity

$$f(x) = \frac{x^2 - 25}{x + 5}$$

 [Watch Video Solution](#)

13. Examine the following functions for continuity

$$f(x) = |x - 5|$$

 [Watch Video Solution](#)

14. Prove that the function  $f(x) = x^n$  is continuous at  $x=n$ , where  $n$  is a positive integer.

 [Watch Video Solution](#)

15. Is the function  $f$  defined by  $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$

continuous at  $x=0$

 [Watch Video Solution](#)

16. Is the function  $f$  defined by  $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$

continuous at  $x=1$ ?

 [Watch Video Solution](#)

17. Is the function  $f$  defined by  $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$

continuous at  $x=2$ ?



Watch Video Solution

18. Find all points of discontinuity of  $f$  where  $f$  is defined

$$\text{by } f(x) = \begin{cases} 2x + 3 & x \leq 2 \\ 2x - 3 & x > 2 \end{cases}$$



Watch Video Solution

19. Find all points of discontinuity of  $f$ , where  $f$  is defined

$$\text{by } f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$

 [Watch Video Solution](#)

20. Evaluate

$$\lim_{x \rightarrow 0} f(x), \text{ where } f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

 [Watch Video Solution](#)

21. Find all points of discontinuity of  $f$ , where  $f$  is defined

$$\text{by } f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

 [Watch Video Solution](#)



22. Find all points of discontinuity of  $f$ , where  $f$  is defined

$$f(x) = \begin{cases} x + 1 & \text{if } x \geq 1 \\ x^2 + 1 & \text{if } x < 1 \end{cases}$$



Watch Video Solution

23. Is the function defined by

$$f(x) = \begin{cases} x + 5, & \text{if } x \leq 1 \\ x - 5, & \text{if } x > 1 \end{cases} \text{ a continuous function?}$$



Watch Video Solution

24. Find the relation between 'a' and 'b' if the function  $f$

defined by

$$f(x) = \begin{cases} ax + 1 & x \leq 3 \\ bx + 3 & x > 3 \end{cases} \text{ is continuous.}$$





[Watch Video Solution](#)

25. Is the function defined by  $f(x) = x^2 - \sin x + 5$  continuous at  $x = \pi$ ?



[Watch Video Solution](#)

26. Discuss the continuity of the following functions:

$$f(x) = \sin x + \cos x$$



[View Text Solution](#)

27. Discuss the continuity of the following functions:

$$f(x) = \sin x - \cos x$$



[View Text Solution](#)

28. Discuss the continuity of the following functions:

$$f(x) = \sin x \cdot \cos x$$



[View Text Solution](#)

29. Find the values of  $k$  so that function  $f$  is continuous at

the indicated points  $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 2ax = 2 & \\ 3 & \text{if } x > 2 \end{cases}$



[Watch Video Solution](#)

30. Find the values of  $k$  so that function  $f$  is continuous at

the

indicated

points

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$$

 [Watch Video Solution](#)

**31.** Prove that the function defined by

$$f(x) = \cos x^2$$

is a continuous function

 [Watch Video Solution](#)

**32.** Show that the function  $f(x) = |\cos x|$  is a continuous function.

 [View Text Solution](#)

**33.** Find all the points of discontinuity of  $f$  defined by

$$f(x) = |x| - |x + 1|$$

 [View Text Solution](#)

**34.** Differentiate  $\sin(x^2 + 5)$  w.r.t.x

 [Watch Video Solution](#)

**35.** Differentiate  $\cos(\sin x)$  w.r.t.x

 [Watch Video Solution](#)

**36.** Differentiate  $\sec(\tan(\sqrt{x}))$  w.r.t.x



Watch Video Solution

37. Differentiate  $\frac{\sin(ax + b)}{\cos(cx + d)}$  w.r.t.  $x$



Watch Video Solution

38. Differentiate  $\cos(x^3)\sin^2(x^5)$  w.r.t.  $x$



Watch Video Solution

39. Differentiate  $2\sqrt{\cot(x^2)}$  w.r.t.  $x$



Watch Video Solution

40. Differentiate  $\cos(\sqrt{x})$  w.r.t.x



Watch Video Solution

41. Differentiate the following w.r.t.x

$$2 \sin^{-1} x + \tan^{-1} x + 1$$



View Text Solution

42. Differentiate the following w.r.t.x  $x \sin^{-1} x$



Watch Video Solution

43. Differentiate the following w.r.t.x  $\frac{\tan^{-1} x}{x}$



[Watch Video Solution](#)

44. Differentiate the following w.r.t.  $x$   $e^{m \tan^{-1} x}$



[View Text Solution](#)

45. Differentiate the following w.r.t.  $x$   $\sin^{-1}(x^3)$



[Watch Video Solution](#)

46. Differentiate the following w.r.t.  $x$   $(\cot^{-1} x)^2$



[Watch Video Solution](#)



47. Find  $\frac{dy}{dx}$  if  $x - y = \pi$



Watch Video Solution

48. Find  $\frac{dy}{dx}$  if  $y + \sin y = \cos x$



Watch Video Solution

49. If  $\sin y = x \sin(a + y)$ , find  $x$  in terms of  $y$  and hence find  $(dx)/(dy)$



Watch Video Solution

50. If  $\sin y = x \sin(a + y)$ , prove that

$$\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$



Watch Video Solution

51. Find  $\frac{dy}{dx}$  if  $2x + 3y = \sin x$



Watch Video Solution

52. Find  $\frac{dy}{dx}$  if  $2x + 3y = \sin y$



Watch Video Solution

53. Find  $\frac{dy}{dx}$  if  $ax + by^2 = \cos y$

 [Watch Video Solution](#)

54. Find  $\frac{dy}{dx}$  if  $xy + y^2 = \tan x + y$

 [Watch Video Solution](#)

55. Find  $\frac{dy}{dx}$ , if  $x^2 + y^2 + xy = 100$

 [Watch Video Solution](#)

56. Find  $\frac{dy}{dx}$  if  $x^3 + x^2y + xy^2 + y^3 = 81$



 [Watch Video Solution](#)

57. Find  $\frac{dy}{dx}$  if  $\sin^2 y + \cos(xy) = \pi$

 [Watch Video Solution](#)

58. Find  $\frac{dy}{dx}$  of the following

$$\sin^2 x + \cos^2 y = 1$$

 [Watch Video Solution](#)

59. Find  $\frac{dy}{dx}$  of the following

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$



[Watch Video Solution](#)

60. If  $y = \tan^{-1} \left( \frac{3x - x^2}{1 - 3x^2} \right)$ ,  $\frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Find  $\frac{dy}{dx}$

 [View Text Solution](#)

61. Find  $\frac{dy}{dx}$  of

$$y = \cos^{-1} \frac{1 - x^2}{(1 + x^2)}, 0 < x < 1$$

 [Watch Video Solution](#)

62. Find  $\frac{dy}{dx}$  if  $y = \sin^{-1} \left( \frac{1 - x^2}{1 + x^2} \right)$ ,  $0 < x < 1$

 [Watch Video Solution](#)

63. If  $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$ ,  $-1 < x < 1$ , Find  $\frac{dy}{dx}$



Watch Video Solution

64. Consider the function

$$f(x) = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{-1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

Find  $f'(x)$ .



Watch Video Solution

65. Find  $\frac{dy}{dx}$  of the following

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$$

 [Watch Video Solution](#)

66. Find  $\frac{dy}{dx}$  of  $y = e^x \log x$

 [Watch Video Solution](#)

67. Find  $\frac{dy}{dx}$  of  $y = \frac{1 - \log x}{1 + \log x}$

 [Watch Video Solution](#)

68. Find  $\frac{dy}{dx}$  of  $y = 4^x$

 [Watch Video Solution](#)

69. Find  $\frac{dy}{dx}$  of  $y = \log_2 x$



[View Text Solution](#)

70. Find  $\frac{dy}{dx}$  of  $y = \log_{10} x$



[View Text Solution](#)

71. Find  $\frac{dy}{dx} y = 2^{\sin^2 x}$



[Watch Video Solution](#)

72. Find  $\frac{dy}{dx}$  of  $y = e^{-x}$



[View Text Solution](#)



 [View Text Solution](#)

73. Find  $\frac{dy}{dx}$  of  $y = \sin(\log x)$

 [Watch Video Solution](#)

74. Find  $\frac{dy}{dx} y = \cos^{-1}(e^2)$

 [View Text Solution](#)

75. Find  $\frac{dy}{dx}$  of  $y = e^{\cos x}$

 [Watch Video Solution](#)

76. Differentiate the following w.r.t.x  $\frac{e^x}{\sin x}$

 [Watch Video Solution](#)

77. Differentiate the following w.r.t.x  $e^{\sin^{-1} x}$

 [Watch Video Solution](#)

78. Differentiate the following w.r.t.x  $e^{x^2}$

 [Watch Video Solution](#)

79. Differentiate the following w.r.t.x  $\sin(\tan^{-1}(e^{-x}))$



 [Watch Video Solution](#)

80. Differentiate the following w.r.t.x  $\log(\cos e^x)$

 [Watch Video Solution](#)

81. Differentiate the following w.r.t.x  $e^x + e^{x^2} + \dots + e^{x^5}$

 [Watch Video Solution](#)

82. Differentiate the following w.r.t.x  $\sqrt{e^{\sqrt{x}}}, x > 0$

 [Watch Video Solution](#)

83. Differentiate the following w.r.t.  $x$   $\log(\log x)$ ,  $x > 1$

 [Watch Video Solution](#)

84. Differentiate the following w.r.t.  $x$   $\frac{\cos x}{\log x}$ ,  $x > 0$

 [Watch Video Solution](#)

85. Differentiate the following w.r.t.  $x$   
 $\cos(\log x + e^2)$ ,  $x > 0$

 [Watch Video Solution](#)

86. Find  $\frac{dy}{dx}$  of

$$y = x^x$$



Watch Video Solution

87. Differentiate  $y = a^x$



View Text Solution

88. Differentiate  $x^{\sin x}$  w.r.t.  $x$



Watch Video Solution

89. Differentiate  $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$  w.r.t.x.

 [Watch Video Solution](#)

90. Find  $\frac{dy}{dx}$ , if  $y^x + x^y + x^x = a^b$

 [Watch Video Solution](#)

91. Differentiate  $\cos x \cdot \cos 2x \cdot \cos 3x$

 [Watch Video Solution](#)

92. Differentiate  $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$  with respect to  $x$ .

 [Watch Video Solution](#)

93. Differentiate  $(\log x)^{\cos x}$  w.r.t.  $x$

 [Watch Video Solution](#)

94. Differentiate  $x^2 - 2^{\sin x}$

 [Watch Video Solution](#)

95. Differentiate  $(x + 3)^2(x + 4)^3(x + 5)^4$  w.r.t.x

 [Watch Video Solution](#)

96. Differentiate  $\left(x + \frac{1}{x}\right)^x + x^{(1+1/x)}$  w.r.t.x

 [Watch Video Solution](#)

97. Differentiate  $(\log x)^x + x^{\log x}$  w.r.t.x

 [Watch Video Solution](#)



98. Find  $\frac{dy}{dx}$  of

$$y = (\sin x)^x + \sin^{-1} \sqrt{x}$$



Watch Video Solution

99. Differentiate  $x^{\sin x} + (\sin x)^{\cos x}$  w.r.t.x



Watch Video Solution

100. Find  $\frac{dy}{dx}$  if  $x^y + y^x = 1$



Watch Video Solution

101. Find  $\frac{dy}{dx}$  of

$$y^x = x^y$$



Watch Video Solution

102. Find  $\frac{dy}{dx}$  of

$$(\cos x)^y = (\cos y)^x$$



Watch Video Solution

103. Find  $\frac{dy}{dx}$  if  $xy = e^{(x-y)}$



Watch Video Solution

**104.** Find the derivative of the function given by

$$f(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8) \text{ and hence}$$

find  $f'(1)$  ?



[Watch Video Solution](#)

**105.** Differentiate  $(x^2 - 5x + 8)(x^3 + 7x + 9)$  by using

product rule



[Watch Video Solution](#)

**106.** Differentiate  $(x^2 - 5x + 8)(x^3 + 7x + 9)$  by

expanding the product

to obtain a single polynomial

 [Watch Video Solution](#)

107. Differentiate  $(x^2 - 5x + 8)(x^3 + 7x + 9)$  by logarithmic differentiation.

 [Watch Video Solution](#)

108. Find  $\frac{dy}{dx}$  of

$$x = 2at^2, y = at^4$$

 [Watch Video Solution](#)

109. Find  $\frac{dy}{dx}$  if  $x = a \cos \theta, y = b \cos \theta$

 [Watch Video Solution](#)

110. Find  $\frac{dy}{dx}$ ,  $x = \sin t$ ,  $y = \cos 2t$

 [Watch Video Solution](#)

111. Find  $\frac{dy}{dx}$ ,  $x = 4t$ ,  $y = \frac{4}{t}$

 [Watch Video Solution](#)

112. Find  $\frac{dy}{dx}$  if  $x = \cos \theta - \cos 2\theta$  and  $y = \sin \theta - \sin 2\theta$

 [Watch Video Solution](#)

113. Find  $\frac{dy}{dx}$  if  $x = a(\theta - \sin \theta)$  and  $y = a(1 + \cos \theta)$

 [Watch Video Solution](#)

114. Find  $\frac{dy}{dx}$  if  $x = a(\cos t + \log \tan(t/2))$  and  
 $y = a \sin t$

 [Watch Video Solution](#)

115. Find  $\frac{dy}{dx}$ ,  $x = a \sec \theta$ ,  $y = b \tan \theta$

 [Watch Video Solution](#)

116. Find  $\frac{dy}{dx}$  of

$$x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta)$$

 [Watch Video Solution](#)

117. If  $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$  Show that  $\frac{dy}{dx} = \frac{-y}{x}$

 [Watch Video Solution](#)

118. Find second order derivative for

$$y = 3x^4 + 4x^2 - x + 1$$

 [Watch Video Solution](#)

119. Find second order derivative for  $y = x \sin x$

 [Watch Video Solution](#)

120. Find second order derivative for  $f(x) = \cos 2x$

 [Watch Video Solution](#)

121. Find second order derivative for  $y = \frac{\log x}{x}$

 [Watch Video Solution](#)

122. Find second order derivative for  $x = ct$  and  $y = \frac{c}{t}$

 [Watch Video Solution](#)



123. Find  $\frac{d^2y}{dx^2}$ , if  $y = x^3 + \tan x$



Watch Video Solution

124.  $y = a \cos x + b \sin x$  is the solution of the differential equation

$$\frac{d^2y}{dx^2} + y = 0$$



Watch Video Solution

125. If  $y = 3e^{2x} + 2e^{3x}$ , prove that

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

 [Watch Video Solution](#)

126. If  $y = \sin^{-1} x$ , then show that

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} = 0$$

 [Watch Video Solution](#)

127. Find the second order derivative of the following functions  $x^2 + 3x + 2$

 [Watch Video Solution](#)

128. Find the second order derivative of the following functions  $x^{20}$



Watch Video Solution

**129.** Find the second order derivative of the following functions  $x \cos x$



Watch Video Solution

**130.** Find the second order derivative of the following functions  $\log x$



Watch Video Solution

**131.** Find the second order derivative of the following functions  $x^3 \log x$



Watch Video Solution

**132.** Find the second order derivative of the following functions  $e^x \sin 5x$



Watch Video Solution

**133.** Find the second order derivative of the following functions  $e^{6x} \cos 3x$



Watch Video Solution

**134.** Find the second order derivative of the following functions  $\tan^{-1} x$



[Watch Video Solution](#)

**135.** Find the second order derivative of the following functions  $\log(\log x)$



[Watch Video Solution](#)

**136.** Find the second order derivative of the following functions  $\sin(\log x)$



[Watch Video Solution](#)

**137.** If  $y = 5 \cos x - 3 \sin x$ , prove that  $\frac{d^2y}{dx^2} + y = 0$



[Watch Video Solution](#)

138. If  $y = \cos^{-1} x$ , find  $\frac{d^2y}{dx^2}$  in terms of  $y$  alone.

[Watch Video Solution](#)

139. Let  $y = 3 \cos(\log x) + 4 \sin(\log x)$

Prove that  $x^2 y_2 + x y_1 + y = 0$

[Watch Video Solution](#)

140. If  $y = ae^{mx} + be^{nx}$ , show that

$$\frac{d^2y}{dx^2} - (m + n) \frac{dy}{dx} + mny = 0$$

[Watch Video Solution](#)

141. If  $y = 500e^{7x} + 600e^{-7x}$ . Show that  $\frac{d^2y}{dx^2} = 49y$

 [Watch Video Solution](#)

142. If  $e^y(x + 1) = 1$ , show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

 [Watch Video Solution](#)

143. If  $y = (\tan^{-1} x)^2$ , show that

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2.$$

 [Watch Video Solution](#)

**144.** Verify Rolle's Theorem for the function

$$f(x) = x^2 + 2x - 8, x \in [-4, 2]$$

 [Watch Video Solution](#)

**145.** Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the converse of Rolle's theorem from these examples?

$$f(x) = [x] \text{ for } x \in [5, 9]$$

 [Watch Video Solution](#)

**146.** Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the



converse of Rolle's theorem from these examples?  $f(x)=[x]$

for  $x \in [-2, 2]$



[Watch Video Solution](#)

**147.** Examine if Rolle's theorem is applicable to any of the following function. Can you say something about the converse of Rolle's theorem from these examples?

$f(x) = x^2 - 1$  for  $x \in [1, 2]$



[Watch Video Solution](#)

**148.** If  $f: [-5, 5] \rightarrow \mathbb{R}$  is a differentiable function and if  $f'(x)$  does not vanish anywhere, then prove that

$f(-5) \neq f(5)$



[Watch Video Solution](#)

**149.** Verify mean value theorem for the function

$f(x) = x^2 - 4x - 3$  in the interval  $[1,4]$  .



[Watch Video Solution](#)

**150.** Verify Lagrange 's mean value theorem, if

$f(x) = x^3 - 5x^2 - 3x$  in the interval  $[a,b]$ , where  $a=1$

and  $b=3$ . Find all  $c \in (1, 3)$  at which  $f'(c)=0$ .



[Watch Video Solution](#)

**151.** Differentiate the following w.r.t  $x$   $(3x^2 - 9x + 5)^9$



Watch Video Solution

152. Differentiate the following w.r.t  $x$   $\sin^3 x + \cos^6 x$



Watch Video Solution

153. Differentiate the following w.r.t  $x$   $(5x)^{3 \cos 2x}$



Watch Video Solution

154. Differentiate the following w.r.t  $x$   $(\log x)^{\log x}, x > 1$



Watch Video Solution

155. Differentiate the following w.r.t.  $x$   $x^x + x^a + a^x + a^a$

for some fixed  $a > 0$  and  $x > 0$



Watch Video Solution

156.

Find

$$\frac{dy}{dx}, \quad \text{if } y = 12(1 - \cos t), x = 10(t - \sin t). \quad \frac{\pi}{2} < t < \frac{\pi}{2}$$



Watch Video Solution

157. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , find  $\frac{d^2y}{dx^2}$



Watch Video Solution

158. If  $y = e^{a \cos^{-1} x}$ , then show that

$$(1 - x^2)y_2 - xy_1 - a^2y = 0$$



[Watch Video Solution](#)