



MATHS

BOOKS - A N EXCEL PUBLICATION

RELATIONS AND FUNCTIONS

Question Type

1. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set

A = {1,2,,3,...,13,14} defined as R= {(x,y) : 3x - y = 0}



2. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation in the set N of natural numbers defined as

$$R = \{(x,y) : y = x + 5 \, \, {
m and} \, \, x < 4\}$$

3. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R un the set A ={1,2,3,4,5,6} as $R = \{(X,Y) : Y | S divisible by x \}$

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4. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set Z of all integers defined as



5. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x and y workattesameplace}

6. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x and y live in the same locality}



7. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a

town at a particular time given by

R={(x,y): x is exactly 7 cm taller than y }



8. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x is wife of y }

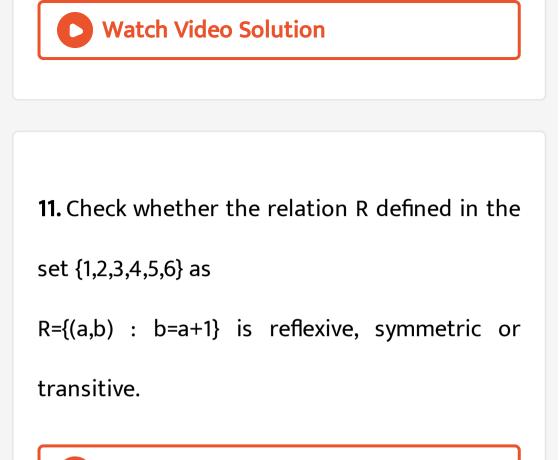
9. Determine whether each of the following relations is reflexive, symmetric and transitive.
Relation R in the set A of human beings in a town at a particular time given by
R={(x,y): x is father of y }

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10. Show that the relation R in the set R of real

number, defined as

 $R = \left\{ (a, b) : a \leq b^2
ight\}$ is neither reflexive nor symmetric nor transitive.



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12. Show that the relation R in R defined as R= $\{(a, b): a \leq b\}$ is reflexive and transitive but

not symmetric.



13. Check whether the relation R in R defined by $R = \left\{(a, b) : a \le b^3
ight\}$ is reflexive, symmetric or transitive.

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14. Show that the relation R in the set {1,2,3} given by R ={(1,2),(2,1)} is symmetric

but neither reflexive nor transitive



15. Show that the relation R in the set A of all the book in a library of a college, given by R = {(x,y): x and y have the same number of pages} is an equivalence relation.



16. Show that the relation R in the set A = (1,2,3,4,5) given by R = $\{(a,b) : |a - b| \text{ is even }\}$, is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2,4\}$.

17. Show that each of the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$,given by R = {(a,b) : |a-b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 in each case.

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18. Show that each of the relation R in the set

 $A=\{x\in Z\!:\!0\leq x\leq 12\},$ given by

R = {(a,b) : a=b} is an equivalence relation. Find

the set of all elements related to 1 in each

case.



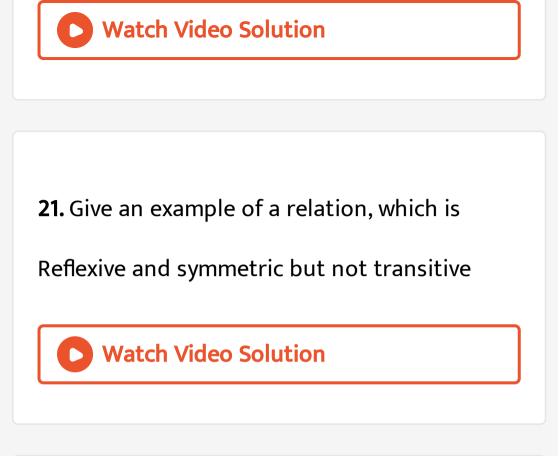
19. Give an example of a relation, which is

Symmetric but neither reflexive nor transitive.

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20. Give an example of a relation, which is

Transitive but neither reflexive nor symmetric.



22. Give an example of a relation, which is

Reflexive and transitive but not symmetric

23. Give an example of a relation, which is

Symmetric and transitive but not reflexive



24. Show that the relation R. in the set A of all points in a place given by R = {(P,Q): the distance of the points P from the point Q from the origin }, is an equivalence relation. Further show that the set of all points related to a point $p \neq (0, 0)$ is the circle passing through P with origin as centre. **25.** Show that the relation R. defined in the set A of all triangle as $R = \{(T_1, T_2) : T_1$ is similar to T_2 , is an equivalence relation. Consider three right angled triangles T_1 with sides 3,4,5, T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangle among T_1, T_2 and T_3 are related ?

26. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2): P_1 \text{ and } P_2 \text{ have the same} \}$ number of sides },is an equivalence relation.What is the set of all elements in A related to the right angled triangle T with sides 3.4 and 5?

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27. Let L be the set of all line in XY place and R

be the relation in L defined as

 $R = \{(L_1, L_2) : L_1 ext{ is parallel to } L_2\}$ Show that

R is an equivalence relation. Find the set of all

line related to the line y=2x+4

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28. Let R be the relation in the set $\{1,2,3,4,\}$ given by R = $\{(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)\}$. Choose the correct answer. a) R is reflexive and symmetric but not transitive b) R is reflexive and transitive but not symmetric c)

R is symmetric and transitive but not reflexive

d) R is an equivalence relation

A. R is reflexive and symmetric but not

transitive.

- B. R is reflexive and transitive but not symmetric.
- C. R is symmetric and transitive but not

reflexive.

D. R is an equivalence relation.

Answer: B



29. Let R be the relation in the set N given by

$$R = \{(a,b) : a = b-2, b > 6\}$$

choose the correct answer

Answer: C

Question Bank

- 1. Let A be the set of all 50 students of class X
- in a school. Let $f\!:\!A o N$
- be function defined by f(x) = roll numbers of
- the student x. Show that
- f is one-one but not onto



2. Show that $f \colon N o N$ given by $f(x) = egin{cases} (x+1) & ext{if} \ xisodd \ (x-1) & ext{if} \ xiseven \end{cases}$

is both one-one and onto.

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3. Consider a function $f: \left[0, \frac{\pi}{2}\right] \to R$ given by f(x) = sin x and $g: \left[0, \frac{\pi}{2}\right] \to R$

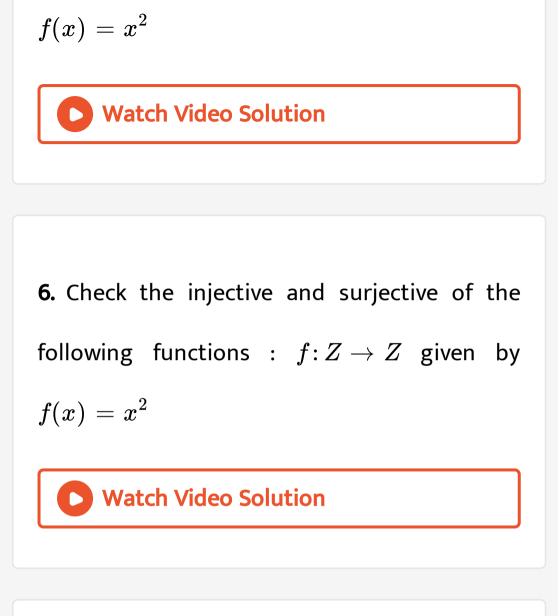
given by $g(x) = \cos x$. Show that f and g are

one-one, but f + g is not one-one.

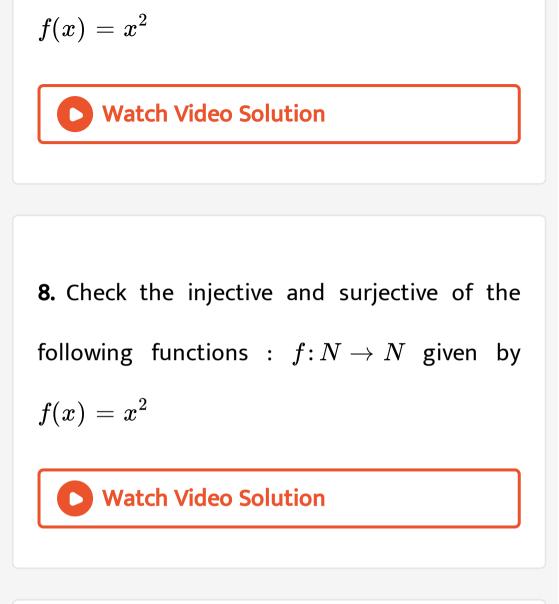
4. Show that the function $f: R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto,where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_* ?



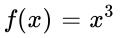
5. Check the injective and surjective of the following functions $: f \colon N o N$ given by



7. Check the injective and surjective of the following functions $: \ f \colon R o R$ given by



9. Check the injective and surjective of the following functions : f: Z o Z given by



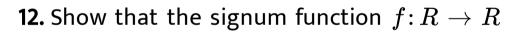
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10. Prove that the greatest integer function $f\colon R o R$ given by f(x)=[x] is neither oneone nor onto, where [x] denotes the greatest integer less than or equal to x.



11. Show that the modulus function $f\colon R o R$ given by f(x)=|x|, is neither one-one nor onto.





given by
$$f(x) = egin{cases} 1 & ext{if} \ x > 0 \ 0 & ext{if} \ x = 0 \ -1 & ext{if} \ x < 0 \end{cases}$$
 is

neither one-one nor onto.

13. Let A = {1, 2, 3}, B { 4, 5, 6} and let f = {(1,4), (2,5)(3,6)} be a function from A to B. Show that f is one-one.



14. In each of the following cases, states whether the function is one-one,

onto or bijective. Justify your answer.

 $f\!:\!R
ightarrow R$ defined by f(x) =3 - 4x

15. In each of the following cases, states whether the function is one-one, onto or bijective. Justify your answer. $f\colon R o R$ defined by f(x) = $1+x^2$

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16. Let A and B be sets. Show that

 $f{:}A imes B o B imes A$ such that

f(a,b)=(b,a) is a bijective function

17. Let f: N to N be defined by

$$f(n) = \left\{egin{array}{c} \left(rac{n+1}{2}
ight) ext{ if } nisodd \ rac{n}{2} ext{ if } niseven \end{array} ext{ for all } n \in N
ight.$$

State whether the function f is bijective. Justify

yur answer.

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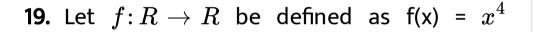
18. Let A = R - {-3} and B = R - {1}

Consider the function $f\!:\!A o B$ defined by

 $f(x) = rac{x-2}{x-3}$. Is f one-one and onto ? Justify

your answer.





Choose the correct answer

A. f is one-one onto

B. f is many one

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D



20. Let $f: R \to R$ be defined as f(x) = 3xChoose the correct answer

A. f is one-one onto

B. f is many one

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A



21. Let A = {a, b, c}, B = {p, q, r} and $f: A \rightarrow B$ be given by f = {(a, q), (b, r), (c, p)}. will f^{-1} exist ? justify your answer.

22. If f: R o R be defined by f(x) = 5x - 3, then prove that f is one-one and onto and find a formula for f^{-1}



23. Let N o R be a function defined as $f(x) = 4x^2 + 12x + 15$ Show that f: N o S, where, S is the range of f, is invertible. Find the inverse of f.

24. Consider of $f: N \rightarrow N$ and $h: N \rightarrow R$ defined as f(x) = 2x, g(y) = 3y + 4and $h(z) = \sin z, \forall, x, y$ and z in N. Show that ho(gof) = (hog)of.

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25. if the function $f \colon R o R$ be defined by f(x)

=
$$x^2 + 5x + 9$$
 find $f^{-1}(9)$.

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26. Let S = \{1, 2, 3\}. Determine whether the
function f \colon S 	o S defined as below have
inverses. Find f^{-1}, if it exists
(a) f = \{(1,1), (2,2), (3,3)\}
(b) f = \{(1,2), (2,1), (3,1)\}
(c) f = \{(1,3), (3,2), (2,1)\}
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27. Let f: \{1, 3, 4\} \rightarrow \{1, 2, 5\} and
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g : {1, 2, 5} \rightarrow (1, 3} be given by

Write down g o f

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28. Let f, g, and h be functions from R to R. Show that

(f+g)oh = foh + goh

29. Let f, g, and h be functions from R to R. Show that

$$(f \cdot g)oh = (foh) \cdot (goh)$$



$$f(x) = |x| \text{ and } g(x) = |5x - 2|$$

31. If $f(x) = 8x^3$ and $g(x) = x^{rac{1}{3}}$, find g(f(x)) and f(g(x))

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32. If f(x) =
$$\frac{4x+3}{6x-4}$$
, $x \neq \frac{2}{3}$, show that (fof)(x) = x, for all $x \neq \frac{2}{3}$.

What is the inverse of f?

33. State with reason whether the following

functions have inverse

f: {1,2,3,4} to {10} with

f = {(1,10),(2,10),(3,10),(4,10)}

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34. State with reason whether the following

functions have inverse

g : {5, 6, 7, 8} \rightarrow {1, 2, 3, 4} with

 $g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$



35. State with reason whether the following functions have inverse h : $\{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with $h = \{(2,7), (3,9), (4, 11), (5, 13)\}$ Watch Video Solution **36.** Show that $f: [-1,1] \rightarrow R$ given by

$$f(x)=rac{x}{x+2}$$
 is one-one.

37. Consider $f: R \rightarrow R$ given f(x) = 4x + 3.

Show that f is invertible. Find the inverse of f.

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38. Consider $f \colon R^+ o [-5,\infty)$ given by

 $f(x) = 9x^2 + 6x - 5$. Show that f is invertible

with
$$f^{-1}(y)=rac{\sqrt{y+6}-1}{3}$$

39. Let $f \colon X \to Y$ be invertible, show that f

has unique inverse

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40. Consider f : {1, 2, 3} \rightarrow {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. find f^{-1} and show that $(f^{-1})^{-1}$ = f

41. Let $f \colon X o Y$ be invertible, show that the

inverse of f^{-1} = f,

i.e.,
$$(f^{-1})^{-1} = f$$
.

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42. If $f: R \to R$ be given by f(x) = $\left(3 - x^3\right)^{rac{1}{3}}$, then (f o f) (x) is

A. $x^{rac{1}{3}}$

C. x

Answer: C

43. f :
$$R - \left\{\frac{-4}{3}\right\} \to R$$
 be a function defined as $f(x) = \frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \to R - \left\{\frac{-4}{3}\right\}$ given by

A.
$$g(y)=rac{3y}{3-4y}$$

$$egin{aligned} \mathsf{B.}~g(y) &= rac{4y}{4-3y}\ \mathsf{C.}~g(y) &= rac{4y}{3-4y}\ \mathsf{D.}~g(y) &= rac{3y}{4-3y} \end{aligned}$$

Answer: B



44. Determine which of the following binary operation on the set R are associative and which are communitative.

a * b = 1



45. Check whether the following binary operation on the set R are associative and commutative. $a * b = \frac{(a+b)}{2} \forall a, b \in R$ **Watch Video Solution**

46. Determine whether or not each of the definitions of * given below gives a binary

operation. In the event that * is not a binary

operation, give justification for this

on Z^+ , define $\ast\,$ by $a \ast b = a - b$

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47. Determine whether or not each of the definitions of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this on Z^+ , define * by a * b = ab



48. Determine whether or not each of the definitions of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this

on R, define * by $a * b = ab^2$

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49. Determine whether or not each of the definitions of * given below gives a binary operation. In the event that * is not a binary

operation, give justification for this

on Z^+ , define $\, * \,$ by $a \, * \, b = |a - b|$



50. Determine whether or not each of the definitions of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this

on Z^+ , define $\, * \,$ by $a \, * \, b = a$

51. For each binary operation * defined below, determine whether * is commutative or associative on Z, define a * b = a - b

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52. For each binary operation * defined below, determine whether * is commutative or associative

on Q, define a * b = ab + 1

53. For each binary operation * defined below, determine whether * is commutative or associative

on Q, define $a * b = 2^{ab}$

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54. For each binary operation * defined below, determine whether * is commutative

or associative

on Z, define a * b = a - b



55. For each binary operation * defined below, determine whether * is commutative or associative

on Z^+ , define $a \ast b = a^b$

56. For each binary operation * defined below, determine whether * is commutative or associative on $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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57. Consider the binary operation \land on the set {1,2,3,4,5} defined by $a \land b=\min\{a,b\}$. Write the operation table of the operation \land .

58. Consider the binary operation * on the set A= {1,2,3,4,5} given by the following multiplication table

Compute (2 * 3) * 4 and 2 * (3 * 4)

*	Г	2	3	4	5
1	1	1	1	i	1
2	. 1	2	1	2	1
3	1	1	3	1	1
4	. 1	2	1	4	1
5	1	.1	1	•1	5

59. Consider the binary operation * on the set A= {1,2,3,4,5} given by the following multiplication table

Is * commutative ?

*	Г	2	3	4	5
1	1	1	1	i	1
2	. 1	2	1	2	1
3	1	1	3	1	1
4	. 1	2	1	4	1
5	1	.1	1	• 1	5

60. Consider the binary operation * on the set A= {1,2,3,4,5} given by the following multiplication table

Compute (2*3)*(4*5)

*	Г	2	3	4	5
1	1	1	1	i	1
2	. 1	2	1	2	1
3	1	1	3	1	1
4	. 1	2	1	4	1
5	1	.1	1	• 1	5

61. Let *' be the binary operation on the set {1,2,3,4,5} defined by a * 'b= H.C.F. of a and b. Is the operation *' same as the operation *

defined in Exercise 4 above ?

Justify your answer.



62. Let * ' be the binary operation on N given by a * 'b= L.c.m. of a and b. Find 5 * 7, 20 * 16

63. Let * ' be the binary operation on N given by a * 'b= L.c.m. of a and b. Is * commutative ?



64. Let * ' be the binary operation on N given

by a * 'b= L.c.m. of a and b. Is * associative ?

65. Let * ' be the binary operation on N given
by a * 'b= L.c.m. of a and b. Find the identify of
* in N

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66. Let * ' be the binary operation on N given by a * 'b= L.c.m. of a and b. Which elements of

N are invertible for the operation * ?

67. Consider an operation * defined on the set

 $A=\{1,2,4,8\}$ by a*b=LCM of a and b.

Show that * is a binary operation.

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68. Let * be a binary operation on N defined

by a * b = HCF of a and b

Is * commutative?

69. Let * be a binary operation on the set Q of rational numbers as follows

a * b = a - b.

Check whether * is commutative and

associative

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70. Let * be a binary operation on the set Q

of rational numbers as follows

 $a \ast b = a^2 + b^2.$

Is the binary operation commutative and associative ?

71. Let * be a binary operation on the set Q of

rational numbers as follows

a * b = a + ab.

Is the binary operation commutative and associative ?

72. Let * be a binary operation on the set Q

of rational numbers as follows

$$a * b = (a - b)^2.$$

Is the binary operation commutative and associative ?



73. Consider the binary operation * on the set R of real numbers, defined by a * b = ab/4Show that * is commutative and associative.

74. Determine whether or not each of the definitions of * given below gives a binary operation. In the event that * is not a binary operation, give justification for this

on R, define * by $a * b = ab^2$

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75. Find which of the following of the operations given above has identity.



76. Find which of the following of the operations given above has identity.

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77. Find which of the following of the

operations given above has identity.

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78. Find which of the following of the

operations given above has identity.



79. Let * be a binary operation on the set Z of integers as a * b = a + b + 1. Then find the

identity element:

80. Let

A = N imes N and $\ st$

be a binary operation on A defined by (a,b)*

(c,d) = (a + c,b + d)

Prove that

*

is associative

81. State whether the following statements are

true or false. Justify

For an arbitary binary operation * on N,

 $a*a=a\,orall a\in N$



82. State whether the following statements are true or false. Justify If * is a commutative binary operation on N, then a * (b * c) = (c * b) * a



83. Consider a binary operation * on N defined as $a * b = a^3 + b^3$. Choose the correct answer. a) both associative and commutative b)commutative but not associative c) associative but not commutative d) neither commutative nor associative

- A. * both associative and commulative
- B. * commutative but not associative
- C. * associative but not commutative

D. * neither commutative nor associative

Answer: B

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84. Let $f: R \to R$ be defined as f(x) = 10x+7 Find the function $g: R \to R$ such that gof = fog = I_R

85. Let $f \colon W \to W$ be defined as f(n) = n-1 if n is odd and

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f(n) = n+1 if n is even.
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Show that f is invertible. Find the inverse of f.

Here W is the set of all whole numbers.



86. If
$$f\!:\!R o R$$
 is defined by

 $f(x)=x^2-3x+2$, find f(f(x))

