MATHS

BOOKS - A N EXCEL PUBLICATION

RELATIONS AND FUNCTIONS

Question Type

1. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set

A = {1,2,,3,...,13,14} defined as R= {(x,y) : 3x -y = 0}

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2. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation in the set N of natural numbers defined as

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

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3. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R un the set A ={1,2,3,4,5,6} as

R = {(X,Y) : Y IS divisible by x }

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4. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set Z of all integers defined as

R = {(x,y) :x-y is an integer}

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5. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x and y workattesameplace}

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6. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x and y live in the same locality}

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7. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x is exactly 7 cm taller than y }

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8. Determine whether each of the following relations is reflexive, symmetric and transitive. Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x is wife of y }

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9. Determine whether each of the following relations is reflexive, symmetric and transitive.

Relation R in the set A of human beings in a town at a particular time given by

R={(x,y): x is father of y }

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10. Show that the relation R in the set R of real number, defined as

$R = \{(a. b) : a \leq b^2\}$is neither reflexive nor symmetric nor transitive.

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11. Check whether the relation R defined in the set {1,2,3,4,5,6} as R={(a,b) : b=a+1} is reflexive, symmetric or transitive.

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12. Show that the relation R in R defined as R= $\{(a, b) : a \leq b\}$ is reflexive and transitive but

not symmetric.

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13. Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

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14. Show that the relation R in the set {1,2,3} given by R ={(1,2),(2,1)} is symmetric but neither reflexive nor transitive

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15. Show that the relation R in the set A of all the book in a library of a college, given by R = {(x,y): x and y have the same number of pages} is an equivalence relation.

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16. Show that the relation R in the set A = (1,2,3,4,5} given by R = {(a,b) : |a - b| is even }, is an equivalence relation. Show that all the elements of {1,3,5} are related to each other and all the elements of {2,4} are related to each other. But no element of {1, 3, 5} is related to any element of {2,4}.

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17. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$,given by R = {(a,b) : |a-b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1 in each case.

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18. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$,given by R = {(a,b) : a=b} is an equivalence relation. Find the set of all elements related to 1 in each case.

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19. Give an example of a relation, which is Symmetric but neither reflexive nor transitive.

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20. Give an example of a relation, which is Transitive but neither reflexive nor symmetric.

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21. Give an example of a relation, which is

Reflexive and symmetric but not transitive

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22. Give an example of a relation, which is

Reflexive and transitive but not symmetric

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23. Give an example of a relation, which is Symmetric and transitive but not reflexive

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24. Show that the relation R. in the set A of all points in a place given by R = {(P,Q): the distance of the points P from the point Q from the origin }, is an equivalence relation. Further show that the set of all points related to a point $p \neq (0, 0)$is the circle passing through P with origin as centre.

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25. Show that the relation R. defined in the set A of all triangle as $R = \{(T_1, T_2) : T_1$ is similar to T_2, is an equivalence relation. Consider three right angled triangles T_1 with sides 3,4,5, T_2 with sides 5,12,13 and T_3 with sides 6,8,10. Which triangle among T_1, T_2 and T_3 are related ?

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26. Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have the same number of sides },is an equivalence relation.What is the set of all elements in A related to the right angled triangle T with sides 3,4 and 5?

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27. Let L be the set of all line in XY place and R be the relation in L defined as

$R = \{(L_1, L_2) : L_1$ is parallel to $L_2\}$Show that R is an equivalence relation. Find the set of all line related to the line y=2x+4

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28. Let R be the relation in the set {1,2,3,4,} given by R = {(1,2),(2,2),(1,1),(4,4),(1,3),(3,3),(3,2)}.Choose the correct answer. a) R is reflexive and symmetric but not transitive b) R is reflexive and transitive but not symmetric c)

R is symmetric and transitive but not reflexive

d) R is an equivalence relation

A. R is reflexive and symmetric but not transitive.

B. R is reflexive and transitive but not symmetric.

C. R is symmetric and transitive but not reflexive.

D. R is an equivalence relation.

Answer: B

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29. Let R be the relation in the set N given by

$R = \{(a, b) : a = b - 2, b > 6\}$

choose the correct answer

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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Question Bank

1. Let A be the set of all 50 students of class X in a school. Let $f: A \rightarrow N$ be function defined by f(x) = roll numbers of the student x. Show that

f is one-one but not onto

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2. Show that $f: N \to N$ given by

$$f(x) = \begin{cases} (x+1) & \text{if } \quad xisodd \\ (x-1) & \text{if } \quad xiseven \end{cases}$$

is both one-one and onto.

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3. Consider a function $f: \left[0, \frac{\pi}{2}\right] \to R$ given by f(x) = sin x and $g: \left[0, \frac{\pi}{2}\right] \to R$ given by g(x) = cos x. Show that f and g are one-one, but f + g is not one-one.

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4. Show that the function $f: R_* \rightarrow R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto,where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_* ?

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5. Check the injective and surjective of the following functions : $f: N \rightarrow N$ given by

$f(x) = x^2$

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6. Check the injective and surjective of the following functions : $f: Z \to Z$ given by $f(x) = x^2$

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7. Check the injective and surjective of the following functions : $f: R \to R$ given by

$f(x) = x^2$

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8. Check the injective and surjective of the following functions : $f: N \rightarrow N$ given by $f(x) = x^2$

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9. Check the injective and surjective of the following functions : $f: Z \rightarrow Z$ given by

$f(x) = x^3$

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10. Prove that the greatest integer function $f: R \rightarrow R$ given by $f(x) = [x]$ is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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11. Show that the modulus function $f: R \to R$ given by $f(x) = |x|$, is neither one-one nor onto.

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12. Show that the signum function $f: R \to R$ given by $f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.

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13. Let A = {1, 2, 3}, B { 4, 5, 6} and let f = {(1,4), (2,5)(3,6)} be a function from A to B. Show that f is one-one.

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14. In each of the following cases, states whether the function is one-one, onto or bijective. Justify your answer. $f: R \rightarrow R$ defined by f(x) =3 - 4x

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15. In each of the following cases, states whether the function is one-one, onto or bijective. Justify your answer. $f: R \to R$ defined by f(x) = $1 + x^2$

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16. Let A and B be sets. Show that $f: A \times B \to B \times A$ such that $f(a, b) = (b, a)$ is a bijective function

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17. Let f: N to N be defined by

$$f(n) = \begin{cases} \left(\frac{n+1}{2}\right) & \text{if} \quad n is odd \\ \frac{n}{2} & \text{if} \quad n is even \end{cases} \text{ for all } n \in N$$

State whether the function f is bijective. Justify yur answer.

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18. Let A = R - {-3} and B = R - {1}

Consider the function $f: A \to B$ defined by

$f(x) = \frac{x-2}{x-3}$. Is f one-one and onto ? Justify your answer.

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19. Let $f: R \to R$ be defined as f(x) = x^4

Choose the correct answer

A. f is one-one onto

B. f is many one

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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20. Let $f: R \rightarrow R$ be defined as f(x) = 3x

Choose the correct answer

A. f is one-one onto

B. f is many one

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: A

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21. Let A = {a, b, c}, B = {p, q, r} and $f: A \to B$ be given by

f = {(a, q), (b, r), (c, p)}. will f^{-1} exist ? justify your answer.

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22. If $f: R \rightarrow R$ be defined by f(x) = 5x - 3, then prove that f is one-one and onto and find a formula for f^{-1}

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23. Let $N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$ Show that $f: N \rightarrow S$, where, S is the range of f, is invertible. Find the inverse of f.

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24. Consider of $f: N \to N$ and $h: N \to R$ defined as f(x) = 2x, g(y) = 3y + 4 and h(z) = sin z, ∀, x, y and z in N. Show that ho(gof) = (hog)of.

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25. if the function $f: R \to R$ be defined by f(x) = $x^2 + 5x + 9$ find $f^{-1}(9)$.

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26. Let S = {1, 2, 3}. Determine whether the function $f: S \rightarrow S$ defined as below have inverses. Find f^{-1}, if it exists

(a) f = {(1,1), (2,2), (3,3)}

(b) f = {(1,2), (2,1), (3,1)}

(c) f = {(1,3), (3,2), (2,1)}

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27. Let f : {1, 3, 4} $\rightarrow$ {1, 2, 5} and

g : {1, 2, 5} $\rightarrow$ (1, 3} be given by f = {(1,2), (3,5), (4,1)} and

g = {(1,3), (2,3), (5,1)}

Write down g o f

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28. Let f, g, and h be functions from R to R.

Show that

$(f + g)oh = foh + goh$

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29. Let f, g, and h be functions from R to R.

Show that

$$(f \cdot g)oh = (foh) \cdot (goh)$$

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30. Find f o g and g o f

f(x) = |x| and g(x) = |5x -2|

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31. If $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$, find $g(f(x))$ and $f(g(x))$

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32. If f(x) = $\frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that (fof)(x) = x, for all $x \neq \frac{2}{3}$.

What is the inverse of f ?

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33. State with reason whether the following functions have inverse

f : {1,2,3,4} to {10} with

f = {(1,10),(2,10),(3,10),(4,10)}

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34. State with reason whether the following functions have inverse

g : {5, 6, 7, 8} $\rightarrow$ {1, 2, 3, 4} with

g = {(5, 4), (6, 3), (7, 4), (8, 2)}

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35. State with reason whether the following functions have inverse

h : {2, 3, 4, 5} $\rightarrow$ {7, 9, 11, 13} with

h = {(2,7), (3,9), (4, 11), (5, 13)}

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36. Show that $f: [-1, 1] \rightarrow R$ given by $f(x) = \dfrac{x}{x+2}$ is one-one.

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37. Consider $f: R \to R$ given f(x) = 4x + 3.

Show that f is invertible. Find the inverse of f.

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38. Consider $f: R^+ \to [-5, \infty)$ given by

$f(x) = 9x^2 + 6x - 5$. Show that f is invertible

with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$

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39. Let $f: X \rightarrow Y$ be invertible, show that f has unique inverse

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40. Consider f : {1, 2, 3} $\rightarrow$ {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. find f^{-1} and show that $\left(f^{-1}\right)^{-1}$ = f

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41. Let $f: X \rightarrow Y$ be invertible, show that the inverse of f^{-1} = f,

i.e.,$\left(f^{-1}\right)^{-1} = f$.

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42. If $f: R \rightarrow R$ be given by f(x) = $\left(3 - x^3\right)^{\frac{1}{3}}$, then (f o f) (x) is

A. $x^{\frac{1}{3}}$

B. x^3

C. x

D. (3 - x^3)`

Answer: C

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43. f : $R-\left\{\frac{-4}{3}\right\} \rightarrow R$ be a function defined as $f(x)=\frac{4x}{3x+4}$. The inverse of f is the map g : Range $f \rightarrow R-\left\{\frac{-4}{3}\right\}$ given by

A. $g(y)=\frac{3y}{3-4y}$

B. $g(y) = \frac{4y}{4 - 3y}$

C. $g(y) = \frac{4y}{3 - 4y}$

D. $g(y) = \frac{3y}{4 - 3y}$

Answer: B

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44. Determine which of the following binary operation on the set R are associative and which are communtative.

$a * b = 1$

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45. Check whether the following binary operation on the set R are associative and commutative.

$$a * b = \frac{(a+b)}{2} \forall a, b \in R$$

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46. Determine whether or not each of the definitions of $*$ given below gives a binary

operation. In the event that $*$ is not a binary operation, give justification for this

on Z^{+}, define $*$ by $a * b = a - b$

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47. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on Z^{+}, define $*$ by $a * b = ab$

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48. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on R, define $*$ by $a*b=ab^2$

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49. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on Z^+, define $*$ by $a * b = |a - b|$

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50. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on Z^+, define $*$ by $a * b = a$

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51. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on Z, define $a * b = a - b$

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52. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on Q, define $a * b = ab + 1$

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53. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on Q, define $a * b = 2^{ab}$

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54. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on Z, define $a * b = a - b$

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55. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on Z^{+}, define $a * b = a^{b}$

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56. For each binary operation $*$ defined below, determine whether $*$ is commutative or associative

on $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

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57. Consider the binary operation $\wedge$ on the set {1,2,3,4,5} defined by $a \wedge b$=min{a,b}. Write the operation table of the operation $\wedge$.

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58. Consider the binary operation $*$ on the set A= {1,2,3,4,5} given by the following multiplication table

Compute $(2*3)*4$ and $2*(3*4)$

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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59. Consider the binary operation $*$ on the set A= {1,2,3,4,5} given by the following multiplication table

Is $*$ commutative ?

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

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60. Consider the binary operation $*$ on the set A= {1,2,3,4,5} given by the following multiplication table

Compute (2*3)*(4*5)

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

61. Let $*'$ be the binary operation on the set {1,2,3,4,5} defined by $a *' b$= H.C.F. of a and b. Is the operation $*'$ same as the operation $*$ defined in Exercise 4 above ?

Justify your answer.

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62. Let $*'$ be the binary operation on N given by $a *' b$= L.c.m. of a and b. Find $5 * 7, 20 * 16$

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63. Let $*$ ' be the binary operation on N given by $a * 'b$= L.c.m. of a and b. Is $*$ commutative ?

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64. Let $*$ ' be the binary operation on N given by $a * 'b$= L.c.m. of a and b. Is $*$ associative ?

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65. Let $*$ ' be the binary operation on N given by $a * 'b$= L.c.m. of a and b. Find the identify of $*$ in N

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66. Let $*$ ' be the binary operation on N given by $a * 'b$= L.c.m. of a and b. Which elements of N are invertible for the operation $*$?

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67. Consider an operation $*$ defined on the set

$A = \{1, 2, 4, 8\}$ by $a * b = LCM$ of a and b.

Show that $*$ is a binary operation.

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68. Let $*$ be a binary operation on N defined by $a * b = HCF$ of a and b

Is $*$ commutative?

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69. Let $*$ be a binary operation on the set Q of rational numbers as follows

$a * b = a - b.$

Check whether * is commutative and associative

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70. Let $*$ be a binary operation on the set Q of rational numbers as follows

$a * b = a^2 + b^2.$

Is the binary operation commutative and associative ?

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71. Let $*$ be a binary operation on the set Q of rational numbers as follows

$a * b = a + ab.$

Is the binary operation commutative and associative ?

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72. Let $*$ be a binary operation on the set Q of rational numbers as follows

$a * b = (a - b)^2$.

Is the binary operation commutative and associative ?

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73. Consider the binary operation $*$ on the set R of real numbers, defined by $a * b = ab/4$

Show that $*$ is commutative and associative.

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74. Determine whether or not each of the definitions of $*$ given below gives a binary operation. In the event that $*$ is not a binary operation, give justification for this

on R, define $*$ by $a * b = ab^2$

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75. Find which of the following of the operations given above has identity.

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76. Find which of the following of the operations given above has identity.

View Text Solution

77. Find which of the following of the operations given above has identity.

View Text Solution

78. Find which of the following of the operations given above has identity.

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79. Let $*$ be a binary operation on the set Z of integers as $a * b = a + b + 1$. Then find the identity element:

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80. Let

$A = N \times N$ and $*$

be a binary operation on A defined by (a,b)*

(c,d) = (a + c,b + d)

Prove that

$*$

is associative

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81. State whether the following statements are true or false. Justify

For an arbitary binary operation $*$ on N, $a * a = a \forall a \in N$

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82. State whether the following statements are true or false. Justify

If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

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83. Consider a binary operation $*$ on N defined as $a * b = a^3 + b^3$. Choose the correct answer. a) both associative and commutative b)commutative but not associative c) associative but not commutative d) neither commutative nor associative

A. $*$ both associative and commulative

B. $*$ commutative but not associative

C. $*$ associative but not commutative

D. $*$ neither commutative nor associative

Answer: B

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84. Let $f: R \rightarrow R$ be defined as f(x) = 10x+7

Find the function $g: R \rightarrow R$ such that gof = fog = I_R

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85. Let $f: W \to W$ be defined as f(n) = n-1 if n

is odd and

f(n) = n+1 if n is even.

Show that f is invertible. Find the inverse of f.

Here W is the set of all whole numbers.

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86. If $f: R \to R$ is defined by

$f(x) = x^2 - 3x + 2$, find f(f(x))

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