MATHS

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CONTINUITY AND DIFFERENTIABILITY

Exercise

1. Consider the function $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$ Redefine the function

in such a way that it becomes continuous at x=0.

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2. Consider the function $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$ Redefine the function in such a way that it becomes continuous at x=0.

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3. Consider the function $f(x) = \begin{cases} 1+x & x \leq 2 \\ 5-x & x > 2 \end{cases}$ Check the differentiability of f(x) at x=2.

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4. Consider the function $f(x) = \begin{cases} 1+x & x \leq 2 \\ 5-x & x > 2 \end{cases}$ Check the differentiability of f(x) at x=2.

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5. If $y = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ Put $x = \tan\theta$ and prove that $y = 2\tan^{-1}x$.

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6. If $y = \cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]$ Find $\frac{dy}{dx}$.

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7. Let siny=xsin(a+y) Express x as a function of y.

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8. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

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9. Consider $f: [3, \infty) \to [1, \infty)$ given by $f(x) = x^2 - 6x + 10$. Find f^{-1}

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10. Find $\frac{dy}{dx}$ if $x^{2/3} + y^{2/3} = a^{2/3}$, where a is a constant.

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11. Find the derivative of $\log(\tan e^x)$ w.r.t.x.

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12. Find $\frac{dy}{dx}$ if $2x + 3y = \sin x$

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13. Consider the parametric functions $x = a\cos^3\theta$ and $y = b\sin^3\theta$. Find $\frac{dx}{d\theta}$.

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14. Consider the parametric functions $x = a\cos^3\theta$ and $y = b\sin^3\theta$. Show that $\frac{dy}{dx} = -\frac{b}{a}\tan\theta$.

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15. Differentiate the following. $e^{ax}.\cos(bx+c)$.

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16. Differentiate the following. $\sin\left[\sqrt{\sin\sqrt{x}}\right]$.

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17. If $y = x^{x^x}$. Prove that $\log y = x^x(\log x)$

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18. If $y = x^{x^x}$ Find $\frac{dy}{dx}$.

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19. Consider $y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$. Hence prove that $y = \frac{\pi}{4} + \frac{x}{2}$.

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20. Consider $y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$ Find $\frac{dy}{dx}$.

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21. $\sin(xy) + \frac{x}{y} = x^2 - y$. Evaluate $\frac{dy}{dx}$

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22. Given $x^y = y^x$. Take logarithm of both sides.

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23. Given $x^y = y^x$ Find dy/dx .

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24. $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right), y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$. Find $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$

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25. If $x = a\cos^3\theta, y = a\sin^3\theta$ Find $\frac{d^2y}{dx^2}$.

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26. If $x^3 + y^3 = 3axy$. Differentiate through out with respect to x.

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27. If $x^3 + y^3 = 3axy$ Show that $\frac{dy}{dx} = \frac{ay - x^2}{y^2 - ax}$.

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28. Consider $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}+1}{x}\right)$. Put $x = \tan\theta$ and

show that $\frac{\sqrt{1+x^2}+1}{x} = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$.

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29. Consider $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$. Find $\frac{dy}{dx}$.

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30. y=tan x+sec x. Find $\frac{dy}{dx}$.

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31. y=tan x+sec x. Prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1+\sin x)^2}$.

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32. If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ Find $\frac{dy}{dx}$.

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33. If $y = \sin^{-1}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{2}\right)$ Find $\frac{dy}{dx}$.

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34. $u = (\sin x)^{\tan x}, v = (\cos x)^{\sec x}$. Find (du)/dx and (dv)/dx.

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35. $u = (\sin x)^{\tan x}, v = (\cos x)^{\sec x}$ Find dy/dx. if $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$

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36. Given that $y = a \sin nx + b \cos nx$. Compute $\frac{dy}{dx}$

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37. Given that $y = a\sin nx + b\cos nx$. Compute $\frac{d^2y}{dx^2}$

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38. Given that $y = a\sin nx + b\cos nx$. Show that $\frac{d^2y}{dx^2} + n^2y = 0$.

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39. Differentiate $\sin\left(\frac{x^2}{3} - 1\right)$ with respect to x.

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40. Differentiate $\log\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}$ with respect to x.

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41. Differentiate $\sqrt{\log\left\{\sin\left(\frac{x^2}{3}-1\right)\right\}}$ with respect to x.

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42. Consider $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ Differentiate x. $\sin^{-1}x$ with respect to x.

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43. Verify mean value theorem for the function $f(x) = (x-2)^2$ in $[1, 4]$.

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44. Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(1, 1)$ and $(4, 4)$

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45. If $y = \sin\left(m \sin^{-1} x\right)$, Find $\frac{dy}{dx}$.

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46. If $y = \sin\left(m \sin^{-1} x\right)$. Show that $\sqrt{1 - x^2} \frac{dy}{dx} = m \cos\left(m \sin^{-1} x\right)$

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47. If $y = \sin\left(m \sin^{-1} x\right)$, Hence prove that $\left(1 - x^2\right) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0.$

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48. Verify Lagrange 's mean value theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a,b], where a=1 and b=3. Find all $c \in (1, 3)$ at which f'(c)=0.

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49. Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n.

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50. Choose the correct answer from the bracket. If x=sin t and y= cos t, then $\frac{dy}{dx}$ is equal to ....

A. tan t

B. cot t

C. -tan t

D. -cot t

Answer:

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51. Find the value of k such that the function f(x) defined by $f(x) = \begin{cases} kx - 1 & \text{if } x \leq \frac{\pi}{2} \\ \sin x & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous function at $x = \frac{\pi}{2}$.

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52. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$.

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53. Find the value of a such that the function f(x) defined by $f(x) = \begin{cases} ax + 3 & \text{if } x \leq 2 \\ a^2x - 1 & \text{if } x > 2 \end{cases}$ is continuous function at $x = 2$.

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54. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$. Prove that $1 + \left(\frac{dy}{dx}\right)^2 = \sec^2\theta$.

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55. Choose the correct answer from the bracket. If $x = at^2$,y=2at, then the value of $\frac{dy}{dx}$ at t=1 is

A. 0

B. 1

C. 2

D. ∞

Answer:

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56. Is the function defined by $f(x) = \begin{cases} x+3 & \text{if } x \leq 1 \\ x-3 & \text{if } x > 1 \end{cases}$ a continuous function? Justify.

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57. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

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58. If $x^2 + 2xy + 2y^2 = 1$, then $\frac{dy}{dx}$ at the point where $y = 1$ is equal to

a)1 b)2 c)-1 d)0

A. 0

B. 1

C. 2

D. -1

Answer:

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59. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 2 \\ bx - 3 & \text{if } x > 2 \end{cases}$ is continous at x=2.

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60. If $x^y y^x = 16$, then find $\frac{dy}{dx}$ at (2,2).

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61. Verify Lagranges' Mean value theorem for the function $f(x) = 2x^2 - 10x + 29$ in $[2, 9]$

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62. If $f(x) = \log\left[e^x\left(\frac{3-x}{3+x}\right)^{1/3}\right]$ then $f'(1)$ is equal to a) $\frac{3}{4}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

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63. Verify Rolle's theorem for the function $f(x) = \sin 3x$ on $\left[0, \frac{\pi}{3}\right]$.

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64. Let $y = \left(x + \sqrt{1 + x^2}\right)^m$ Find $\frac{dy}{dx}$.

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65. Let $y = \left(x + \sqrt{1 + x^2}\right)^m$ Show that $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0.$

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66. Find $\frac{dy}{dx}$ if

$$\sin x + \cos y = xy$$

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67. Find $\frac{dy}{dx}$ if $y = (\sin x)^{\log x}$.

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68. Differentiate the following. $\sin\left[\sqrt{\sin\sqrt{x}}\right]$.

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69. Differentiate the following :

$$y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+1}{x}\right]$$

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70. Let siny=xsin(a+y) Express x as a function of y.

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71. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

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72. Is $f(x) = |x|$ differentiable at $x = 0$?

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73. Differentiate the following function w.r.t.x.

$$(\log x)^x + x^{\log x}$$

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74. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

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75. If $y = (\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$, find $\frac{dy}{dx}$.

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76. Consider the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ Find Rf'(0) and Lf'(0).

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77. Consider the function $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ Find Rf'(0) and Lf'(0).

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78. Let $y = 3\cos(\log x) + 4\sin(\log x)$

find $\frac{dy}{dx}$

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79. Given that $y = 3\cos(\log x) + 4\sin(\log x)$.

What is $x.\ \dfrac{dy}{dx}$?

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80. Given that $\log y = \tan^{-1} x$. Find y_1.

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81. Given that $\log y = \tan^{-1} x$. Show that

$(1 + x^2)y_1 = y$.

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82. Given that $\log y = \tan^{-1} x$. Show that $(1 + x^2)y_2 + (2x - 1)y_1 = 0$

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83. $y = e^{ax} \sin bx$: Find y_1.

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84. $y = e^{ax} \sin bx$: Prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0.$

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85. If y=sin(logx) prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

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86. If $x^y = e^{x-y}$. Express y in terms of x.

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87. If $xy = e^{x-y}$. Find $\frac{dy}{dx}$.

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88. Find $\frac{dy}{dx}$,if $y^x + x^y + x^x = a^b$.

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89. Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w.r.t.x.

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90. Let $u = \sin^2 x \& v = e^{\cos x}$.Find du/dx and dv/dx

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91. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$

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92. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, Express y as a function of x.

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93. if $x\sqrt{1+y}+y\sqrt{1+x}=0$ prove that $\left(\frac{dy}{dx}\right)=-\frac{1}{(1+x)^2}$

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94. $x = a\left(\cos t + \log \tan\left(\frac{t}{2}\right)\right), y = a \sin t$

find $\frac{dx}{dt} \& \frac{dy}{dt}$.

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95. Which among the following is not true

A. A polynomial function is always continuous.

B. A continuous function is always differentiable

C. A differentiable function is always continuous

D. log x is continuous for all x greater than zero.

Answer:

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96. Find $\frac{dy}{dx}$, if $x^2 + y^2 + xy = 100$

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97. Find derivative of $y = \sqrt{\tan x}$

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98. Find $\frac{dy}{dx}$ if $y = x^x + x^{\sin x}$.

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99. if $y = x \cos x$, find $\frac{d^2y}{dx^2}$

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100. Find $\frac{dy}{dx}$ of the following

$\sin^2 x + \cos^2 y = 1$

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101. Find $\frac{dy}{dx}$ for the following. $y = x^2$

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102. Find $\frac{dy}{dx}$ for the following.

$x = a(t - \sin t), y = a(1 + \cos t).$

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103. $\frac{d(a^x)}{dx}$ =... a) a^x b) $\log(a^x)$ c) $a^x \log a$ d) xa^{x-1}

A. a^x

B. $\log(a^x)$

C. $a^x \log a$

D. xa^{x-1}

Answer:

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104. Find $\frac{dy}{dx}$ if $x^y = y^x$

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105. Prove that the function defined by

$$f(x) = \cos x^2$$

is a continuous function

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106. if

$y = e^{a\cos^{-1}x}, \ -1 \leq x \leq 1$, show that

$$\frac{dy}{dx} = \frac{-ae^{a\cos^{-1}x}}{\sqrt{1-x^2}}$$

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107. If $y = e^{a\cos^{-1}x}$, then show that

$$(1-x^2)y_2 - xy_1 - a^2y = 0$$

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108. Find dy/dx if $(\sin x)^{\cos y} = (\cos y)^{\sin x}$

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109. Find all points of discontinuity of f where f is defined by $f(x) = \begin{cases} 2x + 3 & x \leq 2 \\ 2x - 3 & x > 2 \end{cases}$

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110. If $e^{x-y} = x^y$ then prove that $\frac{dy}{dx} = \frac{\log x}{[\log ex]^2}$

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111. Find $\frac{dy}{dx}$ if $x^3 + 2x^2y + 3xy^2 + 4y^3 = 5$

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112. Find $\frac{dy}{dx}$ if $x = 2\cos^3\theta, y = 2\sin^3\theta.$

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113. Find $\frac{dy}{dx}$ of

$$y = \sin^{-1} 2x\sqrt{1-x^2}\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$$

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114. Find the value of k if the function

f(x) = kx+1 if x<5

3x-5 if x>5 is continuous at x=5

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115. Find $\frac{dy}{dx}$, If $x = a(t - \sin t)$, $y = a(1 + \cos t)$

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116. Verify Rolle's theorem for the function

$f(x) = x^2 + 2$ in the interval [-2,2]

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117. Consider $f(x) = \begin{cases} 3x - 8 & x \leq 5 \\ 2k & x > 5 \end{cases}$. Find the value of k if f(x) is continuous at x=5`.

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118. Find $\frac{dy}{dx}$, if $y = (\sin x)^{\log x}$, $\sin x > 0$

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119. If $y = \left(\sin^{-1} x\right)^2$, then show that

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

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120. Determine the value of k so that the function

$$f(x) = \begin{cases} k(x^2 + x + 1) & x < 0 \\ \cos x & x \geq 0 \end{cases}$$

is continuous.

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121. If $y = \left[x + \sqrt{x^2 + 9}\right]^n$, show that

$$\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + 9}}$$

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122. Find $\frac{dy}{dx}$ of

$$x = \sqrt{a^{\sin^{-1} t}} \, y = \sqrt{a^{\cos^{-1} t}}$$

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123. Find the values of k so that function f is continuous at the indicated points

$$f(x) = \begin{cases} kx^2 & \text{if } x \leq 2atx = 2 \\ 3 & \text{if } x > 2 \end{cases}$$

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124. Find $\frac{dy}{dx}$ if $y = x^{\sin x}, x > 0.$

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125. If $y = \sin^{-1} x$,then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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126. Check the continuity of the function given by $f(x) = \begin{cases} x \sin \frac{1}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

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127. Verify mean value theorem for the function $f(x) = x + \frac{1}{x}$ in the interval $[1, 3]$.

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128. Find the derivative of $y = x^a + a^x$ with respect to x.

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129. Find $\frac{dy}{dx}$ if

$x = a\cos^3 t, y = a\sin^3 t$

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130. If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

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131. The function $f(x)=\begin{cases}5 & x\leq 2\\ ax+b & 2\leq x\leq 10\\ 21 & x\geq 10\end{cases}$ is continuous

Find a and b?

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132. Find $\frac{dy}{dx}$ if $y = \sin\left(x^{\sin x}\right)$

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133. If $y = ae^{mx} + be^{nx}$, show that

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

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134. Show that the function $f(x)$ defined by

$f(x) = \sin(\cos x)$ is a continuous function .

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135. If $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$, Show that $\frac{d^2y}{dx^2} = \frac{-\frac{d^2x}{dy^2}}{\left(\frac{dx}{dy}\right)^3}$

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136. Let $y = x^{\sin x} + (\sin x)^x$. Find $\frac{dy}{dx}$

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137. Given, $y = \sqrt{\tan^{-1} x}$ show $2(1+x^2)y\frac{dy}{dx} = 1$

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138. Given, $y = \sqrt{\tan^{-1} x}$ show $(1+x^2)y\frac{d^2y}{dx^2} + (1+x^2)\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx} = 0$

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139. Consider the function $f(x) = |x|, x \in R$

Show that the function is continious at x=0.

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140. Establish that $g(x) = 1 - x + |x|$ is continious at origin.

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141. Check whether $h(x) = |1 - x + |x||$ is continious at origin.

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142. Find $\frac{dy}{dx}$ if $y = \sin\left(x^3 + 7\right)$.

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143. Find $\frac{dy}{dx}$ if $x = a(t - \sin t)$, $y = a(1 - \cos t)$.

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144. If $y = a\cos(\log x) + b\sin(\log x)$, Prove that $x^2 y_2 + x y_1 + y = 0$.

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145. Find $\frac{dy}{dx}$ of e^{x^3}.

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146. If $y = ae^{mx} + be^{nx}$, show that

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$$

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147. If $y = 5\cos x - 3\sin x$, prove that

$$\frac{d^2y}{dx^2} + y = 0$$

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148. Verify mean value theorem for each of the function $f(x) = \frac{1}{4x - 1}$ in [1,4].

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