MATHS

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THREE DIMENSIONAL GEOMETRY

Example

1. Given the straight line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

What are its direction rations?

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2. Given the straight line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Obtain the equation of the straight line passing through (2,4,-1) and line parallel to the given line.

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3. Equation of the plane passing through the intersecton of the

planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and the point (1,1,1) is

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4. Find the equation of the plane through the points (2,2,1),(9,3,6) and perpendicular to the plane 2x+6y+6z-1=0.

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5. Consider the points (-1,2,4) and (1,0,5).Find the direction cosines of the line joining the two points.

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6. The angle between the straight line $r = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $r.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 4$ is

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7. Consider the cartesian equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ Write the co-ordinates of the point,in which the line passes through.

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8. Consider the cartesian equation of the line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ Write the direction ratios of the line.

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9. The cartestian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its vector form.

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10. Find the equation of the plane passing through the line of intersection of the planes 2x-y=0,3z-y=0 and perpendicular to the plane 4x+5y-3z=8

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11. Prove that the points (2,4,4),(1,5,4),(2,6,2)and (0,6,4) are coplanar.

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12. Find the angle between the lines whose direction ratios are a, b, c and b-c, c-a, a-b

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13. Consider the pair of lines whose equations are .

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$$ and

$$\frac{x+1}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

Write the direction ratios of the lines.

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14. Consider the pair of lines whose equations are .

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$$ and

$$\frac{x+1}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

Find the angle between these two lines.

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15. Find the equation of the plane passing through the intersection of the planes

$x+y+4z+5=0$ and $2x-y+3z+6=0$ and contains the point (1,0,0).

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16. Prove that if a plane has the intercepts a, b, c and it is at a distance p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

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17. If O be the origin and the co-ordinates of P be (1,2,-3), then find the equation of the plane passing through P and perpendicular to OP.

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18. Consider the equation of the line 3x+4=4y-1=1-4z.(i) Find the direction ratios of this line.

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19. Consider the equation of the line 3x+4=4y-1=1-4z. What is the vector equation of a line passing through (2,4,6) and parallel to the above line?

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20. Find the equation of the plane through the point (1,2,3) and perpendicular to the plane $x - y + z = 2$ and $2x + y - 3z = 5$

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21. Find the distance between the planes

$x - 2y + 2z - 8 = 0$ and $6y - 3x - 6z = 57$

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22. Consider the vector $4\hat{i} + 8\hat{j} + \hat{k}$ (i) Find the direction cosines of the given vector

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23. Find the angle between the planes 2x-y+z=6 and x+y+2z=7.

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24. Determine the point in XY plane which is equidistant from three point $A(2, 0, 3)$, $B(0, 3, 2)$ and $C(0, 0, 1)$

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25. The perpendicular distance of the point $(6, 5, 8)$ from y-axis is

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26. Find the position vector of the point where the line $\vec{r} = \hat{i} - \hat{j} + \hat{k} + t\left(\hat{i} + \hat{j} - \hat{k}\right)$ meets the plane

$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 5$

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27. Let α, β, γbe the angle made by a line with the positive direction of the X,Y,Z axes respectively.(i) Write the direction cosines of the line.

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28. If a line makes α, β, γ with x,y,z axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

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29. Suppose α, β, γ are the angles made a line with the three axes respectively. Fill in the blank by choosing the correct answer from the barcket $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$= ..... (2,0,1,3)

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30. A straight line is passing through (3,8,3) and is parallel to the vector $3\hat{i} - \hat{j} + k$(i) Form its equation

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31. A straight line is passing through (3,8,3) and is parallel to the vector $3\hat{i} - \hat{j} + k$ Show that it is not

coplanar with the lines $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

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32. A straight line is passing through (3,8,3) and is parallel to the vector $3\hat{i} - \hat{j} + k$.Calculate the shortest distance between these lines $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

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33. A variable plane is at a constant distance P from origin and meets the axes in A(p,0,0),B(0,q,0),C(0,0,r). (ii)Find The centriod of $\triangle ABC$.

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34. Find The equation of the plane passing through the points(2,1,0),(3,-2,-2) and (3,1,7).

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35. Find x such that the four points A(3,2,1)B(4,x,5),C(4,2,-2)and D(6,5-1)are coplanar.

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36. Find the equation of the plane containing the lines $\frac{x-4}{1}=\frac{y-3}{4}=\frac{z-2}{5}$ and

$$\frac{x-3}{1} = \frac{y-2}{-4} = \frac{z+3}{5}$$

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37. Consider the planes 3x-4y+5z=10 and 2x+2y-3z=4 a.Write the equation of the planes through the line of intersection of the above planes.

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38. Write the direction ratios of the line x=2y=3z.

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39. Find the direction cosines of the vector $2\hat{i}+2\hat{j}-\hat{k}$

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40. Find the distance of the point (2,3,4) from the plane $\bar{r}.(3i-6j+2k)=-11$.

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41. Find the shortest distance between the lines $\overrightarrow{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\lambda\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$ and $\overrightarrow{r}=\left(\hat{i}+\hat{j}+\hat{k}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$

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42. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=6$ and $\bar{r}.\left(2\hat{i}+3\hat{j}+4\hat{k}\right)=-5$ at the point (1,1,1) .

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43. Distance between the two planes: '2 x+3 y+4 z=4' and '4 x+6 y+8 z=(12)' is

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44. Find the shortest distance between the skew lines $\vec{r} = s\hat{k}$ and $\vec{r} = (1-t)\hat{i} + t\hat{j}$

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45. If a line makes α, β, γ with x,y,z axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

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46. Find the coordinates of the points where the line through (3,-4,-5) and

(2,-3,1) crosses the plane,passing through the point (2,2,1)

(3,0,1) and

(4,-1,0).

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47. If a line in the space makes angle α, β and γ with the coordinates axes, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is equal to a)1 b)2 c)0 d)3

A. 1

B. 2

C. 0

D. 3

Answer:

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48. The direction ratios of the line $\frac{x-6}{1}=\frac{2-y}{2}=\frac{z-2}{2}$ are a)6,-2,-2 b)1,2,2 c)6,1,-2 d)0,0,0

A. (6,-2,-2)

B. (1,2,2)

C. (6,1,-2)

D. (0,0,0)

Answer:

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49. If the vector equation of a line is

$\bar{r} = i + j + k + \mu(2i - 3j - 4k)$, then the Cartesian equation of the line is

A. $\frac{x+2}{2} = \frac{y+2}{4} = \frac{z+2}{1}$

B. $\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{-4}$

C. $\frac{x+2}{1} = \frac{y+2}{4} = \frac{z+2}{1}$

D. $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{-4}$

Answer:

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50. If the Cartesian equation of a plane is $x + y + z = 12$, then the vector equation of the plane is.... a)$\bar{r}.\,(2i + j + k) = 12$ b)$\bar{r}.\,(i + j + k) = 12$ c)$\bar{r}.\,(i + j + 2k) = 12$ d)$\bar{r}.\,(i + 3j + k) = 12$

A. $\vec{r}.\left(2\hat{i} + \hat{j} + \hat{k}\right) = 12$

B. $\vec{r}.\left(\hat{i} + \hat{j} + 2\hat{k}\right) = 12$

C. $\vec{r}.\left(\hat{i} + \hat{j} + 2\hat{k}\right) = 12$

D. $\vec{r}.\left(\hat{i} + 3\hat{j} + \hat{k}\right) = 12$

Answer:

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51. Cartesian equation of two lines are $\frac{x+2}{2}=\frac{y+2}{4}=\frac{z+2}{1}, \frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-1}{-4}$ .Write the vector equation of the lines.

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52. Cartesian equation of two lines are $\frac{x+2}{2}=\frac{y+2}{4}=\frac{z+2}{1}, \frac{x-1}{2}=\frac{y-1}{-3}=\frac{z-1}{-4}$ .find the shortest distance between the lines.

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53. Consider the lines

$$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - 4\hat{k}\right),$$

$$\bar{r} = \left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} + 3\hat{j} + 2\hat{k}\right)$$

Write the Cartesian equation.

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54. Consider the lines

$$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - 4\hat{k}\right),$$

$$\bar{r} = \left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} + 3\hat{j} + 2\hat{k}\right)$$

Find the angle between the lines.

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55. Let the vector equation of a plane be $\vec{r}.\left(2\hat{i}+4\hat{j}+3\hat{k}\right)=12$(i) Write the cartesian equation of the plane.

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56. Let the vector equation of a plane be $\vec{r}.\left(2\hat{i}+4\hat{j}+3\hat{k}\right)=12$. Find the distance from the point (2,1,3) to the plane

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57. Find the equation of the plane through the point (1,2,3) and perpendicular to the plane $x-y+z=2$

and $2x + y - 3z = 5$

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58. Find the distance between the planes

$x - 2y + 2z - 8 = 0$ and $6y - 3x - 6z = 57$

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59. Consider the Cartesian equation of line $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-5}{-2}$ Find the vector equation of the line.

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60. Consider the Cartesian equation of a line

$$\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-5}{-2}$$

Find the intersecting point with the plane

$$5x + 2y - 6z - 7 = 0$$

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61. Consider the Cartesian equation of line

$$\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-5}{-2}$$

Find the angle made by the line with the plane

$$5x + 2y - 6z - 7 = 0$$

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62. Consider the vector equation of two planes

$\bar{r}.(2i+j+k)=3, \bar{r}(i-j-k)=4$

Find vector equation of any plane through the intersection of the above two planes.

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63. Consider the vector equation of two planes $\bar{r}.(2i+j+k)=3, \bar{r}.(i-j-k)=4$ Find vector equation of the plane through the intersection of the above planes and the point (1,2,-1).

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64. Consider the straight lines

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

Show that these lines are not coplanar.

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65. Consider the straight lines

$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$and

$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

Compute the shortest distance between the lines.

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66. Find the foot and hence the length of the perpendicular from P(5,7,3) to the line $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.Find also the equation of the plane in which the perpendicular and the given straight line lie.

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67. The length of the shortest distance between the two lines $r=(-3i+6j)+s(-4i+3j+2k)$and $r=(-2i+7k)+t(-4i+j+k)$ is

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68. A line passes through the point (3,-2,5) and parallel to the vector $\hat{i} + 2\hat{j} + \hat{k}$ What is the vector equation of the line?

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69. If a plane intersects the co-ordinate axes at a,b,c respectively,write the equation of the plane.

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70. Find the vector and cartesian equations of the plane that passes through

the point (1, 0, -2) and normal to the plane is $\hat{i} + \hat{j} - \hat{k}$

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71. Write all the direction cosines of x-axis.

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72. If a line makes α, β, γ with x,y,z axis respectively, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

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73. A line makes equal angles with the coordinate axis . Find the direction cosines.

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74. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point (2,2,1)`

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75. The Cartesian equation of two lines are given by $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Write the vector equation of these lines.

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76. Find the equation of a plane which makes x,y,z intercepts respectively as 1,2,3

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77. Find the angle between the lines $\frac{x-3}{1}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+1}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

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78. The lines x-1=y=z is perpendicular to the line.....a) $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{-3}$ b)$x-2=y-2=z$ c) $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{3}$ d)x=y=z/2

A. $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{-3}$

B. x-2=y-2=z

C. $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{3}$

D. $x=y=\frac{z}{2}$

Answer:

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79. Find the shortest distance between the lines whose vector equations are $\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

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80. Distance of the point(0,0,1) from the plane $x+y+z=3$ a) $\frac{1}{\sqrt{3}}$ units b) $\frac{2}{\sqrt{3}}$ units c) $\sqrt{3}$ units d) $\frac{\sqrt{3}}{2}$ units

A. $\frac{1}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{\sqrt{3}}{2}$

Answer:

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81. Find the equation of the plane through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to x-y+z=0

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Exercise

1. Find the shortest distance between the lines l_1 and l_2

$\bar{r} = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + k\right)$ and

$\bar{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

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2. Show that the line AB is perpendicular CD if A,B,C,D are the points (2,3,4),(5,4,-1),(3,6,2),(1,2,0) respectively.

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3. Find the values of p so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles.

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4. The foot of the perpendicular drawn from origin to a Plane is (4,-2,5).

How far is the plane from origin?

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5. Find the equation of the plane passing through the point (-1,-1,2) and perpendicular to the plane $3x+2y-3z=1$ and $5x-4y+z=5$.

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