



### MATHS

## **BOOKS - MAXIMUM PUBLICATION**

# **APPLICATION OF DERIVATIVES**



1. Find the equation of tangents and normals to the given curves  $x = \cos t, y = \sin t$ , at  $t = \frac{\pi}{4}$ 

**2.** A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the decreasing when the foot of the ladder is 4m away from the wall.

**3.** Find the points on the curve  $y = x^3$ , the tangents at which are inclined at an angle of 60° to x-axis?



**4.** Find the equation of the tangent to the parabola

 $y^2 = 4x + 5$  which is parallel to y = 2x + 7.

**5.** Find the intervals in which the function f

given  $f(x) = 2x^2 - 3x$  is

Strictly Increasing.

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**6.** Find the intervals in which the function f

given  $f(x) = 2x^2 - 3x$  is

Strictly Decreasing.

7. Find the intervals in which the function  $f(x) = (x+1)^3(x-3)^3$  strictly increasing

or decreasing.



8. Find the intervals in which the function  $f(x) = x + \frac{1}{x}$  strictly increasing or decreasing.

9. Determine whether the  $f(x) = x^2$  function is strictly monotonic on the indicated interval. (-1, 1)



**10.** Determine whether the  $f(x) = x^2$  function

is strictly monotonic on the indicated interval.

 $(\,-1,\,0)$ 

11. Determine whether the  $f(x) = x^2$  function is strictly monotonic on the indicated interval. (0, 1)



12. Determine whether the  $f(x) = x^3 - x$ function is strictly monotonic on the indicated interval.

 $(\,-1,\,0)$ 

13. Determine whether the  $f(x) = x^3 - x$ function is strictly monotonic on the indicated interval.

$$\left(-1,rac{-1}{2}
ight)$$

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14. Determine whether the  $f(x) = x^3 - x$ function is strictly monotonic on the indicated interval.

$$(-1,1)$$



15. Find the approximate change in the volumeV of a cube of side x meters caused byincreasing the side by 1%.

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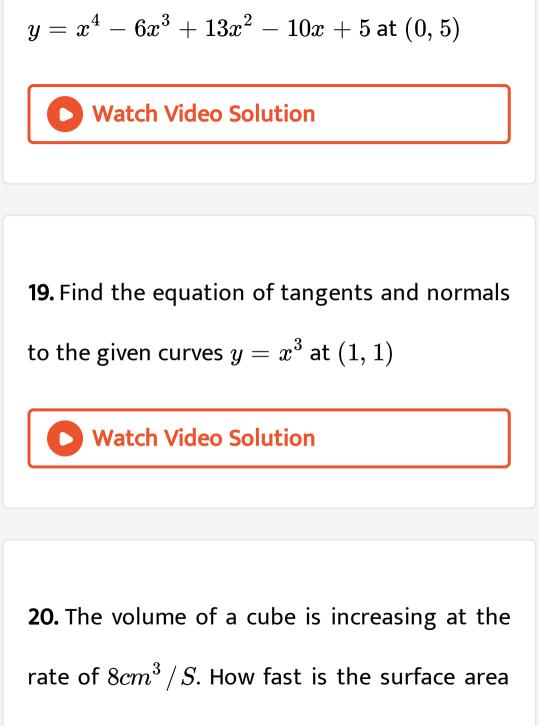
**16.** If the radious of a sphere is measured as 7m with an error of 0.02m then find the approximate error in calculating its volume.

**17.** The length of a rectangle is decreasing at the rate of 5 cm/mi and the width is increasing at the rate of 4cm/min.When length is 8 cm and width is 6 cm, find the rate of change of its area.

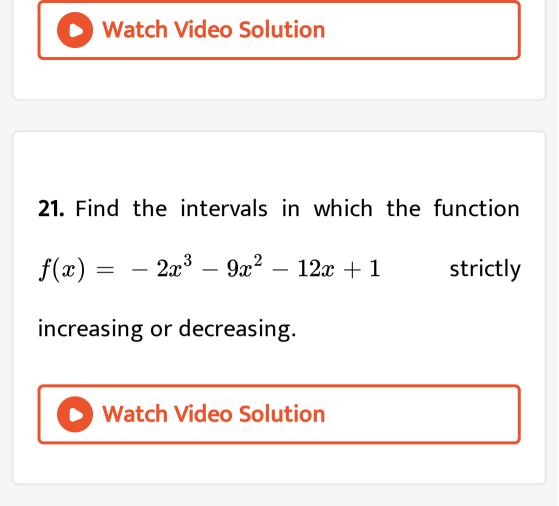
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18. Find the equation on tangents and normals

to the given curves



increasing when the length of an edge is 12cm.



**22.** Find the local maxima and minima of the following functions. Also find the local

maximum and minimum values.

$$f(x) = \sin x + \cos x, 0 < x < rac{\pi}{2}$$



**23.** Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$f(x) = x^3 - 3x$$

**24.** Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$f(x) = x^3 - 6x^2 + 9x + 15$$

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**25.** Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.

$$g(x)=rac{x}{2}+rac{2}{x},x>0$$

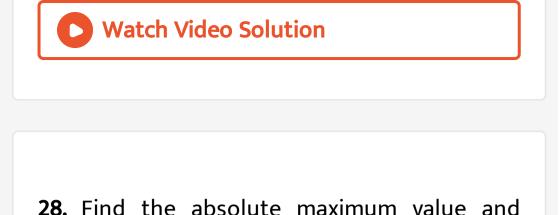


26. Find the local maxima and minima of the following functions. Also find the local maximum and minimum values.  $g(x) = rac{1}{x^2+2}$ 

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**27.** Find the absolute maximum value and minimum value of the following functions.

$$f(x)=x^3, x\in [\,-2,2]$$



minimum value of the following functions.

$$f(x)=4x-rac{x^2}{2}, x\in \left[-2,rac{9}{2}
ight]$$



**29.** A television camera at ground level is filming the lift-off of a space shuttle that is rising vertically according to position equation

 $S = 50t^2$ .

The camera is 2000 feet from the launch pad. Find the rate of change in the angle of elevation of the camera 10 seconds after liftoff.

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**30.** Determine whether the  $f(x) = \sin x$  function is strictly monotonic on the indicated interval.

 $(0, 2\pi)$ 





**31.** Determine whether the  $f(x) = \sin x$  function is strictly monotonic on the indicated interval.

 $(0,\pi)$ 

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**32.** Determine whether the  $f(x) = \sin x$  function is strictly monotonic on the indicated

interval.

$$\left(-rac{\pi}{2},rac{\pi}{2}
ight)$$

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**33.** Find the approximate change in the surface area of a cube of side x meters caused by decreasing the side by 1%.



**34.** The length 'x' of a rectangle is decreasing at the rate of 2cm/s and the width 'y' is increasing at the rate of 2cm/s.

Find the rate of change of perimeter.



**35.** The length 'x' of a rectangle is decreasing at the rate of 2cm/s and the width 'y' is increasing at the rate of 2cm/s. Find  $\frac{dA}{dt}$  when x=12 cm and y = 5 cm.



36. Find the points on the curve

 $x^2+y^2-2x-3=0$  at which the tangent

are parallel to x-axis.

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**37.** Find the equation of the tangent to the curve  $y = \sqrt{3x - 2}$  which is parallel to the line 4x - 2y + 5 = 0.

**38.** Prove that the curve  $x = y^2$  and xy = k cut at right angles, if  $8k^2 = 1$ .



**39.** The gradient at any point (x,y) of a curve is  $3x^2 - 12$  and the curve through the point (2,-7).

Find the equation of the tangent at the point

(2,-7).



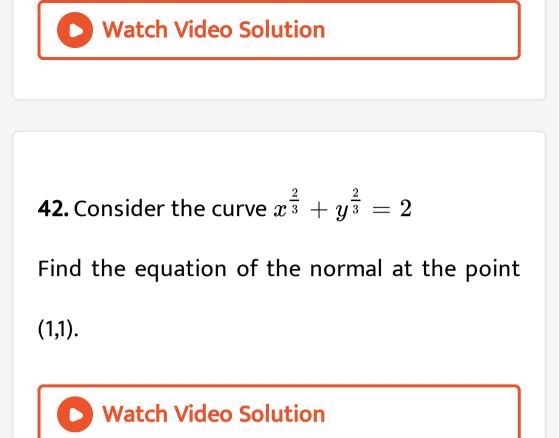
**40.** The gradient at any point (x,y) of a curve is  $3x^2 - 12$  and the curve through the point (2,-7).

Find the equation to the curve.

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**41.** Consider the curve  $x^{rac{2}{3}}+y^{rac{2}{3}}=2$ 

Find the slope of the tangent to the curve at the point (1,1).



**43.** Find the intervals in which the function f given by  $f(x) = 2x^3 - 3x^2 - 36x + 7$  is Strictly Increasing.





**44.** Find the intervals in which the function f

given by  $f(x) = 2x^3 - 3x^2 - 36x + 7$  is

Strictly Decreasing.

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**45.** Use differentials to find the approximate value of  $\sqrt{0.6}$  up to 3 places of decimals.

**46.** Use differentials to find the approximate value of  $(0.999)^{\frac{1}{10}}$  up to 3 places of decimals.

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**47.** Use differentials to find the approximate value of  $(15)^{\frac{1}{4}}$  up to 3 places of decimals.

**48.** Use differentials to find the approximate value of  $(26.57)^{\frac{1}{3}}$  up to 3 places of decimals.



**49.** Find the approximate value of f(5.001) where  $f(x) = x^3 - 7x^2 + 15$ .

**50.** Find the approximate value of f(3.02)

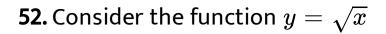
where  $f(x) = 3x^2 + 5x + 3$ .

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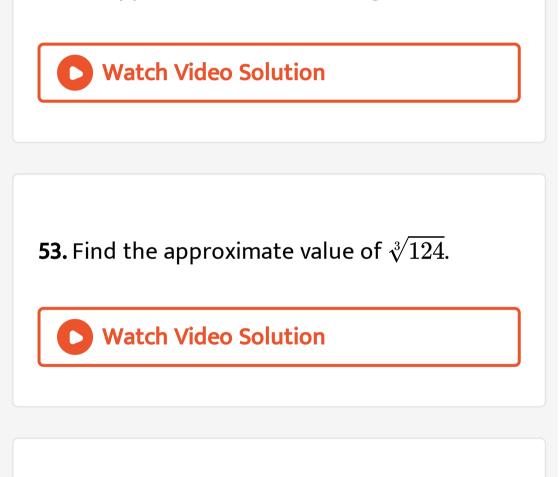
**51.** Consider the function  $y = \sqrt{x}$ 

If x = 0.0036 and riangle x = 0.0001 find riangle y.

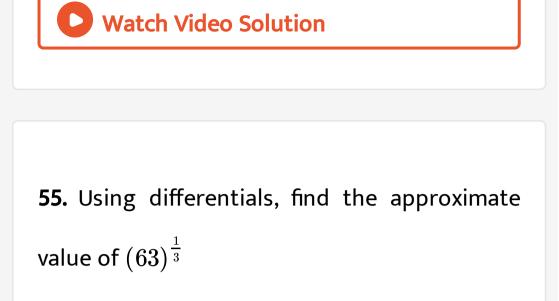




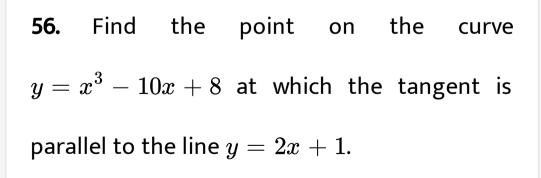
Hence approximate  $\sqrt{.0037}$  using differentials.



**54.** Find two numbers x and y such that their sum is 35 and the product  $x^2y^5$  is a maximum.







57. Find the point on the curve  $y = x^3 - 10x + 8$  at which the tangent is parallel to the line y = 2x + 1. Is the given line tangent to the curve? Why?

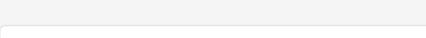
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**58.** Suppose that a spherical balloon is inflated and it has volume 'v' and radius 'r' at time 't'. If the balloon is inflated by pumping 900c.c. of gas per second. Find the rate at which the radius of the balloon is increasing when the

radious is 15 cm.



**59.** Suppose that a spherical balloon is inflated and it has volume 'v' and radius 'r' at time 't'. If the balloon is inflated by pumping 900c.c. of gas per second. Find the rate at which the radius of the balloon is increasing when the radious is 15 cm. **60.** Use differentials to find the approximate value of  $(0.009)^{\frac{1}{3}}$  up to 3 places of decimals.



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**61.** Find the approximate value of  $\sqrt{401}$ .

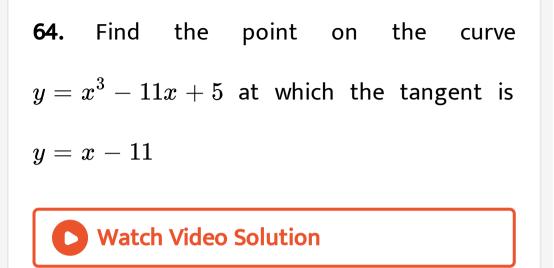
**62.** Consider  $y = \frac{\log x}{x}$ , in  $(0, \infty)$ Find the value of x at which  $\frac{dy}{dx} = 0$ 

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**63.** Consider 
$$y=rac{\log x}{x}$$
, in  $(0,\infty)$ 

Find the maximum value.





**65.** Consider the curve  $y = x^2 - 2x + 7$ 

Find the slope of the tangent of the curve at

$$x = 2.$$

**66.** Consider the curve  $y = x^2 - 2x + 7$ 

Write down the equation of the tangent at

$$x = 2.$$



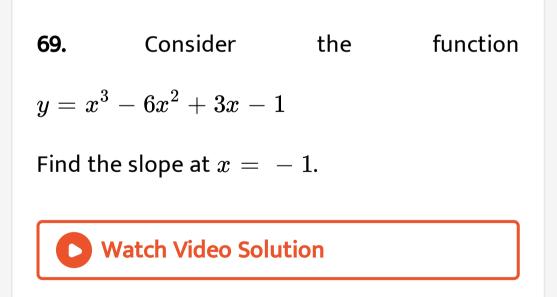
**67.** Find the absolute maximum value and minimum value of the function.

$$f(x)=2x^3-15x^2+36x+1, x\in [1,5]$$

**68.** Find the absolute maximum value and minimum value of the function.

$$f(x)=12x^{rac{4}{3}}-6x^{rac{1}{3}},x\in [\,-1,1]$$





70.Considerthefunction $y = x^3 - 6x^2 + 3x - 1$ Find the minimum gradient of the abovecurve.Watch Video Solution

71. A curve passes through the origin, and its gradient function is  $2x - \frac{x^2}{2}$ . Find its y coordinate when x = 2.

72. A curve passes through the origin, and its gradient function is  $2x - \frac{x^2}{2}$ .

Find the equation of the tangent at x = 2.

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73. Choose the correct answer from the bracket. The slope of the tangent to the curve  $y = x^3 - 2x + 3$  at x=1 is.....

B. 1

C. 2

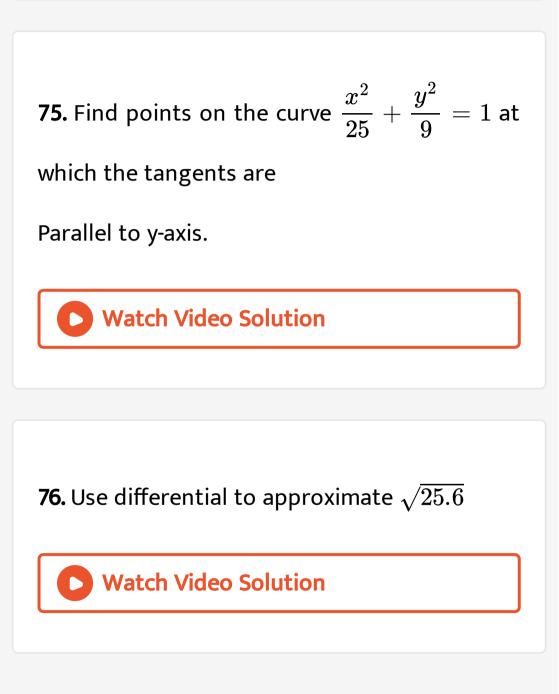
D. 3

### Answer:

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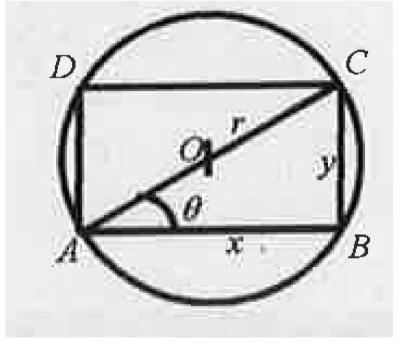
**74.** Find points on the curve 
$$rac{x^2}{25}+rac{y^2}{9}=1$$
 at which the tangents are

Parallel to x-axis.



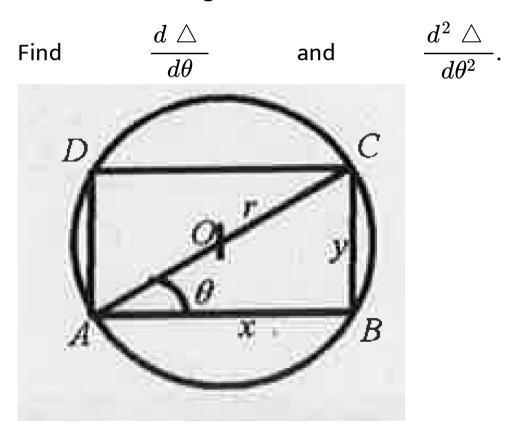
**77.** Let x and y be the length and breadth of the rectangle ABCD in a circle having radius r. Let  $\angle CAB = \theta$  (Ref. Figure). If  $\triangle$  represent area of the rectangle and r is a constant.

Write  $\triangle$  in terms of r and  $\theta$ .





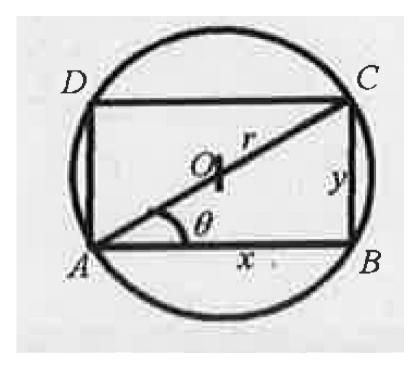
**78.** Let x and y be the length and breadth of the rectangle ABCD in a circle having radius r. Let  $\angle CAB = \theta$  (Ref. Figure). If  $\triangle$  represent area of the rectangle and r is a constant.





**79.** Let x and y be the length and bredth of the rectangle ABCD in a circle having radius r. Let  $\angle CAB = \theta$  (Ref. Figure). If  $\triangle$  represent area of the rectangle and r is a constant.







**80.** Let x and y be the length and bredth of the rectangle ABCD in a circle having radius r. Let

 $\angle CAB = \theta$  (Ref. Figure). If  $\triangle$  represent area of the rectangle and r is a constant. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a  $\sqrt{2}r$ . of side square



81. The second derivative of the equation of a

curve is given by the equation  $xrac{d^2y}{dx^2}=1$ , given by y=1,  $rac{dy}{dx}=0$  when x=1. Find the slope

at x=e.

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82. The second derivative of the equation of a curve is given by the equation  $x\frac{d^2y}{dx^2} = 1$ , given by y=1,  $\frac{dy}{dx} = 0$  when x=1. Find the equation of the curve.

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**83.** The second derivative of the equation of a curve is given by the equation  $x\frac{d^2y}{dx^2} = 1$ , given by y=1,  $\frac{dy}{dx} = 0$  when x=1.Find the

equation of the normal at x=e.

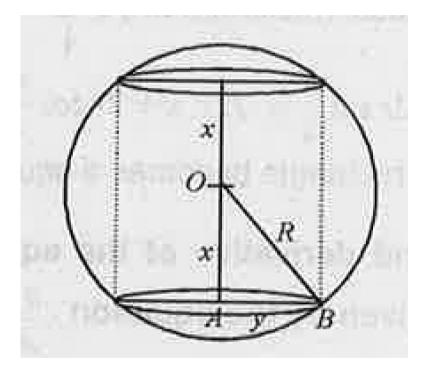
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**84.** The Given figure represents a cylinder inscribed in a sphere.

Figure

### Find an expression for the volume V of the

## cylinder

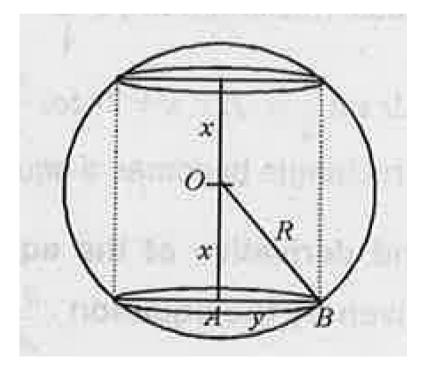


**85.** The Given figure represents a cylinder inscribed in a sphere.

Figure

Find the height of the cylinder when its

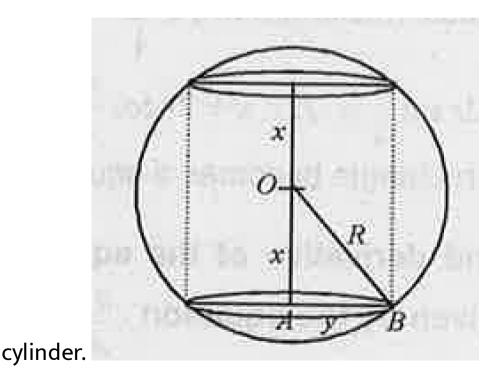
volume V is maximum.





**86.** The Given figure represents a cylinder inscribed in a sphere.

Find the volume and radius of the largest





87. If 
$$f(x) = x^3 + 3x^2 - 9x + 4$$
 is a real

function

Find the intervals in which the fuction is increasing or decreasing.

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88. If 
$$f(x) = x^3 + 3x^2 - 9x + 4$$
 is a real

function

Find the point of local maxima or local minima

of f(x).

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**89.** If 
$$f(x) = x^3 + 3x^2 - 9x + 4$$
 is a real

function

Find the intervals in which the fuction is increasing or decreasing.



**90.** Of all the cylinders with given surface area, show that the volume is maximum when height is equal to the diameter of the base .



**91.** Sand is pouring from a pipe. The falling sand forms a Cone on the ground in such a way that the height of the Cone is always one-sixth of the radius of the base.

and height 'h' of the Cone using the given condition.



**92.** Sand is pouring from a pipe. The falling sand forms a Cone on the ground in such a way that the height of the Cone is always one-sixth of the radius of the base.

if the sand is pouring at the rate of  $12cm^3/s$ 

,Find the rate of change of height of the Cone.



**93.** Sand is pouring from a pipe. The falling sand forms a Cone on the ground in such a way that the height of the Cone is always one-sixth of the radius of the base.

Find 
$$d \frac{h}{dt}$$
 when h=12 cm.

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**94.** Choose the correct answer from the bracket. The rate of change of the area of a circle with respect to its radius r at r =10 cm is

A.  $10\pi$ 

 $\mathsf{B.}\,20\pi$ 

C.  $30\pi$ 

D.  $40\pi$ 

#### Answer:

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**95.** Find the intervals in which the function f

given by  $f(x) = x^2 - 6x + 5$  is

Strictly increasing.



**96.** Find the intervals in which the function f

given by  $f(x) = x^2 - 6x + 5$  is

Strictly decreasing.



**97.** Find the local minimum and local maximum

value, if any, of the function

$$f(x) = x^3 - 6x^2 + 9x + 8.$$

**98.** A wire of length 28 m is cut into two pieces. One of the Pieces is be made into a square and the other in to a circle. What should be the length of the two pieces so that combined area of the square and the circle is minimum using differentiation?



99. An open box of maximum value is to be made from a square piece of tin sheet 24 cm on a side by cutting equal squares from the corners and turning of the sides.

Complete the

following

table.

Height of the box (x. <i>cm</i> )	Width of the box	Volume of the box (V. cm <sup>3</sup> )
1	24 – 2 × 1	$1 \times (24 - 2 \times 1)^2 = 484$
2	24 – 2 × 2	Traine service to seal
3		$2 \times (24 - 2 \times 2)^2 = 800$
4		
5		
6		minimizial e o n e o e

**100.** An open box of maximum volume is to be made from a square piece of tin sheet 24 cm on a side by cutting equal squares from the corners and turning of the sides.

Using the table, express V as a function of x



determine

its

domain.

Height of the box (x. <i>cm</i> )	Width of the box	Volume of the box (V. cm <sup>3</sup> )
1	24 – 2 × 1	$1 \times (24 - 2 \times 1)^2 = 484$
2	24 – 2 × 2	LOUIS PENTY OF ST
3		$\frac{2 \times (24 - 2 \times 2)^2}{$
4		
5		
6		ministed 4 of 1 = 0.4



**101.** A square tank of capacity 250  $m^3$  has to be dug out. The cost of land is Rs. 50 per  $m^2$ . The cost of digging increases with the depth and the whole tank is Rs.  $400 \times (depth)^2$ . Find the expression for the cost of digging the tank.



**102.** A square tank of capacity 250  $m^3$  has to be dug out. The cost of land is Rs. 50 per  $m^2$ . The cost of digging increases with the depth and for the whole tank it is Rs.  $400 \times (depth)^2$ 

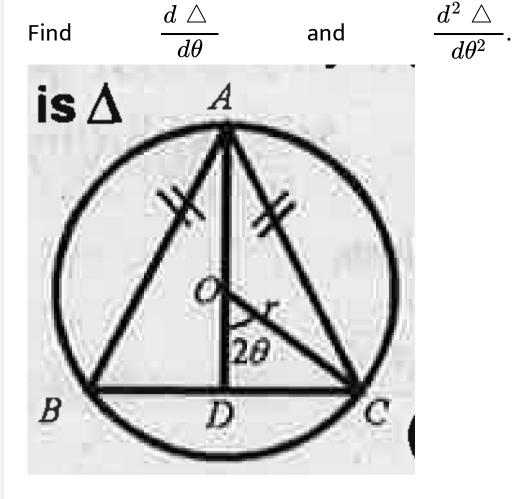
Find the dimension of the tank when the total cost is least.



**103.** Show that the right circular cone of least curved surface and given volume has an altitude equal to  $\sqrt{2}$  times the radius of the base.

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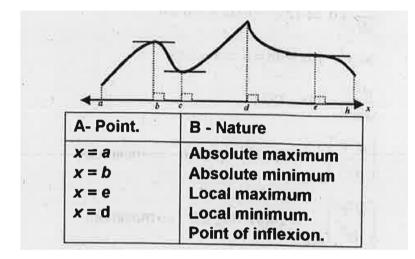
**104.** Let ABCbe an isosceles triangle inscribed ina circle having radius r. Then by figure area of the triangle ABC is  $\triangle$ 

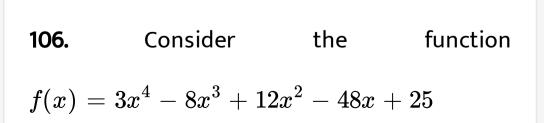




**105.** Using the graph of the function f(x) in

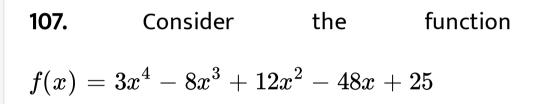
the interval [a, h] match the following.



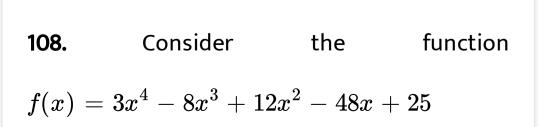


Find the turning point of f(x).





Explain the nature of the turning points.



Find the absolute extreme values of f(x)

 $x \in [0,3]$  are respectively

**109.** An open box with a square base is to be made out of a given quantity of sheet of area  $a^2$ .

If the box has side x units, then show that

volume 
$$V=rac{a^2x-x^3}{4}$$

110. An open box with a square base is to be made out of a given quantity of sheet of area  $a^2$ . Show that the maximum volume is  $\frac{a^3}{6\sqrt{2}}$ . Watch Video Solution the 111. For function,

 $f(x) = \sin 2x, 0 < x < \pi.$ 

Find the point between 0 and  $\pi$  that satisfies f'(x) = 0.





112. For the function,  $f(x) = \sin 2x, 0 < x < \pi.$ Find the point of local maxima and local minima.



Find the local maximum and local minimum

value.



114. A rectangle sheet of tin with adjacent sides 45 cm and 24 cm is to be made into a box without top, by cutting off equal squares from the corners the folding up the flaps. Taking the side of the square cut off as x, express the volume of the box as the function of x.



**115.** An rectangle sheet of tin with adjascent sides 45 cm and 24 cm is to be made into a box without top, by cutting off equal squares of side x from the corners the folding up the flaps.

For what value of x, the volume of the box will be maximum.



116. Find the equation of the tangent to the

curve  $x^{rac{2}{3}} + y^{rac{2}{3}} = 2$  at (1,1).

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117. Find two positive numbers whose sum is

15 and the sum of whose squares is minimum.



**118.** The slope of the tangent to the curve given

- $x=1-\cos heta, y= heta-\sin heta$  by at  $heta=rac{\pi}{2}$ 
  - A. 0
  - B. -1
  - C. 1
  - D. Not Defined

### Answer:

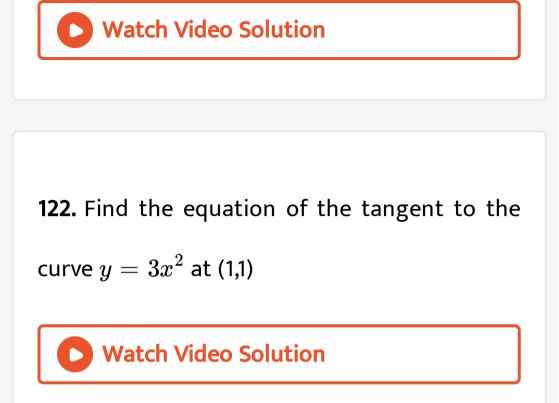


119. Find the intervals in which the function  $f(x) = x^2 - 4x + 6$  is strictly decreasing. Watch Video Solution

120. Find the minimum and maximum value, if any, of the function  $f(x) = \left(2x-1
ight)^2 + 3$ 

121. Which of the following function has  
neither local maxima nor local minima?  
a)
$$f(x) = x^2 + x$$
 b) $f(x) = \log x$  c)  
 $f(x) = x^3 - 3x + 3$   
d) $f(x) = 3 + |x|$   
A.  $f(x) = x^2 + x$   
B.  $f(x) = \log x$   
C.  $f(x) = x^3 - 3x + 3$   
D.  $f(x) = 3 + |x|$ 

### **Answer:**



123. The slope of the normal to the curve, $y = x^3 + x^2$  at (-1,1) is

B. -1

C. 2

D. 0

### Answer:

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124. Find the intervals in which the function  $f(x) = 2x^3 - 24x + 25$  is increasing or decreasing.

# 125. The slope of the normal to the curve,

 $y^2=4x$  at (1,2) is

A. 1

 $\mathsf{B.}\,\frac{1}{2}$ 

C. 2

D. -1

### Answer:



126. Find the intervals in which the function

 $2x^3 + 9x^2 + 12x - 1$  is strictly increasing.

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### 127. Find two positive numbers whose sum is

16 and the sum of whose Cubes is minimum.



128. Show that the function

 $x^3 - 6x^2 + 15x + 4$ 

is strictly increasing in R.



## 129. Find the approximate change in volume of

a cube of side x meters caused by an increase

in the side by 3%.



130. Find the equation of the tangent and normal at the point (1,2) on the parabola $y^2=4x.$ 

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**131.** Consider the parametric forms 
$$x = (t) + \left(\frac{1}{t}\right)$$
 and  $y = (t) - \left(\frac{1}{t}\right)$  of a

curve

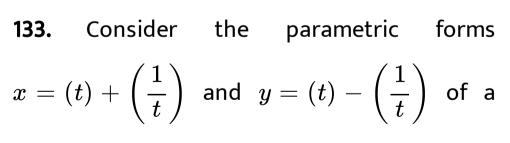
Find 
$$\frac{dy}{dx}$$

132. Consider the parametric forms 
$$x = (t) + \left(\frac{1}{t}\right)$$
 and  $y = (t) - \left(\frac{1}{t}\right)$  of a

curve

Find the equation of the tangent at t=2.





curve

Find the equation of the normal at t=2.



**134.** The radius of a circle is increasing at the rate of 2cm/s. Find the rate at which area of the circle is increasing when radius is 6 cm.

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135. Prove that the function  $f(x) = \log \sin x$ is strictly increasing in  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing in  $\left(\frac{\pi}{2}, \pi\right)$ 

**136.** Find the maximum and minimum value of the function  $f(x) = x^3 - 6x^2 + 9x + 15$ .

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**137.** Find the approximate value of  $(82)^{\frac{1}{4}}$  up to

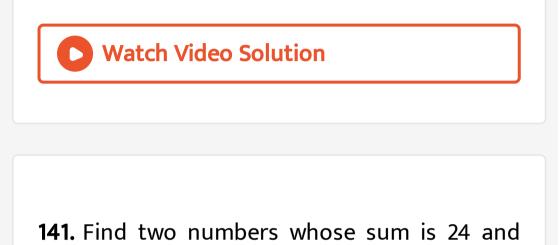
three places of decimals using differentiation.

**138.** Find two positive numbers such that their sum is 8 and the sum of their squares is minimum.



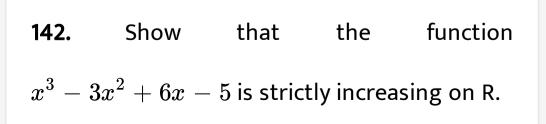
139. The slope of the tangent to the curve 
$$y = x^3 - 1$$
 at x=2 is.....

**140.** Use differentiation to approximate  $\sqrt{36.6}$ .



whose product as large as possible.







143. Find the interval in which the function  $f(x) = \sin x + \cos x, 0 \le x \le 2\pi$  is strictly

increasing or strictly decreasing.

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$$y=\sin heta$$
 at  $heta=rac{\pi}{4}$ 

145. Show that the function

 $f(x) = x^3 - 6x^2 + 15x + 4$  is strictly

increasing in R.

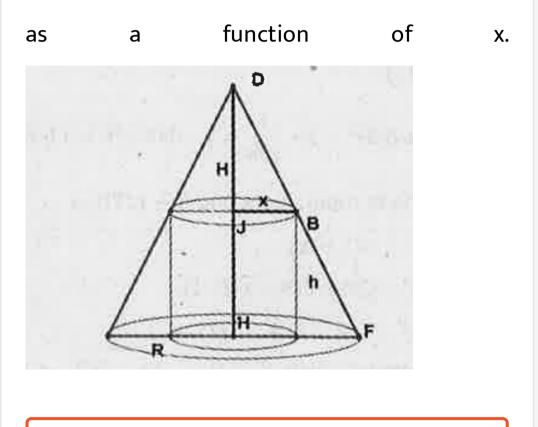
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## 146. Show that all rectangles with a given

perimeter, the square has the maximum area.

**147.** A right circular cylinder is inscribed in a given cone of radius R cm and height H cm as shown in the figure.

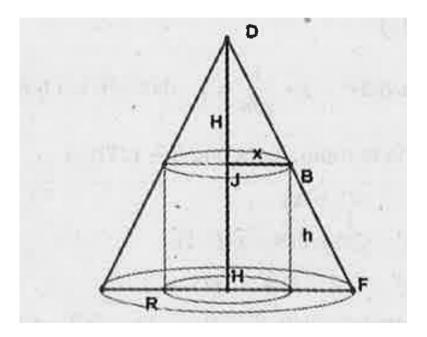
Find the Surface area S of the circular cylinder





**148.** A right circular cylinder is inscribed in a given cone of radius R cm and height H cm as shown in the figure.

Find a relation connecting x and R when S is a maximum.





**149.** Which of the following function is increasing for all values of x in its domain? a)  $\sin x$  b) $\log x$  c) $x^2$  d)|x|

A.  $\sin x$ 

 $B.\log x$ 

 $\mathsf{C}. x^2$ 

D. |x|

### Answer:



**150.** Find a point on the curve  $y = (x - 2)^2$  at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

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151. Find the maximum profit that a company can make, if the profit function is given by  $p(x) = 41 - 24x - 6x^2$ 

152. Find the slope of the tangent to the parabola  $y^2=4ax$  at  $\left(at^2,2at
ight).$ 

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**153.** Find the intervals in which the function

 $x^2 - 2x + 5$  is strictly increasing.

**154.** A spherical bubble is decreasing in volume at the rate of  $2cm^3/\sec$ . Find the rate of which the suraface area is diminishing when the radius is 3 cm.

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155. Which of the following function is always increasing? a) $x + \sin 2x$ b) $x - \sin 2x$  c) $2x + \sin 3x$  d) $2x - \sin x$ 

A.  $x + \sin 2x$ 

 $B.x - \sin 2x$ 

 $\mathsf{C.}\,2x+\sin 3x$ 

D.  $2x - \sin 2x$ 

### Answer:

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**156.** The radius of a cylinder is increasing at a rate of 1cm/s and its height decreasing at a rate of 1cm/s. Find the rate of change of its

volume when the radius is 5 cm and the height

is 5 cm.

