# ©゙" doubtnut 

## MATHS

## BOOKS - NTA MOCK TESTS

## NTA TPC JEE MAIN TEST 49

Mathematics

1. Let $\left(1+x+x^{2}\right)^{3}(1+2 x)=a_{0}+a_{1} x+a_{2} x^{2}+\ldots . . . . . . .+a_{7} x^{7}$
then $\sum_{k=0}^{7} \frac{a_{k}}{k+1}$ is equal to:
A. 16
B. 20
C. 24
D. 28
2. The vertices of a triangle are $3+4 i, 4+3 i$ and $2 \sqrt{6}+i$, then distance between ortho-centre and circum-centre of the triangle is equal to:
A. $\sqrt{137-28 \sqrt{6}}$
B. $\sqrt{137+28 \sqrt{6}}$
C. $\frac{1}{2} \sqrt{137+28 \sqrt{6}}$
D. $\frac{1}{3} \sqrt{137+28 \sqrt{6}}$

Answer: B

## - View Text Solution

3. Let $A$ \& $B$ be $3 \times 3$ symmetric matrices such that $X=A B+B A$ and $Y$ $=\mathrm{AB}-\mathrm{BA}$. Then $(X Y)^{T}$ is equal to $(X Y)^{T}$ is the transpose of matrix XY.$]$
A. XY
B. $Y X$
C. $-X Y$
D. $-Y X$

## Answer: D

## (D) View Text Solution

4. The number of polynomials of the form $x^{3}+a x^{2}+b x+c$ which are divisible $x^{2}+1$ where $a, b c \in\{1,2,3, \ldots \ldots, 10\}$ is equal to
B. 20
C. 30
D. 40

## Answer: A

## - View Text Solution

5. If $\alpha_{1}, \alpha_{2}, \alpha_{3}, \ldots \ldots \ldots, \alpha_{n}$ an are n real roots of equation $\mathrm{f}(\mathrm{x})=0$ and $\mathrm{f}(\mathrm{x})$ satisfies the condition $f(k-x)=f(x+k)$ then the value of $\sum_{i=1}^{n}$ is equal to:
A. $2 n k, \forall n \in N$
B. $n k, \forall n \in N$
C. $n k$, if $n$ is odd
D. $n k$, if n is even
6. If $S$ denotes the sum of first 24 terms of series:
$\frac{1^{2}}{1.3}+\frac{2^{2}}{3.5}+\frac{3^{2}}{5.7}+\ldots \ldots .$. . Then $\mathrm{S}=$
A. $\frac{300}{49}$
B. $\frac{300}{51}$
C. $\frac{295}{49}$
D. None

Answer: A

- View Text Solution

7. In a certain town $25 \%$ families own a phone and $15 \%$ own a car, 65\% families own neither a phone nor a car. 2000 families own both a car and a phone. Consider the following statement in this regard.
8. $10 \%$ families own both a car and a phone
9. $35 \%$ families own either a car or a phone
10. 40000 families live in the town

Which statement(s) is / are correct ?
A. 1 and 2
B. 1 and 3
C. 2 and 3
D. 1,2 and 3

## Answer: C

8. A variable circle passes through the fixed point $A(p, q)$ and touches $x$-axis. The locus of the other end of the diameter through A is:
A. $(y-q)^{2}=4 p x$
B. $(x-q)^{2}=4 p y$
C. $(y-p)^{2}=4 q x$
D. $(x-p)^{2}=4 q y$

## Answer: D

## - View Text Solution

9. A hyperbola having foci $\mathrm{A}(4,-1)$ and $\mathrm{B}(4,5)$ has $x+y-7=0$ as one of its tangent, then the point of contact of this tangent is:
A. $\left(\frac{9}{5}, \frac{5}{2}\right)$
B. $(1,6)$
C. $(0,7)$
D. $(2,5)$

## Answer: C

## - View Text Solution

10. The locus of the middle points of the normal chords of the parabola, $y^{2}=4 a x$ is
A. $x+2 a=\frac{y^{2}}{2 a}+\frac{4 a^{3}}{y^{2}}$
B. $x+2 a=\frac{y^{2}}{2 a}-\frac{4 a^{3}}{y^{2}}$
C. $x-2 a=\frac{y^{2}}{2 a}+\frac{4 a^{3}}{y^{2}}$
D. None of these
11. The angle between the tangents to the curve $y=x^{2}-b x+6$ at points $(2,0)$ and $(3,0)$ is:
A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. $\frac{\pi}{2}$

## Answer: D

## - View Text Solution

12. The equation of the plane through the line of intersection of planes:
$a x+b y+c z+d=0, a x+b^{\prime} y+c z+d^{\prime}=0$ and parallel to the line $y=0$, $z=0$ is:
A. $\left(a b^{\prime}-a^{\prime} b\right) x+\left(b c^{\prime}-b^{\prime} c\right) y+\left(a d^{\prime}-a^{\prime} d\right)=0$
B. $\left(a b^{\prime}-a^{\prime} b\right) x+\left(b c^{\prime}-b^{\prime} c\right) y+\left(a d^{\prime}-a^{\prime} d\right) z=0$
C. $\left(a b^{\prime}-a^{\prime} b\right) y+\left(a c^{\prime}-a^{\prime} c\right) z+\left(a d^{\prime}-a^{\prime} d\right)=0$
D. None of these

## Answer: C

## - View Text Solution

13. If $D, E$ and $F$ are the mid points of the sides $B C, C A, A B$ respectively of a triangle $A B C$ :
$\overrightarrow{A D}+\frac{2}{3} \overrightarrow{B E}+\frac{1}{3} \overrightarrow{C F}=K \overrightarrow{A C}$, then 2 K is equal to:
A. 1
B. 3
C. 2
D. 5

## Answer: A

## - View Text Solution

14. If $y=A \sin \omega t$ then $\frac{d^{5} y}{d t^{5}}=$
A. $A \omega^{5} \cos \left(\omega t-\frac{\pi}{2}\right)$
B. $A \omega^{5} \sin \left(\omega t-\frac{\pi}{2}\right)$
C. $A \omega^{5} \cos \left(\omega t+\frac{\pi}{2}\right)$
D. $A \omega^{5} \sin \left(\omega t+\frac{\pi}{2}\right)$

## Answer: D

15. If the number of points of discontinuity and number of points of non differentiability of $f(x)=\min \left\{\sin x, \sin ^{-1}(\cos x)\right\}$ are p and $q$ respectively, then ordered pair $(p, q)$ is:
A. $(1,2)$
B. $(1,3)$
C. $(0,2)$
D. $(0,3)$

## Answer: D

## - View Text Solution

16. General solution of differential equation:
$\left(e^{y}+1\right) \cos x d x+e^{y} \sin x d y=0$ is:
A. $\left(e^{y}+1\right) \cos x=\lambda$
B. $\left(e^{y}-1\right) \sin x=\lambda$
C. $\left(e^{y}+1\right) \sin x=\lambda$
D. none

## Answer: C

## - View Text Solution

17. $\int \frac{d x}{(x+1)^{\frac{6}{5}}(x-3)^{\frac{4}{5}}}$ is equal to:
A. $\frac{5}{4}\left(\frac{x-3}{x+1}\right)^{1 / 5}+C$
B. $\frac{4}{5}\left(\frac{x-3}{x+1}\right)^{1 / 5}+C$
C. $\frac{5}{4}\left(\frac{x+1}{x-3}\right)^{1 / 5}+C$
D. $\frac{4}{5}\left(\frac{x+1}{x-3}\right)^{1 / 5}+C$

Answer: A
18. If $f(x)=\sin ^{4} x+\cos ^{4} x-\frac{1}{2} \sin 2 x$, then the range of $\mathrm{f}(\mathrm{x})$ is:
A. $\left[0, \frac{3}{2}\right]$
B. $\left[-\frac{1}{2}, \frac{7}{2}\right]$
C. $\left[0, \frac{9}{8}\right]$
D. $\left[\frac{3}{4}, \frac{7}{8}\right]$

## Answer: C

## - View Text Solution

19. If a pair of variable straight lines $x^{2}+A y^{2}+a x y=0$ (where a is a real parameter) cut the ellipse $x^{2}+A y^{2}=4$ at two points, then locus of the point of intersection of tangents at $A$ and $B$ is:
A. $4 x^{2}-y^{2}=0$
B. $x^{2}-4 y^{2}=0$
C. $x^{2}+4 y^{2}-16=0$
D. $x^{2}-4 y^{2}-32=0$

## Answer: B

## - View Text Solution

20. The value of $\theta$ so that $\sin ^{-1}\left(\frac{2}{\sqrt{5}}\right), \cos ^{-1}\left(\frac{1}{\sqrt{10}}\right)$ and $\sin ^{-1} \theta$ are tne angle of a triangle is:
A. $-\frac{1}{\sqrt{2}}$
B. $\frac{1}{2}$
C. $\frac{1}{\sqrt{2}}$
D. $\frac{1}{\sqrt{3}}$

## Answer: C

## - View Text Solution

21. If the system of equations are:
$2 x-y+2 z=2, x-2 y-z=-4 \& x+y+\lambda z=4$, then the value of $A$ such that the given system of equation has no solution, is:

## - View Text Solution

22. Let QP and NM be vertical walls. $\mathrm{NM}=16 \mathrm{~m}$. and $Q N=\frac{27}{4} \mathrm{~m}$. Then the least length of a ladder which reaches the tops $P$ and $N$ of the walls.

## - View Text Solution

23. Consider the triangle $P Q R$, let $a, b$ and $c$ be the lengths of the sides opposite to the angles $P, Q$ and $R$ respectively, and
$\frac{a^{2}-b^{2}}{2 b c}=\cos P$ and $\lambda=\frac{\sin R}{\sin (P-Q)}$. If distance between two parallel lines $2 x+\lambda y-2=0$ and $\lambda x+y+3=0$ is given to be d , then the value of $d^{2}$ equals:

## - View Text Solution

24. The value of $\int_{0}^{1} 4 x^{3} \cdot\left(\frac{d^{2}}{d x^{2}}\left(1-x^{2}\right)^{5}\right) d x$ is:

## - View Text Solution

25. If sum of the solutions of the trigonometric equation $3 \cot ^{2} x+\cot x=-3$ in $[0,2 \pi]$ is $k \pi$ then the value of k is:

## - View Text Solution

26. From each of the three boxes containing 3 white and 1 black, 2
white and 2 black, 1 white and 3 black balls, one ball is drawn at
random. If $\frac{p}{q}$ represents the probability that 2 white and 1 black balls are drawn then evaluate $3 p-q$. [Given that G.C.D. $(\mathrm{p}, \mathrm{q})=1$ ]

## - View Text Solution

27. Let $f:(-1,1) \rightarrow R$ be a differentiable function with
$f(0)=-1$ and $f^{\prime}(0)=1$. If $g(x)=[f\{2 f(x)+2\}]^{2}$, then find find $\left|g^{\prime}(0)\right|$

## - View Text Solution

28. An equilateral triangle has vertices as the points $A\left(\frac{2}{\sqrt{3}}\right) e^{i \frac{\pi}{2}}, B\left(\frac{2}{\sqrt{3}}\right) e^{-I \frac{\pi}{6}}, C\left(\frac{2}{\sqrt{3}}\right) e^{-I \frac{5 \pi}{6}}$. If P be any point on its incircle, then $\left(A P^{2}+B P^{2}+C P^{2}\right)=($ where $i=\sqrt{-1})$

## - View Text Solution

29. If $\int(4 x+1) \sqrt{x^{2}-x-2} d x=\frac{4}{3} f\left(x^{2}-x-2\right)$, then $+\frac{p}{q}(2 x-1) g\left(x^{2}-x-2\right)-\left(\frac{m}{n}\right) h\left(\left|x-\frac{1}{2}+\sqrt{x^{2}-x-2}\right|\right)+c$ evaluate $\left[\frac{m}{n}\right]+f(4)+p q+g(4)+h(1)$, where [ ] represents the greatest integer function, and G.C.D. $(p, q)=1$, G.C.D. $(\mathrm{m}, \mathrm{n})=1$

## - View Text Solution

30. A cube in the first octant has sides $O P, O Q$ and $O R$ of length 1 , along the $z$-axis, $y$-axis and $z$-axis, respectively, where $O(0,0,0)$ is the origin. Let the centre of the cube be at $S\left(\frac{1}{2} \cdot \frac{1}{2}, \frac{1}{2}\right)$ and $T$ be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{S P}, \vec{q}=\overrightarrow{S Q}, \vec{r}=\overrightarrow{S R}$ and $\vec{t}=\overrightarrow{S T}$, then the value of $|(\vec{p} \times \vec{q}) \times(\vec{r} \times \vec{t})|$ is:
