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## MATHS

## BOOKS - NTA MOCK TESTS

## NTA TPC JEE MAIN TEST 53

## Mathematics

1. Let $x_{1}$ and $y_{1}$ be real numbers. If $z_{1}$ and $z_{2}$ are complex numbers such that
$\left|z_{1}\right|=\left|z_{2}\right|=4$, then
$\left|x_{1} z_{1}-y_{1} z_{2}\right|^{2}+\left|y_{1} z_{1}+x_{1} z_{2}\right|^{2}=$
A. $32\left(x_{1}^{2}+y_{1}^{2}\right)$
B. $16\left(x_{1}^{2}+y_{1}^{2}\right)$
C. $4\left(x_{1}^{2}+y_{1}^{2}\right)$
D. $8\left(x_{1}^{2}+y_{1}^{2}\right)$

## Answer: A

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2. Let $A$ and $B$ are two squar matrices of same order such that $A B=B$ and $\mathrm{BA}=\mathrm{A}$, then $A^{2}+B^{2}$ is equal to
A. 2 AB
B. 2BA
C. $A+B$
D. $A B$

## Answer: C

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3. $\sum_{r=1}^{10}\left|\begin{array}{ll}2 r & 2 r+1 \\ 110 & 120\end{array}\right|$ is equal to
A. 0
B. 20
C. 100
D. None

## Answer: D

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4. If Letters $A, A, A, M, N, R, Y$ are arranged in dictionary order then the word whose rank is $629^{\text {th }}$ is :
A. RAMAYAN
B. RAMAYNA
C. RAMAYANA

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5. 

$f(x)=\sin x+\cos x+\tan x+\sin ^{-1} x .+\tan ^{-1} x+\cos ^{-1} x$
If $M$ and $m$ are maximum and minimum vlaue of $f(x)$, then its sum is
A. $\pi+2 \cos 1$
B. $\pi+2 \sin 1$
C. $\frac{\pi}{2}+2 \tan 1$
D. $\pi+\tan 1+\sin 1$

## Answer: A

6. The radical centre of three circles described on the three sides $4 x-7 y+10=0, x+y-5=0$ and $7 x+4 y-15=0 \quad$ of a triangle as diameters is
A. $(2,1)$
B. $(1,2)$
C. $(2,3)$
D. $(-6,-2)$

Answer: B

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7. An ellipse $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1$ and the hyperbola $x^{2}-y^{2}=\frac{1}{2}$ intersect orthogonally. It is given that the eccentricity of the ellipse is reciprocal of that of hyperboa, then $\frac{a^{2}}{b^{2}}$ is equal to
A. $\frac{1}{2}$
B. 2
C. $\frac{1}{4}$
D. 4

## Answer: B

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8. If A, B and C are ehaustive events satisfying $P((A \cup B) \cap \bar{C})=\frac{1}{5}$,
$P(B \cap C)-P(A \cap B \cap C)=\frac{1}{15}$ and $P(A \cap C)=\frac{1}{10} \quad$ then $P(C \cap(\overline{A \cup B}))$ is equal to
A. $\frac{17}{30}$
B. $\frac{18}{30}$
C. $\frac{19}{30}$
D. $\frac{20}{30}$

## Answer: C

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9. The lines $2 x+3 y=6,2 x+3 y=8$ cut the x -axis at A and B A line 'I' is drawn through the point $(2,2)$ meets the $x$-axis at $C$ in such a way that the abscissa of $A, B$ and $C$ are in geometric progression. The equation of the line 'l' is
A. $3 x-5 y=16$
B. $3 x+5 y=16$
C. $5 x+3 y=16$
D. $5 x-3 y=16$

## Answer: B

10. Consider $f:(0,1) \rightarrow R$ is given by $f(x)=\ln \left(1+\sqrt{1-x^{2}}\right)$.

Point $-\sqrt{1-x^{2}}-\ln x$
$A(h, f(h)), 0<h<1$ lies on curve and tangent at point A intersect the $y$-axis at point $B(0, k)$. If distance between pointe $A$ and $B$ is $d$, then which of the following is correct ?
A. $d \propto h$
B. $d<1$
C. $d=1$
D. $d>1$

## Answer: C

11. The value of
$[\vec{A}-\vec{B}, \vec{B}-\vec{C}, \vec{C}-\vec{A}]$
$|\vec{A}|=1,|\vec{B}|=2$ and $|\vec{C}|=3$ is
A. 1
B. 6
C. 0
D. 3

## Answer: C

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12. If $x=\int_{0}^{y} \frac{d t}{\sqrt{1+t^{2}}}$, then $\frac{d^{2} y}{d x^{2}}$ is equal to
A. $y$
B. $\sqrt{1+y^{2}}$
C. $\frac{x}{\sqrt{1+y^{2}}}$
D. $y^{2}$

## Answer: A

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13. Let $f(x)=\frac{x-\{x+1\}}{x-\{x+2\}}$, where $\{\mathrm{x}\}$ is the fractional part of x , then $\lim _{x \rightarrow 1 / 3} f(x)$

$$
x \rightarrow 1 / 3
$$

A. has value 0
B. has value 1
C. has value $-\infty$
D. has value $\infty$

Answer: B
14.
$\lim _{n \rightarrow o} n\left\{\frac{1}{(n+1)(n+2)}+\frac{1}{(n+2)(+4)}+\frac{1}{(n+3)(n+6)}+\ldots=\right.$
A. $\log \left(\frac{3}{2}\right)$
B. $\log \left(\frac{2}{3}\right)$
C. $\frac{1}{3} \log 2$
D. $\frac{1}{2} \log 3$

## Answer: A

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15. If the tangent to the curve $y=e^{k x} \operatorname{atM}(0,1)$ meet the $x$-axis at
$\mathrm{N}(\mathrm{a}, 0)$ where $a \in[-2,-1]$ then k belong to
A. $\left[\frac{-1}{2}, 0\right]$
B. $\left[-1, \frac{-1}{2}\right]$
C. $[0,1]$
D. $\left[\frac{1}{2}, 1\right]$

## Answer: D

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16. The value of $\int \frac{\cot x}{\sqrt{5+9 \cot ^{2} x}} d x$ is equal to (where C is constant of integration.)
A. $\frac{1}{2} \sin ^{-1}\left(\frac{2 \sin x}{3}\right)+C$
B. $\frac{1}{2} \sin ^{-1}\left(\frac{3 \sin x}{2}\right)+C$
C. $\frac{1}{3} \sin ^{-1}\left(\frac{3 \sin x}{2}\right)+C$
D. $\frac{1}{3} \sin ^{-1}\left(\frac{2 \sin x}{3}\right)+C$
17. The value of
$\cot ^{4} \frac{\pi}{16}-4 \cot ^{3} \frac{\pi}{16}-6 \cot ^{2} \frac{\pi}{16} i s+4 \cot \frac{\pi}{16}+2$
A. 0
B. -1
C. 2
D. 1

## Answer: D

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18. If $\theta_{1}, \theta_{2}, \theta_{3} \in[0,3 \pi]$, then the number of ordered triplets
$\left(1+\operatorname{cosec}{ }^{4} \theta_{1}\right)\left(2+\cot ^{4} \theta_{2}\right)\left(4+\sin 4 \theta_{3}\right) \leq 12 \sin ^{2} \theta_{1}$ are
A. 18
B. 36
C. 72
D. 48

## Answer: B

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19. The $x$ satisfying
$\sin ^{-1} x+\sin ^{-1}(1-x)=\cos ^{-1} x$ are
A. 1,0
B. $1,-1$
C. $0, \frac{1}{2}$
D. None of these

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20. Which of the following is a contradiction ?
A. $(p \wedge q) \wedge \sim(p \vee q)$
B. $p \vee(-p \wedge q)$
C. $(p \Rightarrow q) \Rightarrow p$
D. None of these

## Answer: A

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21. The postivie integer which is just greater than $(1+0.001)^{1000}$ is
22. The number of integers in the range of 'a' for which roots of $x^{2}-2 x-a^{2}+1=0$ lie between the roots of the equation $x^{2}-2(a+1) x+a(a-1)=0$, is

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23. Let $u$,v,w be three real numbers in geometric progression such that $u>v>w$. Suppose
$u^{40}=u^{n}=w^{60}$. Then the value of $\frac{n}{6}$ is equal to

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24. 

$\frac{x-1}{2}=\frac{y-2}{x_{1}}=\frac{z-3}{x_{2}}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5} \quad$ lie in the same plane, then for equation
$x_{1} t^{2}+\left(x_{2}+2\right) t+a=0$ sum of roots is $\alpha$, then $|\alpha|$ is

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25. Let $f(x)=\left\{\begin{array}{ll}x-1, & x<0 \\ x^{2}-2 x, & x \geq 0\end{array}\right.$ and $h(x)=|f(x)|$. Find the number of points at which function h is not differentiable.

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26. If $(\hat{i}+\hat{j}+3 \hat{k}) x+(3 \hat{i}-3 \hat{j}+\hat{k}) y+(-4 \hat{i}+5 \hat{j}+0 \hat{k}) z$
$=\lambda(x \hat{i}+y \hat{j}+z \hat{k})$ where
$(x, y, z) \neq(0,0,0)$. Find how many values of $\lambda$ exists.

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27. Total number of solutions of

$$
\log _{5}\left(x^{2}-4 x+3\right)=\log _{5}(3 x+21)
$$

28. In a group of 6 people, each person sends an email and receives an email amongst themselves. The different number of ways of doing this is $N$ then the last digit of $N$ is (if it is given that no one sends email back to himself)

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29. The weighted means of first 7 natural numbers whose weights are equal to the square of corresponding numbers. Find the weight mean.

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30. A point moves in the $X-Y$ plne such that the sum of its distance form two mutally perpendicular lines is always equal to 3 . The area (in square unit) enclosed by the locus of the point is
