MATHS

BOOKS - ARIHANT PUBLICATION

APPLICATION OF DERIVATIVES

Sample Question

1. What is the acceleration, at the end of 2 s of the particle that moves with rule $s = \sqrt{t} + 1$?

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2. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm $/$ sec. Find the rate of increase of its surface area, when the radius is 7 cm. (π = 3.141 approx)

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3. The radius of a circle increases at a rate of 0.2 cm/s. Calculate the rate of the increase of the area, when the radius is 5 cm.

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4. Find the slope of the tangent to the curve $y = x^3 - \mathrm{x}$ at $\mathrm{x} = 2$.

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5. Find the slope of the tangent and normal to the following curve.

(i) $y = 2x^2 + 3\sin \mathrm{x}$ at x = 0

(ii) $y = (\sin 2x + \cot x + 2)^2$ at $\mathrm{x} = \frac{\pi}{2}$

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6. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are

(i) Parallel to X-axis.

(ii) Parallel to Y-axis.

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7. Find the equations of the tangent and normal to the curve $x^{2/3} + y^{2/3} = 2$ at (1, 1).

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8. The tangent to the curve $y = 2x^2 - 7x + 3$ at certain point is parallel to the straight line y = x + 2. Find the equation of this tangent and the point, where it cuts Y-axis.

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9. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles, if $8k^2 = 1$.

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10. Show that $f(x) = 3x + 5$ is a strictly increasing function on R.

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11. If a is a real number such that $0 < a < 1$, show that the function $f(x) = a^x$ is strictly decreasing on R.

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12. Show that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is an increasing function on R.

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13. Show that the function $f(x) = \cos^2 x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

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14. Prove that the function f given by f(x) = logsin x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$

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15. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is increasing and decreasing.

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16. Write the interval in which the function $\sin^2 x - x$ is increasing.

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17. Find interval (s) in which the function $f(x) = \sin x + \cos x, x \in (0, \pi/2)$ is increaing or decreasing.

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18. Find the extreme points of the function $f(x) = x^3 - 6x^2 + 9x - 8$. Specify if the extremum is a maximum or minimum. Also, find the extreme values.

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19. Find the extreme points of the function $f(x) = (x-1)^3$ and also find the extreme values.

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20. Find the extremum, if any of the function $f(x) = \frac{1}{x^2 + 2}$.

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21. Find maximum and minimum values of the following functions

(i) $f(x) = x\sqrt{1 - x}$, where x > 0

(ii) $f(x) = \frac{x}{(x - 1)(x - 4)}$, where $1 < x < 4$

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22. Test the function $y = x^8 + 6$ for extreme values.

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23. Test the function $f(x) = x^5$ for extreme values.

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24. Find the absolute maximum value of $f(x) = 2x^3 - 24x + 107$ in the interval [1, 3].

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25. Find the greatest and least values of $f(x) = -x + 2\sin x \text{ on} [0, 2\pi]$

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26. Find two positive numbers x and y such that x + y = 60 and xy^3 is maximum.

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27. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Find the volume of the largest cylinder inscribed in a sphere of radius R.

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28. Show that the semi-vertical angle of right circular cone of given surface area and maximum volume is $\operatorname{cosec}^{-1}(3)$.

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29. If $y = 3x^2 + 5x + 3$, then find Δy and dy, when x = 3 and $\Delta x = 0.02$

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30. Use differential to approximate $(25)^{1/3}$.

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31. Find the approximate value of $\log_{10} 10.1$, it is being given that $\log_{10} e = 0.4343$.

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32. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

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Question For Practice Part I Very Short Answer Type Questions

1. What is the acceleration at the end of 3s of the particle that moves with the rule $S = 3t^2 + 5t + 2$?

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2. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference ?

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3. The side of an equilateral triangle is increase at the rate of 2 cm/s. Find the rate at which the area increases, when the side of the triangle is 10 cm ?

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4. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of bubble increasing, when the radius is 1 cm ?

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Question For Practice Part I Short Answer Type Questions

1. The length x of a rectangle is decreasing at the rate of 5 cm/min and the width y is increasing at the of 4 cm/min. When x = 8 cm and y = 6 cm, find the rate of change of

(i) the perimeter.

(ii) the area of the rectangle.

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2. The volume of a cube is increasing at a rate of 9 cm^3/s. How fast is the surface area increasing when the length of an edge is 10 cm ?

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3. An Airforce plane is ascending vertically at the rate of 100 km/h. If the radius of the Earth is r km, then how fast is the area of the Earth visible from the plane increasing at 3 min after it started ascending ? Given that the visible area A at height h is given by $A = 2\pi r^2 \frac{h}{r+h}$.

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4. Car starts from a point P at time t = 0 second and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$. Find the time taken by it to reach at Q and also find distance between P and Q.

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Question For Practice Part Ii Very Short Answer Type Questions

1. Find the slope of the tangent to the curve $y = 3x^4 - 4x$ at x = 4.

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2. Find the slope of the normal to the curve $x = a\cos^3\theta$ and $y = a\sin^3\theta \text{at} \theta = \frac{\pi}{4}$

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3. Write the equation of the tangent to the curve y=[x] at the point (-2,2).

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4. Determine the point on the curve y = ln x, at which the tangent will be parallel to the chord joining the points P(1, 0) and Q(e, 1).

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Question For Practice Part Ii Short Answer Type Questions

1. Find the equation of the tangent and normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $\left(\sqrt{2}a, b\right)$.

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2. Find the points on the curve $4x^2 + 9y^2 = 1$, where the tangents are perpendicular to the line 2y + x = 0.

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3. Find the equations of the normal to the curve $y = x^3 + 2x + 6$, which are parallel to line $x + 14y + 4 = 0$.

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4. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$, which is

(i) parallel to the line $2x - y + 9 = 0$.

(ii) perpendicular to the line $5y - 15x = 13$.

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5. Find the equation of the tangent to the curve $y = \sqrt{5x - 3} - 2$, which is parallel to the line $4x - 2y + 3 = 0$.

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6. Find the equations of the tangent and the normal to the curve $y = \frac{x - 7}{(x - 2)(x - 3)}$ at the point, where it cuts the X-axis.

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7. Find the equation of tangents to the curve $y = (x^3 - 1)(x - 2)$ at the points, where the curve cuts the X-axis.

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8. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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9. Find the points on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts on the axes.

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Question For Practice Part Ii Long Answer Type Questions

1. Find the tangent to the curve

$y = \cos(x + y), 0 < x < 2\pi$

which is parallel to the line $x + 2y = 0$.

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2. If $x \cos \alpha + y \sin \alpha = p$ is a tangent to the curve

$\left(\frac{x}{a}\right)^{\frac{n}{n}-1} + \left(\frac{y}{b}\right)^{\frac{n}{n}-1} = 1$then so that

$(a \cos \alpha)^n + (b \sin \alpha)^n = p^n$.

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3. Prove that the sum of the cubes of the intercepts on the coordinate axes of any tangent to the curve $x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}}$ is a constant.

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4. Show that the equation of normal at any point on the curve $x = 3\cos\theta - \cos^3\theta, y = 3\sin\theta - \sin^3\theta \text{is} 4\left(y\cos^3\theta - x\sin^3\theta\right) = 3\sin 4\theta$

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5. Prove that all normals to the curve $x = a\cos t + at\sin t, y = a\sin t - at\cos t$ are at a distance a from the origin.

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6. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1, 2). Also, find the equation of the corresponding tangent.

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7. Find the value of p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cuts each other at right angles.

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8. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.

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9. Find the condition that curves $2x = y^2$ and $2xy = k$ intersect orthogonally.

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10. Show that the curves $4x = y^2$ and $4xy = k$ cuts at right angles, if $k^2 = 512$.

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11. Show that the condition that the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ should intersect orthogonally such that $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$.

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Question For Practice Part Iii Very Short Answer Type Questions

1. Prove that $f(x) = \frac{3}{x} + 7$ is strictly decreasing for $x \in R, (x \neq 0)$.

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2. Find the intervals in which the function $y = \frac{Inx}{x}$ is increasing and decreasing.

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3. etermine the sub-interval of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, in which f(x)=tan x-4x is increasing.

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Question For Practice Part Iii Short Answer Type Questions

1. Find the value(s) of x for which $y = [x(x-2)]^2$ is an increasing function.

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2. Show that for $a > 1, f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing in R.

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3. Find the intervals in which the following functions is increasing or decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

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4. Find the intervals in which the following functions is increasing or decreasing.

$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$

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5. Find the intervals in which the following functions is increasing of decreasing.

$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36x}{5} + 11$

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6. Find the intervals in which the following functions is increasing or decreasing.

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

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7. Find the intervals in which the following functions is increasing or decreasing.

$f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$

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8. Find the intervals in which the following functions is increasing of decreasing.

$f(x) = \sin^4 x + \cos^4 x, 0 \leq x \leq \frac{\pi}{2}$

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9. Prove that $\frac{x}{1+x} < \log(1+x) < \text{x for x} > 0$

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Question For Practice Part Iv Very Short Answer Type Questions

1. Write the set of points, where the function $f(x) = x^3$ has relative (local) extreme.

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2. Answer with reasons, whether the following function has a relative (local) maximum at x = 2 or not.

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x \leq 2 \\ 3 - x & 2 < x \leq 3 \end{cases}$$

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3. Prove that, if $f(x) = a_0 + a_1x^2 + a_2x^4$ and $0 < a_0 < a_1 < a_2$, then f(x) has only one minima at x = 0.

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4. It is given that at x = 1, the function $x^4 - 62x^2 + ax + 9$ attains maximum value on the interval |0, 2|. Find the value of a.

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5. Find the least value of $f(x) = e^x + e^{-x}$.

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6. Prove that $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $x = \frac{\pi}{6}$.

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7. Show that $\sin x(1+\cos x), x \in |0, \pi|$ is maximum value at $x = \frac{\pi}{3}$.

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Question For Practice Part Iv Short Answer Type Questions

1. Find all the points of local maxima and local minima of the function

$$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105.$$

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2. Find the number of local maxima and local minima points in

$$f(x) = 2x^3 - 3x^2 - 12x + 4.$$

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3. Show that $\left(x^5 - 5x^4 + 5x^3 - 1\right)$ has local maximum value when x = 1, a local minimum value when x = 3 and neither when x = 0.

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4. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has

(i) local maxima.

(ii) local minima.

(iii) point of inflection.

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5. Find the maximum and minimum values, if any of the following functions.

$f(x) = |x+2| - 1$ or $g(x) = -|x+1| + 3$

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6. Find the local maximum and local minimum values of $\frac{(x-1)(x-6)}{x-10}, x \neq 10.$

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7. Find the local maximum and local minimum values of the function $f(x) = \frac{4}{x+2} + x.$

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8. Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{e}}$.

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9. Find the point of local maxima and local minima of the function $f(x) = \sin x - \cos x, 0 < x < 2\pi.$

Also, find the local maximum and local minimum values.

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10. Find the local maximum and local minimum value of the function

$f(x) = (2\cos x + x)$, where $0 < x < \pi$

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11. Find the maximum and minimum values of

$f(x) = \sec x + \log \cos^2 x, 0 < x < 2\pi.$

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12. Show that $\sin^P \theta \cos^q \theta$ attains a maximum value, when

$\theta = \tan^{-1} \sqrt{\frac{p}{q}}.$

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13. Find the absolute maximum and minimum values of $f(x) = 2x^3 - 24x + 57$ in the interval [1, 3].

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14. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$, on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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15. Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$.

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16. Find two positive numbers whose sum is 16 and the sum of whose squares is minimum.

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17. Prove that the largest rectangle with a given perimeter is a square.

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Question For Practice Part Iv Long Answer Type Questions

1. Find the point P on the curve $y^2 = 4ax$, which is nearest to the point (11a, 0).

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2. A jet of enemy country is flying along the curve $x^2 = 4y$. A soldier placed at point (-1, 2) wants to shoot down the jet of enemy, when it is nearest to him. Find the nearest point to the soldier.

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3. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.

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4. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

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5. Prove the semi-vertical angle of the right circular cone of given volume and least curved surface area is $\cot^{-1}\sqrt{2}$.

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6. Show that the semivertical angle of a cone of given slant height is $\tan^{1}\sqrt{2}$ when its volume is maximum.

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7. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\frac{\cos^{-1}(1)}{\sqrt{3}}$.

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8. Show that the radius of the right cicular cylinder of greatest curved surface that can be inscribed in a given cone is half the redius of the

base of the cone.

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9. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Also, show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

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10. Find the area of greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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11. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a. Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$.

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Question For Practice Part V Very Short Answer Type Questions

1. What is the principal part ?

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2. Absolute error

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3. Percentage error

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4. Relative error and

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5. Find the relation between the relative error and percentage error.

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6. Write df, when $f(x) = \sqrt{x}, x = 4$ and $\Delta x = 0.2$.

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Question For Practice Part V Short Answer Type Questions

1. If $y = 3x^2 + 5x + 3$, then find Δy and dy, when x = 3 and $\Delta x = 0.02$

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2. Using differentials, find approximate value

$\sqrt{49.5}$

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3. Find an approximate value of $\sqrt{16.04}$ using differential.

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4. Using differentials, find approximate value

$\left(\frac{17}{81}\right)^{1/4}$

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5. Using differentials, find approximate value

$(3.968)^{3/2}$

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6. Using differentials, find approximate value

$(255)^{1/4}$

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7. Using differentials, find approximate value

$(0.0037)^{1/2}$

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8. Using differentials, find approximate value

$(0.999)^{\frac{1}{10}}$

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9. Using differentials, find approximate value

$(1.999)^5$

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10. Using differentials, find approximate value

$\sqrt{0.082}$

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11. Using differentials, find approximate value

$(81.5)^{1/4}$

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12. Using differentials, find the approximate value of $\tan 46^\circ$, if it being given that $1^\circ = 0.01745$ radian.

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13. Using differential, find approximately the difference between the volumes of two cubes of sides 2 cm and 2.01 cm.

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14. Find approxiamately the difference between the volumes of two cubes of sides 4 cm and 4.03 cm.

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15. Find the approximate change in the volume V of a cube of side x m caused by decreasing the side by 1%.

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16. Find the approximate change in the volume V of a cube of side x m caused by increasing the side by 2%.

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17. Find the approximate change in the volume V of a cube of side x m caused by increasing the side by 2%.

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18. A circular metal plate expands under heating, so that its radius increase by 2%. Find the approximate increase in the area of the plate,

if the radius of the plate before heating is 10 cm.

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19. Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

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Odisha Bureau S Textbook Solutions Exercise 8 A

1. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules. $s = 2t^2 + 3t + 1$

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2. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules.$s = \sqrt{t} + 1$

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3. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules. $s = \frac{3}{2}t + 1$

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4. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules. $s = t^3 - 6t^2 + 15t + 12$

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5. The sides of an equilateral triangle are increasing at the rate of $\sqrt{3}$ cm/sec. find the rate at which the area of the triangle is increasing when the side is 4 cm long.

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6. Find the rate at which the volume of a spherical balloon will increase when its radius is 2 meters if the rate of increase of its redius is 0.3 m/min.

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7. The surface area of a cube is decreasing at the rate of 15 sq. cm/sec. Find the rate length of the edge is 5 cm.

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Odisha Bureau S Textbook Solutions Exercise 8 B

1. Find the equations to the tangents and normals to the following curves at the indicated points. $Y = 2x^2 + 3\ atx = \ -1$

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2. Find the equations to the tangents and normals to the following curves at the indicated points. $y = x^3 - x\ atx = 2$

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3. Find the equations to the tangents and normals to the following curves at the indicated points. $y = \sqrt{x} + 2x + 6\ atx = 4$

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4. Find the equations to the tangents and normals to the following curves at the indicated points. $Y = \sqrt{3}\sin x + \cos x at x = \frac{\pi}{3}$

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5. Find the equations to the tangents and normals to the following curves at the indicated points. $y = (\log x)^2$ at x = 1/e`

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6. Find the equations to the tangents and normals to the following curves at the indicated points. $y = \frac{1}{\log x}$ at x = 2`

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7. Find the equations to the tangents and normals to the following curves at the indicated points. $y = xe^x \; at x = 0$

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8. Find the equations to the tangents and normals to the following curves at the indicated points. $y = a(thita - \sin\theta), y = a(1 - \cos\theta)$ $at\theta = \frac{\pi}{2}$

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9. Find the equations to the tangents and normals to the following curves at the indicated points. (x/a)^2/3 +(y/b) 2/3 = 1at(a cos^3 theta, b sin^3 theta)`

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10. Find the point on the curve $y^2 - x^2 + 2x - 1 = 0$

where the tangent is parallel to the x - axis.

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11. Find the point (S) on the curve

$$x = \frac{3at}{1+t^2}, y = \frac{3at^2}{1+t^2}$$

where the tangent is perependicular to the line 4x+3y+5=0.

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12. Find the point on the curve $x^2 + y^2 - 4xy + 2 = 0$

where the normal is paralell to the x-asis.

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13. Show that the line y = mx + c touches the parabola y^2 = 4ax if c $= \frac{a}{m}$.

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14. Show that the line y = mx + c touches the ellips

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{if} \quad c^2=a^2m^2+b^2.$$

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15. Show that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ is constant.

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16. Show that the curves $y=2^x$ and $y=5^x$ intersect at an angle $\tan^{-1}\left|\frac{1n\left(\frac{5}{2}\right)}{1+1n21n5}\right|$.

Note Angle between two curves is the angle between their tangents at the point of intersection.

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17. Show that the curves $ax^2 + by^2 = 1$ and $a'x^2 + b'y^2 = 1$. $Inter\sec tatright\angle s$ if 1/a - 1/b = 1/a' - 1/b' cdot

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18. Find the equasion of the tangents drawn from the point (1,2) to the curve.

$y^2 - 2x^3 - 4y + 8 = 0$

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19. Show that the equation of the normal to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is y cos theta - x sin theta = a cos 2"theta"$where$theta` is the inclination of the normal to x-axis.

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20. Show that the length of the portion of the tangent to `x^(2/3) + y^(2/3) = a^(2/3)

intercepted between the axes is constant.

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21. Find the tangent to the curve $y = \cos(x + y), 0 \leq x \leq 2\pi$ which is parallel to the line x + 2y = 0

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22. If tangents are drawn from the origin to the curve y = sin x, then show that the locus of the points of contact is $x^2y^2 = x^2 - y^2$.

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23. Find the equation of the normal to the curve given by

$x = 3\cos\theta - \cos^3\theta$y = 3 sin theta - sin^3 theta`

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24. If $x\cos\alpha + y\sin\alpha = p$ is a tangent to the curve

$\left(\frac{x}{a}\right)^{\frac{n}{n}-1} + \left(\frac{y}{b}\right)^{\frac{n}{n}-1} = 1$then so that

$(a\cos\alpha)^n + (b\sin\alpha)^n = p^n.$

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25. Show that the tangent to the curve

$x = a(t - \sin t), y = at(1 + \cos t)$ at

$t = \frac{\pi}{2}$ has slope.(1- pi/2)`

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Odisha Bureau S Textbook Solutions Exercise 8 C

1. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \sin x, \ \in [0, 2\pi]$

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2. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = Inx, x \in R_+$

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3. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = a^x, a > 0, x \in R$

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4. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \sin x + \cos x, x \in [0, 2\pi]$

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5. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = 2x^3 + 3x^2 - 36x - 7$

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6. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \frac{1}{x-1}, x \neq 1$

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7. Find the intervals where the following functions are (a) increasing and (b) decreasing.

$$y = \begin{cases} x^2 + 1 & x \leq -3 \\ x^3 - 8x + 13 & x > -3 \end{cases}$$

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8. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = 4x^2 + \frac{1}{x}$

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9. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = (x-1)^2(x+2)$

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10. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \frac{Inx}{x}, x > 0$

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11. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \tan x - 4(x-2), x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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12. Find the intervals where the following functions are (a) increasing and (b) decreasing. $y = \sin 2x - \cos 2x, x \in [0, 2\pi]$

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13. Show that the function $\frac{e^x}{x^p}$ is strictly increasing for $x > p > 0$.

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14. Show that `2 sin x + than x ge 3x all x in (0, pi/20).

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Odisha Bureau S Textbook Solutions Exercise 8 D

1. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = x^2 + 2x + 3$

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2. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = 5x^2 - 2x^5$

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3. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = \dfrac{3x}{x^2 + 1}$.

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4. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = x^2\sqrt{1 - x^2}$

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5. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = 2x^3 - 15x^2 - 36x + 18$

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6. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = 60/\left(x^4 - x^2 + 25\right)$

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7. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = (x-1)^3$

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8. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = (x-2)^3(x+3)^4$

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9. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = x + \frac{1}{x}$

Watch Video Solution

10. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = 4\cos 2x - 3\sin 2x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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11. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = \sin x \cos x, x \in \left[\frac{\pi}{8}, \frac{\pi}{2}\right]$

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12. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = \cos x(1 + \sin x), x \in [0, 2\pi]$

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13. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = \sin px \cos qx, p, q > 0, x \in \left[0, \frac{\pi}{2}\right]$

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14. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = xe^{-x}, x \in (-2, 2)$

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15. Shows that the following functions do not possess maximum or minimum. x^3

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16. Shows that the following functions do not possess maximum or minimum. x^5

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17. Shows that the following functions do not possess maximum or minimum. $3x^3 - 12x^2 + 16x - 15$

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18. Shows that the following functions do not possess maximum or minimum. $4 - 3x + 3x^2 - x^3$

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19. Shows that the following functions do not possess maximum or minimum. $In|x|, x \neq 0$

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20. Use the function $f(x) = x^{1/x}, x > 0$ to show that $e^{\pi} > \pi^{e}$.

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21. Prove the inequality

$x^2 e^{-x^2} \leq e^{-1}, x \in R.$

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22. If f(x) = a In $x + bx^2 + x$ has extreme values at $x = -1$ and $x = 2$ then find a and b.

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23. Show that $\frac{x}{1 + x \tan x,} x \in \left(0, \frac{\pi}{2}\right)$ is maximum when x = cos x.

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24. Determine the absolute maximum and absolute minimum of the following function on [-1, 1].

$$f(x) = \begin{cases} (x+1)^2 & x \leq 0 \\ (x-1)^2 & x > 0 \end{cases}$$

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25. Find extreme value of $f(x)$. $= \begin{cases} \frac{x}{1-x^2} & -1 < x < 0 \\ x^3 - x & 0 \leq x < 2 \end{cases}$ on (-1,2)

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26. Find two numbers x and y whose sum is 15 such that xy^2 is maximum.

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27. If then sum of two positive numbers in constant then show that their product is maximum when they are equal.

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28. Determine a rectangle of area 25sq. Units which has minimum perimeter.

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29. Find the altitude of a right circular cylinder of maximum volume inscribed in a sphere of radius r.

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30. Show that the radius of the right cicular cylinder of greatest curved surface that can be inscribed in a given cone is half the redius of the base of the cone.

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31. Show that the semivertical angle of a cone of given slant height is $\tan^1 \sqrt{2}$ when its volume is maximum.

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32. A cylindrical open water tank with a circular base is to be made out of 30 sq metres of metal sheet. Find the dimensions so that it can hold maximum water. (Neglect thickness of sheet).

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33. A cylindrical vessel of capacity 500 cubic metres open at the top is to be constructed. Find the dimensions of the vessel if the material used is minimum given that the thickness of the material used is 2 cm.

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34. Find the coordinates of the point on the curve $x^2y - x + y = 0$ where the slope of the tangent is maximum.

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35. Find the points on the curve $y = x^2 + 1$ which are nearest to the point (0,2).

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36. Show that the minimum distance of a point on the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$ from the origin is a + b.

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37. Show that the vertical angle of a right circular cone of minimum curved surface that circumscribes a given sphere is $2\sin^{-1}\left(\sqrt{2}-1\right)$.

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38. Show that the semi-vertical angle of a right circular cone of minimum volume that circumscribes a given sphere is $\sin^{-1}\left(\frac{1}{3}\right)$.

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39. Show that the shrtest distance of the point (0, 8a) from the curve $ax^2 = y^3$ is $2a\sqrt{11}$.

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40. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

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Odisha Bureau S Textbook Solutions Exercise 8 E

1. Determine the differentials in each of the following cases. $y = x^3 - 1$

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2. Determine the differentials in each of the following cases. $y = \sin^2 x$

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3. Determine the differentials in each of the following cases. $y = \dfrac{1+\sqrt{x}}{1-\sqrt{x}}$

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4. Determine the differentials in each of the following cases. $z = \cos 2t - 2\cot t$

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5. Determine the differentials in each of the following cases. $r = \dfrac{4}{1+\sin\theta}$

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6. Determine the differentials in each of the following cases. $x^2y = 2$

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7. Determine the differentials in each of the following cases. $xy^2 + yx^2 = 1$

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8. Find δf and df when $f(x) = 2x^2 - 1, x = 1, \delta x = 0 \cdot 02$

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9. Find δf and df when $f(x) = \sqrt{x}, x = 16, \delta x = 0.3$

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10. Find δf and df when $f(x) = (x+1)^3$, $x = 8$, $\delta x = 0.04$

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11. Find δf and df when f(x) = In (1+x), x = 1, deltax = 0.04

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12. Find approximate values of the following : $\sqrt[3]{28}$

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13. Find approximate values of the following : $\sqrt[6]{63}$

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14. Find approximate values of the following : $\sqrt{48.96}$

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15. Find approximate values of the following : $(1.99)^7$

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16. Find approximate values of the following : $2^{3.02}$

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17. Find approximate values of the following : $\sin 59^\circ$

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18. find the percentage of error in calculation of the surface area of a sherical ballon of diameter 14.02 m. if the true diameter is 14m.

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19. Find approximately the difference between the volumes of two cubes of sides 3cm and 3.04 cm.

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20. The height of a regular cone is 3 times the radius of its base. The radius of the base was wrongly measured to be 5 cm. where as its true radius is 4.88 cm. Find the relative error in measuring the curved surface area of the cone.

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Chapter Practice Very Short Answer Type Questions

1. The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square.

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2. The radius of a circle is increasing at the rate of 0.9 cm/s. What is the rate of increase of its circumference ?

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3. If the radius of a circle increasing from 5 cm to 5.1 cm, then find the increase area.

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4. Find the slope of tangent to the curve $y = x^2 + x + 1$ at x = 1.

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5. Find the slope of normal to the curve $y = \sin^2 x$ at x = $\pi/4$.

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6. Find the equation of normal to the curve $y = x^3 + 5x^2 + 2$ at (1, -1).

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7. Find a point on the curve $y = x^2 - 4x$ at which tangent to the curve is parallel to the X-axis.

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8. Show that $f(x) = e^{1/x}$ is a strictly decreasing function for all $x > 0$.

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9. Show that f(x) = x - sin x is increasing for all $x \in R$.

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10. Show that $f(x) = \left(x - \frac{1}{x}\right)$ is increasing for all $x \in R, x \neq 0$.

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11. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

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12. Show that the function $f(x) = \cos^2 x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

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Chapter Practice Short Answer Type Questions

1. The side of an equilateral triangle is increasing at the rate of 2 cm/s. At what rate is its area increasing, when the side of the triangle is 20 cm ?

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2. An edge of variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing, when the edge is 10 cm long ?

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3. A stone is dropped into a quiet lake and waves moves in circles at a speed of 5 cm/s. At the instant when the radius of the circular wave is 8 cm, how fast is the enclosed area increasing ?

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4. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/s. Find the rate at which the volume of the bubbles is increasing at the instant, when its radius is 6 cm.

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5. The volume of a spherical balloon is increasing at the rate of 25 cm^3/s. Find the rate of change of its surface area at the instant, when its radius is 5 cm.

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6. A balloon which always remains sperical is being inflated by pumping in gas at the rate of 800 cm^3/s. Find the rate at which the radius of the balloon is increasing, when the radius is 20 cm.

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7. If the slope of the curve $y = \frac{ax}{b-x}$ at the point (1, 1) is 2. Then, find the value of a and b.

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8. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which the tangents to the curve is parallel to the line y-2x+3 = 0.

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9. Find a point on the curve f(x)=$(x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

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10. Find the points on the curve $x^2+y^2-2x-3=0$ at which the tangents are parallel to X-axis.

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11. Find the equation of tangent and normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,\text{at}(x_1,y_1)$.

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12. Find the equation of the tangent and normal to the curve $16x^2+9y^2=144at(x_1,y_1)$, where $x_1=2$ and $y_1>0$. Also, find the

points of intersection, where both tangent and normal cuts X-axis.

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13. Find the equations of tangent and normal to curve $x = 1 - \cos\theta$ and $y = \theta - \sin\theta \text{at} \theta = \frac{\pi}{4}$.

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14. Find the equation of the normal lines to the curve $3x^2 - y^2 = 8$, which are parallel to the line x + 3y = 4.

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15. Find the equations of all lines having slope -1, that are tangent to the curve $y = \frac{1}{x-1}, x \neq 1$.

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16. Show that the line $\frac{x}{a}+\frac{y}{b}=1$, touches the curve $y=be^{-x/a}$ at point, where curve intersects the axes.

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17. If the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then prove that $a^2\cos^2\alpha+b^2\sin^2\alpha=p^2$.

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18. Prove that the curves $y^2=4ax$ and $xy=c^2$ cut at the right angles, if $c^4=32a^4$.

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19. Find the intervals in which the function given by $f(x) = \sin 3x, x \in \left[0, \frac{\pi}{2}\right]$ is

(i) increasing

(ii) decreasing

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20. Find the intervals of the function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly decreasing.

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21. Find the values of k for which $f(x) = kx^3 - 9kx^2 + 9x + 3$ is increasing on R.

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22. Determine the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is strictly increasing and strictly decreasing.

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23. Find the interval(s) in which the following functions are

(i) increasing

(ii) decreasing

$$f(x) = \log(1 + x) - \frac{x}{1 + x}, x \neq -1$$

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24. Find the interval(s) in which the following functions are

(i) increasing

(ii) decreasing

$$f(x) = (x + 2)e^{-x}$$

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25. Find the points of local maxima, local minima and the points of inflection of the function $f(x) = x^5 - 5x^4 + 5x^3 - 1$. Also, find the corresponding local maximum and local minimum values.

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26. The sum of two numbers is 24. Find the numbers, so that their product is maximum.

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27. Find the approximate value of the following using derivatives.

$(82)^{1/4}$

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28. Find the approximate value of the following using derivatives.

$\sqrt{36.6}$

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29. Find the approximate value of the following using derivatives.

$(26.57)^{1/3}$

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30. If $y = \left(x^4 - 12\right)$ and if x changes from 2 to 1.99, then what is the approximate change in y ?

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Chapter Practice Long Answer Type Questions

1. Find the equations of tangents to curve $3x^2 - y^2 = 8$, which passes through point $\left(\frac{4}{3}, 0\right)$

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2. Find the point on the curve $y^2 = 2x$, which is at a minimum distance from point (1, 4).

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3. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet ?

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4. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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5. Show that a cylinder of a given volume, which is open at the top has minimum total surface area, when its height is equal to the radius of its base.

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6. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ with its vertex at one end of the major axis.

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