MATHS

BOOKS - ARIHANT PUBLICATION

CONTINUITY AND DIFFERENTIABILITY

Part I Sample Questions

1. Examine the continuity of the following functions at gives points

$f(x) = x^2 \text{ at } x = -25$

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2. Examine the continuity of the following functions at gives points

$f(x) = |x - 5| \text{ at } x = 5$

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3. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{x^2 - x - 6}{x + 2}, & \text{if } x \neq -2 \\ -5, & \text{if } x = -2 \end{cases} \quad \text{at } x = -2.$$

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4. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases} \quad \text{at } x = 0.$$

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5. Show that the function f(x) given by

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ is continuous at x =0

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6. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1 & , \text{if } x \leq 3 \\ bx+3 & , \text{if } x > 3 \end{cases}$ is contunuous at x=3.

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7. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$$

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8. Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} -2, \text{if} x \leq -1 \\ 2x, \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$$

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9. Discuss the continuity of sine function.

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10. Discuss the continuity of $f(x) = |\sin x + \cos x|$ at $x = \pi$.

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Part I Question For Practice 1 Mark

1. Examine the continuity of the following function at the given point :

$f(x) = 5x - 3 \quad \text{at} \quad x = -3$

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2. Examine the continuity of the following function at the given point :

$f(x) = x^2 + 5 \quad \text{at} \quad x = -1$

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3. Examine the continuity of the following function at the given point :

$f(x) = x^2 + 3x + 4 \quad \text{at} \quad x = 1$

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4. Is the function f defined by $f(x) = \begin{cases} x, \text{ if } & x \leq 1 \\ 5, \text{ if } & x > 1 \end{cases}$ continuous at x=0, at x=1 and at x=2 ?

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5. Examine the following functions for continuity

$$f(x) = \frac{1}{x-5}, x \neq 5$$

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6. Examine the following functions for continuity

$$f(x) = \frac{1}{x+3}, x \in R$$

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7. Examine the following functions for continuity

$$f(x) = \frac{x^2 - 25}{x+5}, x \neq -5$$

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8. Examine the following functions for continuity

$f(x) = x - 5$

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Part I Question For Practice 4 Mark

1. Show that the function $f(x) = 2x - |x|$ is continuous at $x = 0$.

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2. Let $f(x) = x - \left|x - x^2\right|, x \in [-1, 1]$ find the point of discontinuity, (if any) of this function on [-1,1]

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3. Show that the function

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$.

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4. Discuss the continuity of the following function at $x = 0$.

$$f(x) = \begin{cases} \frac{x^4 + 2x^3 + x^2}{\tan^{-1} x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

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5. If $f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$. Then, find value of $f\left(\frac{\pi}{4}\right)$, so that f(x) becomes continuous at $x = \frac{\pi}{4}$.

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6. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points.

Here, [x] denotes the greatest integer less than or equal to x.

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7. Determine f(0) so that the function f(x) defined by $f(x) = \frac{(4^x - 1)^3}{\sin \frac{x}{4} \log\left(1 + \frac{x^2}{3}\right)}$ becomes continuous at $x = 0$.

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8. Examine the continuity of a function

$$f(x) = \begin{cases} |x| \cos \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ at $x = 0$.

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9. Show that the function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is discontinuous at $x = 0$.

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10. For what value of λ is the function

$f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$ continuous at x=0 ?

What about continuity at x=1 ?

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11. Find the value of k, so that the function defined by

$f(x) = \begin{cases} kx + 1, & \text{if } x \leq \pi \\ \cos x, & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$.

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12. For what value of k, the function

$$f(x) = \begin{cases} \frac{\tan 2x}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$ is continuous at x=0 ?

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13. For what value of k the following function is continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

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14. Find the value of a, so that the function f(x) is defined by

$$f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$ may be continuous at x=0.

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15. Show that the function

$$f(x) = \begin{cases} |x-a|\frac{\sin(1)}{x-a}, & \text{if } x \neq a \\ 0, & \text{if } x = a \end{cases}$$ is continuous at x=a.

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16. A car driver is driving a car on the dangerous path given by

$$f(x) = \begin{cases} \frac{1-x^m}{1-x}, & \text{if } x \neq 1 \\ m-1, & \text{if } x = 1 \end{cases} \quad m \in N.$$

Find the dangerous point (point of discontinuity) on the path.

Whether the driver should pass that point or not ? Justify your answer.

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17. For what value of k, the function

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$ is continuous at x=0 ?

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18. Find the value of k for which

$$f(x) = \begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$$ is continuous at x = 0

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19. find the value of k, so that the function

$$f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases}$$ is continuous at x=2.

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20. Find the values of a and b such that the function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a+b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$$ is a

continuous function at $x = 4$.

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21. Discuss the continmuity of function f defined by

$$f(x) = \begin{cases} \frac{1}{2} - x, & \text{if } 0 \leq x \leq \frac{1}{2} \\ 1, & \text{if } x = \frac{1}{2} \\ \frac{3}{2} - x, & \text{if } \frac{1}{2} < x \leq 1 \end{cases}$$

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22. Discuss the continuity of the function $f(x) = \sin x - \cos x$.

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23. Given, $f(x) = \dfrac{1}{x-1}$. Find the points of discontinuity of composite function $y = f\{f(x)\}$.

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24. Examine that $\sin|x|$ is a continuous function.

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25. Show that the function f defined by $f(x) = |1 - x + |x\ |\ |$, where x is any real number is a continuous function.

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26. Show that tangent function is continuous in their domain

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Part Ii Sample Question

1. Discuss the differentiability of $f(x) = x^2 \quad \text{at} \quad x = 1$

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2. Prove that $f(x) = |x|$ is not differetiable at $x = 0$

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3. Examine the following function f(x) for continuity at $x = 1$ and differentiability at $x = 2$,

$$f(x) = \begin{cases} 5x - 4, & \text{if } 0 < x < 1 \\ 4x^2 - 3x, & \text{if } 1 \leq x < 2 \\ 3x + 4, & \text{if } x \geq 2 \end{cases}$$

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4. Differentiate $\frac{1}{\sqrt{x}}$ from definition.

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5. Differentiate e^{x^2} from definition

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6. Find the derivative of $\left(x^2+\cos x\right)$.

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7. If $y=(2+3\sin x)(3-2\cos x)$, then find $\frac{dy}{dx}$.

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8. Differentiate $\frac{\sec x-1}{\sec x+1}$ w.r.t. x.

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9. Differentiate $\sin\left(x^2+5\right)$ w.r.t. x.

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10. Differentiate $\sin 5x \cdot \cos 7x$ w.r.t. x.

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11. Differentiate $2\sqrt{\cos\left(x^2\right)}$ w.r.t. x.

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Part Ii Question For Practice 1 Mark

1. Differentiate the following functions :

$\tan\left(x^2+5\right)$

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2. Differentiate the following functions :

$\tan(\sin x)$

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3. Differentiate the following functions :

$\text{cosec}\left(\sqrt{x^2+2}\right)$

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4. Differentiate the following functions :

$\sec\left(\frac{5}{x}\right)$

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5. If $f(x) = |\cos x|$, then find $f'\left(\frac{3\pi}{4}\right)$

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6. If $x^{2/3} + y^{2/3} = a^{2/3}$, then find $\frac{dy}{dx}$

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7. Prove that the function f gives by $f(x) = |x - 1| x \in R$ is not differentiable at $x = 1$.

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8. Show that the function $f(x) = |x - 1| + |x + 1|$

$\forall x \in R$ is not differentiable at the points $x = -1$ and $x = 1$.

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9. A function f(x) is defined as follows :

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

show that f(x) is differentiable at x=0.

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10. Let $f(x) = x|x|, \forall x \in R$. Discuss the derivability of $f(x)$ at $x = 0$.

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11. Examine the differentiability of the function f defined by

$$f(x) = \begin{cases} 2x + 3, \text{ if } -3 \leq x < -2 \\ x + 1, \text{ if } -2 \leq x < 0 \\ x + 2, \text{ if } 0 \leq x \leq 1 \end{cases}$$

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12. Discuss the continuity and differentiability of function $f(x) = |x| + |x - 1|$ in the interval (-1,2).

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13. Prove that the greatest interger function defined by $f(x) = [x], 0 < x < 3$ is not differentiable at $x = 1$.

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14. If $f(x) = |\cos x - \sin x|$ then find f ' $\left(\frac{\pi}{6}\right)$

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15. Differentiate $\sqrt{\tan\sqrt{x}}$ w.r.t. x.

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16. Differentiate $\cos x^3 \cdot \sin^2\left(x^5\right)$ w.r.t.x.

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17. If $y = \sin\left[\sqrt{\sin\sqrt{x}}\right]$, then find $\frac{dy}{dx}$.

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18. If $y = x\tan x + \sec x$, then find the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$.

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19. If $y = \sin^3\sqrt{ax^2 + bx + c}$ then find $\frac{dy}{dx}$

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20. If $y = \left[x + \sqrt{x^2 + a^2}\right]^n$, then prove that $\frac{dy}{dx} = \frac{ny}{\sqrt{x^2 + a^2}}$

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21. If $y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$, then show that $\frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0, x \in \left[0, \frac{\pi}{4}\right]$

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22. Prove that $y = \tan(x+y)$, then $\frac{dy}{dx} = \frac{1+y^2}{-y^2}$

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23. Differentiate from definition

$\left(x^2 + 3x + 5\right)$

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24. Differentiate from definition

$e^{\sin x}$

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25. Differentiate from definition $\sqrt{\cos x}$

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26. Differentiate from definition

$$\frac{\sin x}{x}$$

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Part Iii Sample Questions

1. Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt{a^2 - x^2}}$

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2. Differentiate $\cos^{-1}\left(5x^2+4\right)$ w.r.t. x.

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3. Differentiate $\tan^{-1}(\sec x+\tan x)$ w.r.t. x.

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4. Show that $\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]=\sqrt{a^2-x^2}$

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5. If $y=\sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$, then find $\frac{dy}{dx}$.

Here $f^{-1}(x)$ expression is of the form $\frac{x^2+a^2}{x^2-a^2}$, so we substitute $x=\tan\theta$ and then use suitable trigonometrical formula to write it in simplest form and then differentiate

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6. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

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7. Find $\frac{dy}{dx}$, if $y=\sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$.

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8. Differentiate $\tan^{-1}\left(\frac{a\cos x-b\sin x}{b\cos x+a\sin x}\right)-\frac{\pi}{2}<x<\frac{\pi}{2}$ and $\frac{a}{b}\tan x>-1$.

Firstly, convert the given inverse trigonometic function into the simplest form and the differentiate it.

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Part Iii Question For Practice 1 Mark

1. If $f(x) = \cos^{-1}(\sin x)$, then find $f'.(x)$

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2. If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then find $\frac{dy}{dx}$

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3. Differentiate $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ w.r.t. x.

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4. Differentiate $\tan^{-1}\left[\frac{\sqrt{x}+\sqrt{a}}{1-\sqrt{ax}}\right]$ w.r.t. x.

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5. Differentiate $\cos^{-1}\sqrt{\frac{1+\cos x}{2}}$ w.r.t. x.

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6. If $y = \tan^{-1}\left(\frac{3x - x^3}{1 - x^2}\right)$, find $\frac{dy}{dx}$.

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7. Find the derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \frac{-\pi}{4} < x < \frac{\pi}{4}$.

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8. If $y = \tan^{-1}\left[\frac{\sqrt{1 + a^2x^2} - 1}{ax}\right]$, then find $\frac{dy}{dx}$.

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Part Iii Question For Practice 4 Mark

1. Find $\frac{dy}{dt}$, when $y = \sin^{-1}\left(2\frac{\sqrt{t^2-1}}{t^2}\right)$

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2. Differentiate $y = 2^{x^2} + \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$ w.r.t. x.

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3. Prove that $y = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$

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4. Find the derivative of $\tan^{-1}\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$ w.r.t. x.

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5. Find the derivative of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ w.r.t. x.

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6. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, 0<x<\frac{\pi}{2}, \text{ or } x\in\left(0,\frac{\pi}{4}\right)$

.

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7. Find the derivative of $\cos^{-1}\left(\frac{3\cos x-4\sin x}{5}\right)$ w.r.t. x.

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8. If $y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$ then find $\frac{dy}{dx}$

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9. If the derivative of $\tan^{-1}(a+bx)$ takes the value $\frac{dy}{dx}=1$ at $x=0$, then prove that $b=1+a^2$

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10. Differentiate $y=\tan^{-1}.\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$

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11. Differentiate $y=\tan^{-1}.\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}$

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12. Write the simplest form $\tan^{-1}\left[\frac{3a^2x - x^3}{a^3 - 3ax^2}\right], a > 0, -\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$.

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13. If $y = \cos^{-1}\left[\frac{2x - 3\sqrt{1 - x^2}}{13}\right]$ then find $\frac{dy}{dx}$

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Part Iv Sample Question

1. If $x^3 + y^3 = 3axy$, then find $\frac{dy}{dx}$.

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2. Find $\frac{dy}{dx}$ for the equation $x^3 + y^3 = \sin(x + y)$

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Part Iv Question For Practice 1 Mark

1. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$

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2. If $y = \tan(x + y)$, then find $\frac{dy}{dx}$.

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3. Find the derivative of $\left(x^2 + y^2\right)^2 = xy$ w.r.t. x.

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4. Find $\frac{dy}{dx}$, when $\sin(x+y) = x^2 + y^2$

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Part Iv Question For Practice 4 Mark

1. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then show that $\frac{dy}{dx} \cdot \frac{dx}{dy} = 1$

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2. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for $-1 < x, y < 1$ then prove that $\frac{dy}{dx} = \frac{-1}{(x+1)^2}$

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3. Find the derivative of $\sin(xy) + \frac{x}{y} = x^2 - y$ w.r.t. x.

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4. If cos y = x cos(a+y) then prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

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Part V Sample Question

1. Differentiate the following w.r.t x:

$\log(\cos e^x)$

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2. Differentiate the following w.r.t x:

e^{x^3}

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3. Differentiate each of the following w.r.t. x.

$e^{\sqrt{\cot x}}$

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4. Differentiate each of the following w.r.t. x.

$\log \tan \frac{x}{2}$

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5. Differentiate a^x w.r.t. x, where a is a positive constant.

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6. If $y = \log \sqrt{\frac{1+\sin^2 x}{1-\sin x}}$, then find $\frac{dy}{dx}$.

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7. Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ with respect to x.

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Part V Question For Practice 1 Mark

1. Differentiate $e^{\sin^{-1} x}$ w.r.t.x.

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2. If $f(x) = x\cos x + e^x$, then find $f.\ (0)$.

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3. If $y = \frac{\log x}{x}$, then find $\frac{dy}{dx}$

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4. Find $\frac{dy}{dx}$, if $y = \log_{x^2} 3$.

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5. Find $\frac{dy}{dx}$, if $y = 10^{\log_e \sin x}$

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Part V Question For Practice 4 Mark

1. If $\log\sqrt{x^2+y^2} = \tan^{-1}\left(\frac{x}{y}\right)$, then show that $\frac{dy}{dx} = \frac{y-x}{y+x}$.

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2. If $(x-y)\cdot e^{\frac{x}{x-y}} = a$, then prove that $y\frac{dy}{dx} + x = 2y$.

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3. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$.

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4. If $\frac{x}{x-y} = \log\left(\frac{a}{x-y}\right)$, then prove that $\frac{dy}{dx} = 2 - \frac{x}{y}$.

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5. If $y^x = x^y$, then find $\frac{dy}{dx}$.

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6. Find the derivative of $y = (x+3)^2(x+4)^3(x+5)^4$

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Part V Question For Practice 6 Mark

1. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

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2. Differentiate the following function w.r.t. x,

$(\log x)^x + x^{\log x}$.

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3. If $y = x^{\sin x} + (\sin x)^{\cos x}$, then find $\frac{dy}{dx}$.

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4. If $x^p y^q = (x+y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$.

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5. Find $\frac{dy}{dx}$, if $y = \cot^{-1}\left(\log \cos e^{-x}\right) + \frac{x \sin^{-1} x}{\sqrt{1-x^2}}$.

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6. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$.

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Part Vi Sample Questions

1. Find $\frac{dy}{dx}$, if $x = a \log t$ and $y = b \sin t$

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2. If $x = \cos\theta - \cos 2\theta$ and $y = \sin\theta - \sin 2\theta$, then find $\frac{dy}{dx}$.

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3. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, then prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

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Part Vi Question For Practice 4 Mark

1. Find $\frac{dy}{dx}$, when $x = \cos^2\theta$ and $y = \sin^2\theta$

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2. Find $\frac{dy}{dx}$, if$x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$

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3. If $x = t + \frac{1}{t}, y = t - \frac{1}{t}$, then find $\frac{dy}{dx}$.

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4. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$.

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5. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\frac{dy}{dx} = -\frac{y}{x}$.

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6. If $x = e^{\theta}\left(\theta + \frac{1}{\theta}\right)$, $y = e^{-\theta}\left(\theta - \frac{1}{\theta}\right)$, then find $\frac{dy}{dx}$.

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7. If $x = a\sin 2t(1 + \cos 2t)$ and $y = b\cos 2t(1 - \cos 2t)$, then show that at $t = \frac{\pi}{4}$, $\frac{dy}{dx} = \frac{b}{a}$

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8. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, then prove that $\frac{dy}{dx} = \frac{-y \log x}{x \log y}$.

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9. If $x = a \sec \theta$, $y = b \tan \theta$, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

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Part Vi Question For Practice 6 Mark

1. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, then find $\frac{dy}{dx}$.

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2. Show that $\frac{dy}{dx}$ is independent of t . $\cos x = \sqrt{\frac{1}{1+t^2}}$ and $\sin y = \frac{2t}{1+t^2}$

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Part Vii Sample Questions

1. Write the derivative of sin x w.r.t. cos x.

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2. Differentiate $\log\left(1+x^2\right)$ w.r.t. $\tan^{-1}$ x.

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3. Find derivative of $\sin^{-1}$ x w.r.t. $\tan^{-1}$x.

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Part Vii Question For Practice 1 Mark

1. Write the derivative of $e^{3\log x}$ w.r.t.x^2

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2. What is the derivative of $\sin^{-1}\left(3x-4x^3\right)$ w.r.t $\sin^{-1}x$?

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Part Vii Question For Practice 4 Mark

1. Differentiate $\sin x^2$ w.r.t x^2.

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2. Differentiate $(\sin x)^{\log x}$ w.r.t. $\sin x$.

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3. find the derivative of $\tan^{-1}\left[\frac{\sqrt{1-x^2}}{x}\right]$ w.r.t. $\tan^{-1}\frac{2x}{1-x^2}$.

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4. Find the drivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x=\frac{1}{2}$

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5. Find the derivative of $\tan^{-1}\left(1+x^2\right)$ with respect to $\log\left(x^2+1\right)$

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Part Viii Sample Questions

1. Find the second order derivative of

$x^4 + \cot x$

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2. Find the second order derivative of

$\sin(\log x)$

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3. If $y = e^x \sin 5x$, the find $\frac{d^2y}{dx^2}$.

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4. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$.

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5. If $y = x^x$, then prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

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6. If $y = \sin^{-1} x$, then show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

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7. If $y = \mathrm{Pe}^{ax} + \mathrm{Qe}^{bx}$, then show that $\frac{d^2y}{dx^2} - (a + b)\frac{dy}{dx} + aby = 0$

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8. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then show that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

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Part Viii Question For Practice 1 Mark

1. Find the second order derivative of the following : $x^3 - x^2 + 2$

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2. If $f(x) = \tan^{-1} x$, then find f''(x).

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Part Viii Question For Practice 4 Mark

1. If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2} = \frac{\cos x}{(1 - \sin x)^2}$

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2. If $y = \tan^{-1} x$, then show that $(1 + x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$

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3. $x = \tan\left(\frac{1}{2}\log y\right)$, then show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 2)\frac{dy}{dx} = 0$

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4. If $y = \sqrt{x + 1} - \sqrt{x - 1}$, then prove that $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{1}{4}y = 0.$

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5. If $y = a\sin x + b\cos x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

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6. If $y = \log\left[x + \sqrt{x^2+1}\right]$, then prove that $\left(x^2+1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$

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7. If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y.$

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8. If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2 y_2 + x y_1 + y = 0$.

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9. If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$.

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10. If $y = a\cos t$, $x = a\sin t$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$.

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11. If $y = 5\cos x - 3\sin x$,then prove that $\frac{d^2y}{dx^2} + y = 0$

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12. If $(x-a)^2+(y-b)^2=c^2$, then prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant and independent of a and b.

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13. If $f(x)=|x|^3$, then show that $f''(x)$ exists for all real x and find it.

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14. If $x=a\sec^3\theta, y=a\tan^3\theta$ then find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{4}$.

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Part Ix Sample Questions

1. Verify Rolle's theorem for the function $y = x^2 + 2, a = -2$ and $b = 2$.

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2. Verify Rolle's theorem for the function $f(x) = \sin 2x$ in $\left[0, \frac{\pi}{2}\right]$

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3. If $f[-5, 5] \to R$, is a differentiable function and if f'(x) does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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4. Verify LMVT, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a,b], where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$

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5. Verify the LMVT for the function $f(x) = \frac{1}{4x-1}, 1 \leq x \leq 4.$

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Part Xi Question For Practice 4 Mark

1. find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $\left[0, \sqrt{3}\right]$.

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2. Verify the Rolle's theorem for the function $f(x) = \log\left(x^2 + 2\right) - \log 3$ in $[-1, 1]$

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3. It is given that for the function f given by $f(x) = x^3 + bx^2 + ax, x \in [1, 3]$. Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$. Find the values of a and b.

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4. Verify, MVT, if $f(x) = x^2 - 4x - 3$ in the interval [a,b], where a =1 and b=4.

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5. Verify Cauchy's mean value for the functions $f(x) = \sin x, g(x) = \cos x$ in $\left[0, \frac{\pi}{2}\right]$

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6. Verify mean value theorem for the function $f(x) = \sin x - \sin 2x$ in $[0, \pi]$

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7. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1, 0)$ and $B(2, 1)$ where tangents are parallel to the chord. Also, find that point.

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Chapter Practice 4 Mark

1. Find the derivative of $\cos\left(\sqrt{x}\right)$.

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2. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$

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3. Show that the function $f(x) = |x - 3|, x \in R$ is continuous but not differentiable at $x = 3$

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4. Differentiate $x^{\sin x}$ w.r.t x.

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5. Examine the continuity of the function

$$f(x) = \begin{cases} \frac{|\sin x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ at } x = 0$$

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6. Show that the function $f(x) = \begin{cases} \frac{|x-a|}{x-1}, & x \neq a \\ 1, & x = a \end{cases}$ is discontinuous at $x = 0$.

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7. Discuss the continuity of, the function $f(x) \begin{cases} \frac{x}{|x|+2x^2}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ at $x = 0$.

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8. If the function $f(x) = \begin{cases} 3ax + b, & \text{if } x > 1 \\ 11, & \text{if } x = 1 \\ 5ax - 2b, & \text{if } x < 1 \end{cases}$ is continuous at $x = 1$, then find the values of a and b

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9. Find all points of discontinuity of f, where f is defined by

$$f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$$

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10. Find the value of k so that the function f is continuous at $x = \pi/2$, $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 5, & \text{if } x = \frac{\pi}{2} \end{cases}$

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11. The function $f(x) = \begin{cases} \frac{x^2}{a}, & \text{if } 0 \leq x < 1 \\ a, & \text{if } 1 \leq x < \sqrt{2} \\ \frac{2b^2 - 4b}{x^2}, & \text{if } \sqrt{2} \leq x < \infty \end{cases}$ is continuous on $(0, \infty)$. Find the most suitable value of a and b.

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12. If the function f(x) defined by

$$f(x) = \begin{cases} \frac{\log(1+ax) - \log(1-bx)}{k}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$ is continuous at $x = 0$, then find the value of k.

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13. Examine the continuity of the following function

$$f(x) = \begin{cases} \frac{x}{2|x|}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$$ at x=0.

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14. Find the point of discontinuity, if any of the following function

$$f(x) = \begin{cases} x^{10} - 1, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}.$$

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15. Find the point of discontinuity of the function $f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{if } x < 0 \\ x + 2, & \text{if } x \geq 0 \end{cases}$

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16. If function $f(x) = |x - 3| + |x - 4|$, then show that f(x) is not differentiable at $x = 3$ and $x = 4$.

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17. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ w.r.t. $\cos^{-1} x^2$.

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18. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$

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19. If $y = (\sin x)^x + \sin^{-1}\sqrt{x}$, then find $\frac{dy}{dx}$.

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20. Find $\frac{dy}{dx}$, if $y = (\cos x)^x + (\sin x)^{1/x}$.

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21. If $x = \alpha \sin 2t(1 + \cos 2t)$ and $y = \beta \cos 2t(1 - \cos 2t)$, then show that $\frac{dy}{dx} = \frac{\beta}{\alpha}\tan t$.

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22. Find the derivative of the function w.r.t. x at $x = 1, \cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right] + x^x$.

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23. If $y = e^{m \sin^{-1} x}$, then show that $\left(1 - x^2\right) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0.$

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24. Test the continuity and differentiability at $x = 1$ of the function f defined by

$$f(x) = \begin{cases} 3^x, & -1 \leq x < 1 \\ 4 - x, & -1 \leq x < 4 \end{cases}$$

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25. Show that the function $f(x) = |x - 1| + |x - 2|$ is not differentiable at $x = 2$.

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26. If a function f is differentiable at a point c, then prove that it is also continuous at that point.

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27. If $x=a\cos^3\theta, y=a\sin^3\theta$, then find $\frac{d^2y}{dx^2}$

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28. Find the derivative of $\frac{x+\sin x}{x+\cos x}$ w.r.t. x.

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29. Differentiate w.r.t. x, $\sin^{-1}\left[\frac{2^{x+1}\cdot 3^x}{1+(36)^x}\right]$.

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30. If $y = \dfrac{\sin(ax+b)}{\cos(cx+d)}$, the find $\dfrac{dy}{dx}$.

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31. Differentiate $\tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$ w.r.t. $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

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32. If $y = \tan^{-1}\left(\dfrac{a}{x}\right) + \log\sqrt{\dfrac{x-a}{x+a}}$, then prove that $\dfrac{dy}{dx} = \dfrac{2a^3}{x^4-a^4}$.

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33. If $x = a\left(\dfrac{1-t^2}{1+t^2}\right)$ and $y = \dfrac{2t}{1-t^2}$, then find $\dfrac{dy}{dx}$.

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34. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, then find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

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35. Differentiate $x^{x\cos x} + \frac{x^2+1}{x^2-1}$ w.r.t. x.

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36. If $y = (\sin x - \cos x)^{(\sin x - \cos x)}$, $\frac{\pi}{4} < x < \frac{3\pi}{4}$, then find $\frac{dy}{dx}$.

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37. If $\log\left(\sqrt{1+x^2} - x\right) = y\sqrt{1+x^2}$, then show that $\left(1+x^2\right)\frac{dy}{dx} + xy + 1 = 0$.

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38. Discuss the continuity of secant.

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39. Find the set of points, where the function f given by $f(x) = |2x - 1|\sin x$ is differentiable.

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40. If $y = e^x \sin x$, then prove that $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$,

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41. Find $\frac{dy}{dx}$, where $y = x^{\cot x} + \frac{2x^2 - 3}{x^2 + x + 2}$.

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42. Find the derivative of $(\sin x)^x + \sin\sqrt{x}$ w.r.t. x.

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43. For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2+2), & \text{if } x \leq 0 \\ 4x+6, & \text{if } x > 0 \end{cases}$ is continuous $x = 0$? Hence check the differentiablity of f(x) at x=0.

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44. Find the value of k, the function,

$f(x) = \begin{cases} \frac{1-\sin x}{(\pi - 2x)^2}, & x \neq \frac{\pi}{2} \\ k, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

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45. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, the prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - y = 0.$

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46. Differentiate $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right)$ w.r.t. $\sin\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)$.

[Hint put $x = \sin\theta$ in $\tan^{-1}$ and $x = \cos\theta$ in $\cot^{-1}$]

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47. If $y = \left(x + \frac{1}{x + \frac{1}{x+\ldots\infty}}\right)$ find $\frac{dy}{dx}$, the rhs being a valid expression.

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Chapter Practice 1 Mark

1. Differentiate $\sin\left[\cos\left(x^2\right)\right]$ w.r.t. x.

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2. Find the derivative of log sin x w.r.t. x.

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3. Differentiate $e^{\sqrt{x}}$ w.r.t. x.

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4. Differentiate $e^{3x} \cdot \cos 2x$ w.r.t. x.

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5. If $x = a\cos\theta$ and $y = b\cos\theta$, then find $\frac{dy}{dx}$.

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6. Find $\frac{dy}{dx}$, when $2x + 3y = \sin y$.

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7. Find the second order derivative of log x.

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8. Find the derivative of $\sin\left(\tan^{-1} x\right)$ w.r.t. x.

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