MATHS

BOOKS - ARIHANT PUBLICATION

RELATIONS AND FUNCTIONS

Sample Question Part I

1. If R = {(x, y): x + 2y = 8) is a relation on a set of natural numbers (N), then write the domain, range and codomain of R.

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2. Give an example of a relation, which is

(i) symmetric but neither reflexive nor transitive.

(ii) transitive but neither reflexive nor symmetric.

(iii) reflexive and symmetric but not transitive.

(iv) reflexive and transitive but not symmetric.

(v) symmetric and transitive but not reflexive.

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3. Show that the relation R on the set {1,2,3) given by R={(1,1), (2, 2), (3, 3), (1, 2), (2, 3)) is reflexive but neither symmetric nor transitive.

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4. Check whether the relation R defined in the set A = {1,2,3,...,13,14 as R = {(x,y):3x - y =0} is reflexive, symmetric and transitive.

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5. Check whether the relation R defined on the set A = {1,2,3,4,5,6} as R = {(x,y):y is divisible by x} is reflexive, symmetric and transitive.

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6. Let T be the set of all triangles in a plane with R is a relation in T given by R=$\{(T_1, T_2) : T_2$ is congruent to T_2 and $T_1, T_2 \in T)$. Show that R is an equivalence relation.

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7. Prove that the relation 'Congruence modulo, m' on the set Z of all integers is an equivalence relation.

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8. Let R is the equivalence in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a - b)}. Write the equivalence class [0].

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9. Let A={1,2,3,...,9) and R be the relation on A $\times$ A defined by (a, b) R (c,d), if a+d=b+c for (a, b), (c,d) in A $\times$ A. Prove that R is an equivalence relation and also obtain the equivalence class (2,5).

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10. Show that the relation R is in the set $A = \{1, 2, 3, 4, 5\}$ given by R={(a, b): |a-b| is divisible by 2}, is an equivalence relation. Write all the equivalence classes of R.

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Sample Question Part Ii

1. Let X = { 1,2,3 } and Y = { 2,4,6,8}. Consider the rule f: X $\rightarrow Y$ defined as f (x) = 2x $\forall x \in X$. Find the domain codomain and range of f .

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2. Check which of the following function is onto and into.

(i) f :X $\rightarrow$ Y, given by f(x) = 3x, where X = {0, 1, 2 and Y = {0,3,6).

(ii) f: Z $\rightarrow$ Z, given by f(x) = 3x + 2, (Z = set of integers).

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3. Determine whether the function f:X $\rightarrow$ Y defined by f(x) = 4x + 7, x $\in X$ is one-one.

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4. Show that the function f: $R \rightarrow R$ given by f(x) = $\begin{cases} 1 & \text{if} x > 0 \\ 0 & \text{if} x = 0 \\ -1 & \text{if} x < 0 \end{cases}$ is not one - one

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5. Let R be the set of all non -zero real numbers. Then show that f: R $\rightarrow$ R given by f(x) = $\frac{1}{x}$ is one- one and onto.

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6. Show that the function f : $R \rightarrow R$ defined as f(x) = x^2 is neither one-one nor onto.

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7. Show that the function f: N $\rightarrow$ N, given by f(1) f(2)= 1 and f(x) = x - 1 for every x $>$ 2, is onto but not one-one.

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Sample Question Part Iii

1. If $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

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2. Find gof and fog, if f :R $\rightarrow$ R and g:R $\rightarrow$ R are given by

(i) f(x) = sinx and g(x) = $4x^2$.

(ii) f(x) = x^2 and g(x) = 2x +1.

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3. Let f : R $\rightarrow$ R be the signum function defined as f(x)= $\begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$ and g :R $\rightarrow$ be the greatest integer function given by g(x) = [x] . Then prove that fog and gof coincide in [-1,0).

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4. Show that if f : R - $\left\{\frac{7}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined by f (x) = $\frac{3x+4}{5x-7}$ andc g:R - $\left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{7}{5}\right\}$ is defined by g(x) = $\frac{7x+4}{5x-3}$ then fog $= I_A$ and gof = I_B where A = R - $\left\{\frac{3}{5}\right\}$ B=R - $\left\{\frac{7}{5}\right\}, I_A(x) = x \,\forall x \in A$ and $I_B(x) = x \,\forall x \in B$ are called identity functions on sets A and B respectively.

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5. Consider f : N $\rightarrow N$,g : $N \rightarrow N, g\colon N \rightarrow N$ and h : $N \rightarrow R$ defined as f(x) = 3x g(y) = 2y +3 and h(z) = cos z $\forall x, y, z \in N$. Show that (hog) of = ho (gof).

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Sample Question Part Iv

1. Find the inverse of the function f(x) = $(x-3)^3$

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2. Let Y = $\{n^2 n \in N\} \subset$ N. Consider f:N $\rightarrow$ Y as f(n) = n^2. Show that f is invertible. Also, find the inverse of f.

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3. Let f : N $\rightarrow$ Y be a function defined as f(x) = 4x +3, where Y = {Y $\in$ N: y = 4x +3 for some x $\in$ N). Show that f is invertible. Find the inverse.

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4. Let S = {1,2,3). Determine whether the functions f :'S $\rightarrow$ S defined as below have inverses. Find f^{-1}, if it exists.

(i) f= {(1, 1), (2, 2), (3, 3)]

(ii) f = {(1,2),(2,1),(3, 1),

(iii) f = {(1,3), (3, 2), (2,1)]

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5. Let A = R-{2} and B = R - {1}. If f : A $\rightarrow$ B is a function defined by f(x)= $\frac{x-1}{x-2}$ then show that f is one-one and onto. Hence, find f^{-1}.

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6. If the function f:[1, ∞) $\rightarrow [1, \infty)$ defined by f(x) = $2^{x(x-1)}$ is invertible, then find f^{-1} (x).

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Sample Question Part V

1. Let *:$R \times R \rightarrow R$ given by (a,b) $\rightarrow$ a + $4b^2$ is a binary operation compute (-5) *(2*0) .

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2. Let be a binary operation on N given by a *b = HCF (a,b) , a,b, $\in$ N . Find n12 *4 and 7 *5.

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3. Show that addition, subtraction and multiplication are binary operations on R. Also, show that division is not a binary operation on the set R.

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4. Test for commutative and associative of a binary operation on Z^+ defined by a b= a^b .

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5. On the set Q+ of all positive rational numbers, define a binary operation * on Q+ by a*b = $\frac{ab}{3} \forall (a, b) \in Q^{+}$. Then, find the identity element in Q for*.

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6. On the set Q^{+} of all positive rational numbers define a binary operation * on Q^{+} by a * b= $\frac{ab}{3}$ $\forall$ (a,b) $\in Q^{+}$. Then what is the inverse of a $\in Q^{+}$?

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7. Consider the binary operation * on the set{1,2,3,4,5}defined by a*b=min(a,b). Write operation table for operation *.

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8. A binary operation $\cdot$ on the set {0,1,2,3,4,5} is defined as

$$a \cdot b = \begin{cases} a+b & \text{if} a+b < 6 \\ a+b-6 & \text{if} a+b \geq 6 \end{cases}$$

Find the composition table for $\cdot$ Also, show that zero is the identity for this operation and each non-zero element a of the set is invertible with 6-a, being the inverse of a.

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Part I Question For Practice Part I Relations Very Short Answer Type Questions

1. Set A and B have respectively m and n elements. The total number of relations from A to B is 128. If m $\neq$ 1, write the values of m and n, respectively.

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2. If R is a relation 'is divisor of'from the set A={1, 2, 3) to B = {4, 10, 15), then write down the set of ordered pairs.corresponding to R.

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3. Let R={(a, a^3): a is a prime number less than 5) be a relation. Find the range of R.

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4. For real numbers x and y, define x R y if and only if $x-y+\sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

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5. Let A = {a,b,c) and the relation R be defined on A as follows:

R={{a,a), (b,c),(a,b)}.

Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

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Part I Question For Practice Part I Relations Short Answer Type Questions

1. Let A = {1, 2, 3). Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

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2. Show that the relation R on the set A of real numbers defined as R = {(a,b): a $\leq b$).is reflexive. and transitive but not symmetric.

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3. Let a relation R on the set A of real numbers be defined as (a, b) $\in R \Rightarrow 1 + ab > 0, \ \forall a, b \in A$. Show that R is reflexive and symmetric but not transitive.

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4. Let A be the set of all points in a plane and R be a relation on A defined as R={(P,Q): distance between P and Q is less than 2 units). Show that R is reflexive and symmetric but not transitive.

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5. Let R be a relation defined on the set of natural numbers N as follows: R = {(x,y): x $\in$ N,y $\in$ N and 2x + y = 24). Then, find the domain and range of the relation R. Also, find whether R is an equivalence relation or not.

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6. Let A (1,2,3). Then, find the number of equivalence relations containing (1,2).

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7. Show that the relation R on the set Z of integers given by R = {(a,b): 2 divides (a - b)} is an equivalence relation.

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8. Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v), if and only if xv = yu. Show that R is an equivalence relation.

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Part I Question For Practice Part I Relations Long Answer Type Questions

1. Let. X = {1, 2, 3, 4, 5, 6, 7, 8,9}. Let R_1, be a relation on X given by R_1={(x, y): x - y is divisible by 3)} and R_2, be another relation on X given by R_2={(x, y): {x,y) $\subset$ (1,4,7) or (x, y) $\subset$ (2,5,8) or (x, y) $\subset$ (3, 6, 9)}. Show that $R_1 = R_2$.

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2. Show that the relation R on the set A = {1,2,3,4,5) given by R = {(a,b): la -bl is even)} is an equivalence relation. Also, show that all elements of (1, 3, 5) are related to each other and all the elements of (2, 4) are related to each other, but no element of (1, 3, 5) is related to any element of (2, 4).

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3. If N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b+c) = bc(a+d)$. Show that R is an equivalence relation.

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4. Show that the relation S in set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4} is an equivalence relation. Find the set of all elements related to 1.

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5. Show that the relation R on the set A of points in a plane given by R ={(P,Q): distance of the point P from the origin is same as the distance of the point Q from the origin) is an equivalence relation. Further, show that the set of all points related to a point $P \neq (0,0)$ is the circle passing through P with origin as centre.

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6. The relation 'less than' in the set of natural numbers is

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Part Ii Question For Practice Part Ii Very Short Answer Type Questions

1. If A={1,2,3) and f g are relations corresponding to the subset of A $\times$ A indicated against them, which off, g is a function? Why? f = {(1, 3), (2, 3), (3, 2)), g = {(1,2), (1, 3), (3, 1)}

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2. Let f:R $\rightarrow R$ be defined by f(x) = x^2 +1 Then, find pre-images of 17 and - 3.

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3. State whether the function f : R $\rightarrow$ R, defined by f(x) = 3 - 4x is onto or not.

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4. Find whether the function f:$Z \rightarrow Z$, defined by f(x) = $x^2 + 5, \forall x \in Z$ is one-one or not.

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5. If A = {1, 2, 3), B = {4, 5, 6, 7) and f = {(1,4),(2,5), (3,6)} is a function from A to B.b State whether f is one-one or not.

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6. Show that the function f :R $\rightarrow$ R, given by f(x) = cos x,$\forall x \in R$ is neither one-one nor onto.

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Part Ii Question For Practice Part Ii Short Answer Type Questions

1. Check the injectivity of the following function f: $R \rightarrow R$ is given by f(x) = x^3

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2. Consider a function f : $\left[0, \frac{\pi}{2}\right] \rightarrow R$ given by f(x) = sinx and g : $\left[0, \frac{\pi}{2}\right] \rightarrow R$ given by g(x) = cos x . Show that f and g are one -one but f+g is not one - one .

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3. Prove that the greatest integer function f:R $\rightarrow$ R, given by f(x) = [x] is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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4. Show that the signum function f:RR, given by f(x) = $\begin{cases} 1 & \text{if} x > 0 \\ 0 & \text{if} x = 0 \\ -1 & \text{if} x < 0 \end{cases}$ is neither one -one nor onto.

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5. Given, A = {2,3,4), B = (2, 5, 6, 7). Construct an example of each of following:

(i) An injective mapping from A to B.

(ii) A mapping from A to B, which is not injective.

(iii) A mapping from B to A.

(iv) A surjective mapping from A to B.

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6. Let f : N $\rightarrow$ N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Show that f is many one and onto function.

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7. Let A and B be sets.

Show that f : $A \times B \rightarrow B \times A$ such that f (a,b) = (b,a) is bijective function .

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8. Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x^2+1}$ is neither one-one nor onto.

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Part Ii Question For Practice Part Ii Long Answer Type Questions

1. Show that the function f : R $\rightarrow \{x \in R : -1 < x < 1\}$ defined by f (x) = $\frac{x}{1+|x|}, x \in R$ is one - one and onto function.

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2. Given a function defined by y = f(x) = $\sqrt{4-x^2} 0 \leq x \leq 2, 0 \leq y \leq 2.$

Show that f is bijective function .

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Part Iii Question For Practice Part Iii Very Short Answer Type Questions

1. If f(x) = x +7 and g(x) = x-7, x $\in$ R then find fog (6).

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2. Let f: R $\rightarrow$ R be defined by f(x) = 2x - 3 and g: R $\rightarrow$ R be defined by g(x) = $\frac{x+3}{2}$, show that fog'= I_R

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3. If F : R $\rightarrow$ R is given by f(x) = $\left(3-x^3\right)^{1/3}$ then find (fof) x .

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4. Let f: [0,1) $\rightarrow$ [0,1] be defined by

f(x) = $\begin{cases} x & \text{if x is if x is rational} \\ 1-x & \text{if x is irrational} \end{cases}$

Then find fof (x) .

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5. If f the greatest integer function and g be the modulus function. Find the value of gof $\left(\frac{-1}{3}\right)-fog\left(\frac{-1}{3}\right)$.

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Part Iii Question For Practice Part Iii Short Answer Type Questions

1. If the function'f : R $\rightarrow$ R is given by f(x) = x^2+2 and g:R $\rightarrow$ R is given by g(x) =$\frac{x}{x-1}, x \ne 1$ then find fog and gof and hence find fog (2) and gof (-3) .

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2. Let f : Z $\rightarrow Z$ be a function defined by f(n) =3n $\forall n \in Z$ and g: Z $\rightarrow Z$ be defined by

g(n) = $\begin{cases} \frac{n}{3} & \text{if n is a multiple of 3} \\ 0 & \text{if n is not a multiple of 3} \end{cases}$

Show that gof = I_z and fog $\neq I_z$.

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3. Let f : R $\rightarrow R$ be defined by f (x) = $\dfrac{x}{\sqrt{1+x^2}}$ $\forall x \in R$. Then find (fofof) (x) . Also show that fof $\neq f^2$.

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4. Let f ,g: R $\rightarrow R$ be two functions defined as f(x) = |x| +x and g(x) = |x| -x $\forall x \in R.$

Then find fog and gof .

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5. Let f : R $\rightarrow R$ be a function given by f(x) = px +q $\forall x \in R$. Find constants p and q such that fof = I_R

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Part Iv Question For Practice Part Iv Very Short Answer Type Questions

1. Consider f : (1,2,3) $\rightarrow$ {a,b,c) given by f(1) = a, f(2)=b, f(3) = c . Find f^{-1}. Show that $\left(f^{-1}\right)^{-1}$ = f.

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Part Iv Question For Practice Part Iv Short Answer Type Questions

1. Let R^{+} be the set of all positive real numbers. If f:$R^{+} \rightarrow R^{+}$ is defined as f(x)=e^{x}, $\forall x \in R^{+}$, then check whether f is invertible or not.

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2. Let f be an invertible function . Then prove that $\left(f^{-1}\right)^{-1}$ = f.

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Part Iv Question For Practice Part Iv Long Answer Type Questions

1. Let f : W $\rightarrow$ W be defined as f (x) = x -1 if x is odd and f(x) = x +1 if x is even then show that f is invertible. Find the inverse of f where W is the set of all whole numbers.

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2. If f(x) = $\frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$ then find fof and the inverse of f.

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3. Consider f : R $\rightarrow(-9, \infty)$ given by f(x) = $5x^2+6x-9$. Prove that f is invertible with

$$f^{-1}(y)=\left(\frac{\sqrt{54+5y}-3}{5}\right)$$

where R^+ is the set of all positive real numbers.

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4. Show that f: R $\rightarrow(-1, 1)$ is defined by f(x) $=\frac{10^x-10^{-x}}{10^x+10^{-x}}$ is invertible also find f^{-1}.

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Part V Question For Practice Part V Very Short Answer Type Questions

1. Let be a binary operation defined by a*b = 7a+9b. Find 3*4.

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2. If * defined on the set A ={1,2,3,4,5} by $a * b = LCM$ of a and b a binary operation ? Justify your answer.

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3. Find the number of binary operations on the set {a,b}.

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4. Is the binary operation defined on Z . (set of integers) by m* n=m-n+mn, ∀ m,n ∈ z commutative?

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5. Let *be a binary operation defined on R, the set of all real numbers, by a*b = $\sqrt{a^2 + b^2}, \forall a, b \in R.$ Show that is associative on R

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6. Let * be a binary operation on the set S of all non-negative real numbers defined by $a * b = \sqrt{a^2 + b^2}$. Find the identity elements in S with respect to *.

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Part V Question For Practice Part V Short Answer Type Questions

1. (i) Is the binary operation defined on set R, given by $a \cdot b = \frac{a+b}{2} \forall a, b \in R$ commutative?

(ii) Is the above binary operation associative?

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2. Let * be a binary operation on Q defined by $a * b = ab + 1$.

Determine whether * is commutative but not associative.

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3. Let* be a binary operation on R - (-1) defined by a*b= $\frac{a}{b+1}$ for all a, b $\in$ R-{-1} is

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4. Give an example of a binary operation

(i) which is commutative as well as associative.

(ii) which is neither commutative nor associative.

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5. If S is the set of all rational numbers except 1 and * be defined on S by $a * b = a + b - ab$, for all $a, b \in S$.

Prove that

(i) * is a binary operation on S.

(ii) * is commutative as well as associative.

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6. Determine the total number of binary operations on the set S = (1, 2) having 1 as identity element.

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7. Let S =Q $\times$ Q and let * be a binary operation on S defined by (a, b)* (c,d) = (ac, b+ad) for (a, b), (c,d) $\in$ S. Find the identity element in S and the invertible elements of S.

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Part V Question For Practice Part V Long Answer Type Questions

1. Let * be the binary operation on N given by a*b = LCM of a and b. Find

(i) 5*7,20*16

(ii) Is* commutative?

(iii) Is * associative?

(iv) Find the identity of* in N.

(v) Which elements of N are invertible for the operation ?

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2. Let * be the binary operation on N defined by a* b = HCF of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

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3. Let A=R $\times$ R and be the binary operation on A defined by (a, b) * (c,d) = (a + c, b + d). Show that *is commutative and associative. Find the identity element for* on A, if any.

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4. Let A=N $\times$ N and let* be a binary operation on A defined by (a, b) * (c,d) = (ad + bc, bd), $\forall$ (a, b), (c,d) $\in$ N $\times$ N. Show that

(i) * is commutative.

(ii) * is associative.

(iii) A has no identity element.

5. A binary operation * is defined on the set

$X = R - \{-1\}$ by $x * y = x + y + xy, \forall x, y \in X$.

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

6. Let Abe set of all cube roots of unity and let multiplication operation (x) be a binary operation on A. Construct the composition table for (x),on A. Also, find the identity element for (x) on A. Also, check its commutativity and prove that every element of A is invertible.

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Odisha Bureau S Textbook Solutions Exercise 1 A

1. If A-{a,b,c,d) mention the type of relations on A given below, which of them are equivalence relations?

{(a, a),(b,b)}

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2. If A-{a,b,c,d) mention the type of relations on A given below, which of them are equivalence relations?

{(a, a),(b,b), (c,c), (d, d)}

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3. If A-{a,b,c,d) mention the type of relations on A given below, which of them are equivalence relations?

{(a,b), (b, a),(b,d),(d, b)}

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4. If A={a,b,c,d} mention the type of relations on A given below, which of them are equivalence relations?$\{(b, c), (b, d), (c, d)\}$

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5. If A={a,b,c,d} mention the type of relations on A given below, which of them are equivalence relations?

$\{(a, a), b, b), (c, c), (d, d), (a, d), (a, c), (d, a), (c, a), (c, d), (d, c)\}$

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6. Write relations in tabular form and determine their type for

$R = \{(x, y) : 2x - y = 0\} on A = \{1, 2, 3, \ldots, 13\}$

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7. Write relations in tabular form and determine their type for

$R = \{(x, y) : x \text{divides} y\} on A = \{1, 2, 3, 4, 5, 6\}$

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8. Write relations in tabular form and determine their type for

$R = \{(x, y) : x \text{divides} 2 - y\} \text{on} A = \{1, 2, 3, 4, 5\}$

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9. Write the relations in tabular form and determine their type.

R= {(x, y): y $\leq$ x $\leq$ 4} on A = {1,2,3,4,5)

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10. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n):m-n $\geq$ 7) on Z.

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11. Test wheter relations are reflexive, symmetric or transitive on the sets specified for $R = \{(m, n\} : 2 \mid (m + n)\} \text{on} Z.$

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12. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R= {(m,n): m +n is not divisible by 3) on Z.

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13. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R={(m,n): $\frac{m}{n}$ is a power of 5} on Z - {0}.

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14. Test whether the relations are reflexive, symmetric or transitive on the sets specified.

R=(m,n}: mn is divisible by 2) on Z.

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15. Test wheter relations are reflexive, symmetric or transitive on the sets specified for $R = \{(m, n) : 3 \text{divides} m - n\} on \{1, 2, 3. \ldots, 10\}$.

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16. List the members of the equivalence relation defined by $\{\{1\}, \{2\}, \{3, 4\}\}$ partitins on X={1,2,3,4}.Also find the equivalence classes of 1,2,3 and 4.

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17. List the members of the equivalence relation defined by $\{\{1, 2, 3\}, \{4\}\}$ partitins on X={1,2,3,4}.Also find the equivalence classes of 1,2,3 and 4.

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18. List the members of the equivalence relation defined by $\{\{1, 2, 3, 4\}\}$ partitins on X={1,2,3,4}.Also find the equivalence classes of 1,2,3 and 4.

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19. Show that if R is an equivalence relation on X, then Dom R = Rng R = X.

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20. Given an example of a relation which is

reflexive, symmetric but not transitive.

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21. Given an example of a relation which is

reflexive, transitive but not symmetric.

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22. Given an example of a relation which is

symmetric, transitive but not reflexive.

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23. Given an example of a relation which is

reflexive but neither symmetric nor transitive.

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24. Given an example of a relation which is

transitive but neither reflexive nor symmetric.

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25. Given an example of a relation which is

an empty relation.

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26. Given an example of a relation which is a universal relation.

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27. Let R be a relation on X, If R is symmetric then $xRy \Rightarrow yRx$. If it is also transitive then xRy and $yRx \Rightarrow xRx$.So whenver a relation is symmetric and transitive then it is also reflexive. What is wrong in this argument ?

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28. Suppose a box contains a set of n balls $(n > 4)$(denoted by B)of four different colours (many have different sizes), viz,red, blue, green and yellow. Show that a relation R defined on B as $R = \{(b_1, b_2) : \text{balls} b_1 \text{and} b_2$ have the same colour} is an equivalence relation on B. How many equivalence classes can you find with respect ot R ?

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29. Find the number of equivalence, relations on X ={1,2,3),

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30. Let R be the relation on the set R of real numbers such that aRb iff a-b is and integer. Test whether R is an equivalence relation. If so find the equivalence class of $1 \text{and} \frac{1}{2}$ wrt. This equivalence relation.

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31. Find the least positive integer r such that $185 \in [r]_7$

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32. Find the least positive integer r such that $-375 \in [r]_{11}$

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33. Find the least positive integer r such that $-12 \in [r]_{13}$

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34. Find least non negative integer r such that $7 \times 13 \times 23 \times 413 \equiv r (\bmod 11)$

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35. Find least non negative integer r such that

$6 \times 18 \times 27 \times (-225) \equiv r \pmod 8$

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36. Find least non negative integer r such that

$1237 \pmod 4 + 985 \pmod 4 \equiv r \pmod 4$

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37. Find least non negative integer r such that

$1936 \times 8789 \equiv r (\text{mod} 4)$

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38. Find least positive integer x, satisfying

$276x + 128 = 4$ (mod 7).

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39. Find three positive integers

$x_i, i = 1, 1, 3 \text{satisfying} 3x \equiv 2 (\bmod 7)$

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Odisha Bureau S Textbook Solutions Exercise 1 B

1. Let X ={x,y} and Y ={u,v}. Write down all the functions that can be defined from X To Y. How many of these are (i) one-one (ii) onto and (iii)one-one and onto ?

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2. Let X and Y be sets containing m and n elements respectively.

(i) What is the total number of functions from X to Y.

(ii) How many functions from X to Y are one-one according as $m < n, m > n$and m=n?

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3. Examine $f: R \to R, f(x) = x^2$functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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4. Examine $f: R \to [-1, 1], f(x) = \sin x$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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5. Examine $f: R_{+} \to R+, f(x) = x + \frac{1}{x}$ where $R_{+} = \{x \in R : x > 0\}$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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6. Examine $f: R \to R, f(x) = x^3 + 1$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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7. Examine $f: (-1, 1) \to R, f(x) = \frac{x}{1 - x^2}$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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8. Examine $f: R \to R, f(x) = [x] = \text{the greatest integer} \leq x.$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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9. Examine $f: R \to R, f(x) = |x|$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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10. Examine $f: R \to R, f(x) = \text{sgn}x$ functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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11. Examine $f: R \rightarrow R, f = id_R =$ the identity function or R. functions if it is (i) injective (ii) surjective, (iii) bijective and (iv) none of the three.

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12. Show that the functions are injective

f(x) = sinx on $\left[0, \frac{\pi}{2}\right]$

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13. Show that $f(x) = \cos x[0, \pi]$ functions are injective.

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14. Show that $f(x) = \log_a x \text{on}(0, \infty), (a > 0 \text{and} a \neq 1)$ functions are injective.

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15. Show that the function $f(x) = a^x, x \in R$ is injective, where $(a > 0 \text{ and } a \neq 1)$.

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16. Show that functions f and g defined by f(x)=2 log x and g(x) $= \log x^2$ are not equal even though log `x^2 =2 log x.

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17. Give an example of a function which is Surjective but not injective.

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18. Prove that the following sets are equivalent

{1, 2, 3, 4, 5, 6.....}, (2, 4, 6, 8, 10,...)

(1,3,5, 7, 9,...), {1, 4, 9, 16, 25...)

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19. Let f={(1,a),(2,b),(3,c),(4,d)} and g={(a,x),(b,x),(c,y),(d,x)} Determine gof and fog if possible. Test whether fog=gof.

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20. Let f= {(1,3),(2,4),(3,7)} and g ={(3,2),(4,3),(7,1)}

Determine gof and fog if possible . Test whether fog =gof.

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21. Let $f(x) = \sqrt{x} \text{and} g(x) = 1 - x^2$. Find natural domains of f and g.

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22. Let $f(x) = \sqrt{x} \text{and} g(x) = 1 - x^2$. Compute fog and gof and find their natural domains.

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23. Let $f(x) = \sqrt{x}$and$g(x) = 1 - x^2$. Find natural domain of h(x) = 1-x.

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24. Let f(x) = $\sqrt{x}$ and g(x) = $1 - x^2$.

Show that hgof only on $R_0 = \{x \in R : x \geq 0\}$ and not on R..

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25. Find the composition fog and gof and test whether fog = gof when f and g are functions or R given by $f(x) = x^3 + 1, g(x) = x^2 - 2$

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26. Find the composition fog and gof and test whether fog = gof when f and g are functions or R given by $f(x) = \sin x, g(x), g(x) = x^5$

Watch Video Solution

27. Find the composition fog and gof and test whether fog = gof when f and g are functions or R given by $f(x) = \cos x, g(x) = \sin x^2$

Watch Video Solution

28. Find the composition fog and gof and test whether fog = gof when f and g are functions or R given by $f(x) = g(x) = \left(1 - x^3\right)^{\frac{1}{3}}$

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29. Let f be a real function. Show that h(x) =f(x)=f(-x) is always an even function and g(x) = f(x) -f(-x) is always an odd fuction.

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30. Express each of e^x function as the sum of an even function and an odd function.

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31. Let $X = \{1, 2, 3, 4\}$Determine whether f:$X \rightarrow X$defined as given below have inverses. Find f^{-1} if it exist $f = \{(1, 4,), (2, 3), (3, 2), (4, 1)\}$

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32. Let $X = \{1, 2, 3, 4\}$Determine whether f:$X \rightarrow X$defined as given below have inverses. Find f^{-1} if it exist $f = \{(1, 3)(2, 1), (3, 1), (4, 2)\}$

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33. Let $X = \{1, 2, 3, 4\}$Determine whether f:$X \rightarrow X$defined as given below have inverses. Find f^{-1} if it exist $f = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$

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34. Let X $= \{1, 2, 3, 4\}$Determine whether f:$X \rightarrow X$defined as given below have inverses. Find f^{-1} if it exist $f = \{(1, 1), (2, 2), (2, 3), (4, 4)\}$

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35. Let X ={1,2,3,4}Determine whether f:$X \rightarrow X$defined as given below have inverses. Find f^{-1} if it exist

$f = \{(1, 2), (2, 2), (3, 2), (4, 2)\}$

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36. Let f: $X \rightarrow Y$

If there exists a map g:Y $\rightarrow$ X such that gof = id_X and fog = id_y,

then show that

f is bijective and (ii) $g = f^{-1}$

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37. Construct an example to show that $f(A \cap B) \neq f(A) \cap f(B)$ where $A \cap B \neq \theta$.

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38. Prove that for any $f: X \rightarrow Y$, $foid_x = f = id_Y$ of.

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Odisha Bureau S Textbook Solutions Exercise 1 C

1. Show that the operation * given by x*y=x+y+ -xy is a binary oeration on Z,Q and R but not on N.

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2. Determine whether$a * b = 2a + 3b\text{on}Z$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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3. Determine whether $a * b = ma - nb\text{on}Q + \text{where}m\text{and}n \in N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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4. Determine whether $a * b = a + b(\text{mod}7)\text{on}\{0, 1, 2, 3, 4, 5, 6\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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5. Determine whether $a * b = \min\{a, b\} \text{on} N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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6. Determine whether $a * b = \text{GCD}\{a, b\} \text{on} N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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7. Determine whether $a * b = \text{LCM}\{a, b\} \text{on} N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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8. Determine whether $a * b = \text{LCM}\{a, b\}\text{on}\{0, 1, 2, 3, 4. \ldots . , 10\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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9. Determine whether $a * b = \sqrt{a^2 + b^2} on Q_+$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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10. Determine whether $a * b = a \times b(\text{mod}5)\text{on}\{0, 1, 2, 3, 4\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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11. Determine whether $a * b = a^2 + b^2 \text{on} N$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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12. Determine whether $a * b = a + b - ab \text{on} R - \{1\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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13. (Z,*) where a* b = a+b-ab for all $a, b \in Z$ prove that the given binary operation * is associative and commutative.

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14. Constract the composition table/multiplication table for the binary operation * defined on {0,1,2,3,4}by $a * b = a \times b(\text{mod} = 5)$. Find the identity element if any. Also find the inverse elements of 2 and 4.

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Chapter Practice Very Short Answer Type Questions

1. Let A = {1,2,3,4,...., 15,16} and let R be a relation in A given by $R = \{(a, b)\}: b = a^2\}$, then find domain and range of relation R.

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2. If the relation R is defined on the set A={1,2,3,4,5} by R={a,b} : $|a^2 - b^2| < 8$. Then, find the relation R.

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3. State the reason for the relation R in the set {1, 2, 3} given by R ={(1, 2), (2, 1)} not to be transitive.

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4. Show that the function f: R $\rightarrow$ R , f(x)=x^4 is many-one and into.

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5. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$. Then, find gof(x).

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6. If f: R $\rightarrow$ R is defined by f(x) = 3x + 2 define f (f (x))

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7. Write fog if f:R $\rightarrow$ R and g: $R \rightarrow R$ is given by f(x) = | x| and g(x) = |5x - 2|.

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8. Find gof and fog, if f: $R \rightarrow R$ and g: R $\rightarrow$ R are given by f(x) = cos x and g(x) = $3x^2$, then show that gof $\neq$ fog.

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9. If A= {a,b,c,d) and the function f = {(a,b),(b,d), (c,a), (d,c)}, then write f^{-1}.

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10. If f is an invertible function, defined as f(x)= $\frac{3x-4}{5}$ then write f^{-1}(x).

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11. Let* is a binary operation on the set of all non-zero real numbers, given by a*b =$\frac{ab}{5}$, $\forall a, b \in R - \{0\}$. Find the value of x, given that 2*(x*5) = 10.

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12. If the binary operation on the set of integers Z, defined by a b= $a + 3b^2$, then find the value of 8*3.

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13. Show that * On Z^+ defined by a*b = |a-b| is a binary operation or not.

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14. Show that * On R defined by a*b = a +4b^2 is a binary operation or not.

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15. For each binary operation defined below, determine whether * is the commutative or not.

On R defined a *b = a + $4b^2$.

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16. For each binary operation defined below, determine whether * is the commutative or not.

On N defined a*b =a+b +2 .

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17. Prove that for binary operation defined on R such that ab=a+ $4b^2$ is not associative

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18. For each binary operation * defined below, determine the identity element.

On Q -{1} such that a*b= a +b -ab .

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19. For each binary operation * defined below, determine the identity element.

On Q^+ , all positive rational numbers defined by a*b= $\frac{ab}{2}$.

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Chapter Practice Short Answer Type Questions

1. Let N be the set of all positive integers and R be a relation on N, defined by R = {(a,b): a is a factor of b), show that R is reflexive and transitive.

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2. If A = {1,2,3,4} define relations on A which have properties of being

(i) reflexive, transitive but not symmetric,

(ii) symmetric but neither reflexive nor transitive.

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3. Let n be a fixed positive integer. Define a relation R on Z as follows for all a, b $\in$ Z, aRb, if and only if a-bis divisible by n. Show that R is an equivalence relation.

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4. If $f: X \rightarrow Y$ is a function. Define a relation R on X given by R= {(a, b): f(a)=f(b)}. Show that R is an equivalence relation on X.

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5. Let CandR denote the set of all complex numbers and all real numbers, respectively. Then, show that f: C $\rightarrow$ R is given by f(z)=|z|, $\forall z \in C$, is neither one-one nor onto.

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6. If $f: R \rightarrow R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection.

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7. If A = R -{3} and B = R -{1}. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$, for all $x \in A$. Then, show that f is bijective. Find $f^{-1}(x)$.

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8. Consider f:R_+ $[4, \infty]$ is given by f(x)= $x^2 + 4$. Show that f is invertible with the inverses f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R_+, is the set of all non-negative real numbers.

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9. Consider the binary operation on the set {6, 7, 8, 9, 10), defined by a*b = min (a, b}. Write operation table of operation *.

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Chapter Practice Long Answer Type Questions

1. Show that the relation R, defined in the set of A all triangles as R = $\{(T_1, T_2) : T_1,$ is similar to $T_2\}$, is equivalence relation. Consider three right angle triangles T_1, with sides 3, 4, 5, T_2, with sides 5, 12, 13 and T_3 with sides 6, 8, 10, which triangle among T_1, T_2, and T_3 are related?

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2. Let A=(-1,1), then discuss whether the functions defined on A are one-one and onto or bijective.

$f(x) = x/2$

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3. Let A=(-1,1), then discuss whether the functions defined on A are one-one and onto or bijective.

$g(x) = |x|$

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4. Let A=(-1,1), then discuss whether the functions defined on A are one-one and onto or bijective.

$h(x) = x|x|$

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5. Let A=(-1,1), then discuss whether the functions defined on A are one-one and onto or bijective.

k(x) =x^2

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6. Show that the function f:N $\rightarrow$ N, given by f(n)=n-$(-1)^n, \forall n \in N$ is a bijection.

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7. Using the definition, prove that the function f: A $\rightarrow$ B is invertible, if and only if f is both one-one and onto.

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8. Let A $= \left\{x : x \in R, \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$ and B = $(y : y \in R, \ -1 \leq y \leq 1\}$. Show that the function f : A $\rightarrow$ B given by f(x) = sin x is invertible andc hence find f^{-1} .

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9. Let* be the binary operation defined on Q. Which of the binary operations are commutative?

a*b=a-b, $\forall a, b \in Q$

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10. Let* be the binary operation defined on Q. Which of the binary operations are commutative?

$a \cdot b = a^2 + b^2, \ \forall, a, b \in Q$

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11. Let* be the binary operation defined on Q. Which of the binary operations are commutative?

a*b=a +ab , $\forall a, b, \in Q$

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12. Let* be the binary operation defined on Q. Which of the binary operations are commutative?

a*b=$(a-b)^2, \forall a, b \in Q$

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13. Let* be a binary operation on Q, defined by a*b=$\frac{3ab}{5}$ .Show that is commutative as well as associative. Also, find its identity, if it exists.

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14. Given a non-empty set X, Let *: $P(x) \times P(x)$ be defined as $A * B = (A - B) \cup (B - A), \forall A, B \in P(x)$. Show that the empty set ϕ is the identity for the operation * and all the elements A of p(x) are invertible with $A^{-1} = A$.

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