MATHS

BOOKS - ARIHANT PUBLICATION

SAMPLE PAPER 2

Very Short Answer Type Questions

1. Answer all the questions

Given a square matrix A of order 3×3, such that $|A| = 12$, find the value of $|A. AdjA|$.

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2. Answer all the questions

If A and B are two non-zero vectors such that $|A \times B| = A. B$, then the angle between A and B.

3. If F : R $\rightarrow$ R is given by f(x) = $\left(3 - x^3\right)^{1/3}$ then find (fof) x .

4. If $\tan^{-1}x + \tan^{-1}y = \dfrac{\pi}{4}, xy < 1,$

then write the value of $x + y + xy$.

5. Show that the function $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$

is discontinuous at $x = 0$.

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6. Answer all the questions

Find the slope at the tangent to the curve

$x = 3t^2 + 1, y = t^3 - 1$ at $x = 1$.

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7. Answer all the questions

Evaluate $\int_0^1 [2x] dx$ (where $[.]$ is greatest integer function).

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8. Answer all the questions

Write the sum of the order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + x^4 = 0$

9. Answer all the questions

Find the angle between the planes $r.\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$ and $r.\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0$.

10. Answer all the questions

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \dfrac{5}{13}$ and $P\left(\dfrac{A}{B}\right) = \dfrac{2}{5}$.

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Short Answer Type Questions

1. An animal feed company must produce $200kg$ of a mixture consisting of ingredients A and B. The-ingredient A costs Rs.3 per kg and B costs 5 per kg. No more than 80 kg of A can be uséd and at least 60 kg of B must be used. Formulate the problem to minimise thé cost of mixture.

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2. Answer any three questions

Prove that: $2\tan^{-1}\left(\dfrac{1}{5}\right) - \tan^{-1}\left(\dfrac{1}{4}\right) = \tan^{-1}\left(\dfrac{8}{53}\right)$

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3.

Let $f : R \to R$ be defined by $f(x) = 3x + 5$ Show that f is bijective.∴

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4.

Solve: $\cos^{-1}x + \sin^{-1}\left(\dfrac{1}{5}\right) = \dfrac{\pi}{2}$.

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5. Check whether the relation R defined in the set A = {1,2,3,...,13,14 as R = {(x,y):3x - y =0} is reflexive, symmetric and transitive.

6. Find $\dfrac{dy}{dx}$ if $y = \sin^{-1}\left(\dfrac{1 - x^2}{1 + x^2}\right)$

$0 < x < 1$

7. Answer any three questions

Using properties of determinants, prove the following

$$\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = \left(1 + a^2 + b^2\right)^3.$$

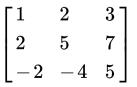**8.** Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

9. Using elementary transformation , find the inverse of the following matrices .

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & 5 \end{bmatrix}$$

10. Answer any three questions

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $f(x) = x^2 + x - 1$, then find $f(A)$.

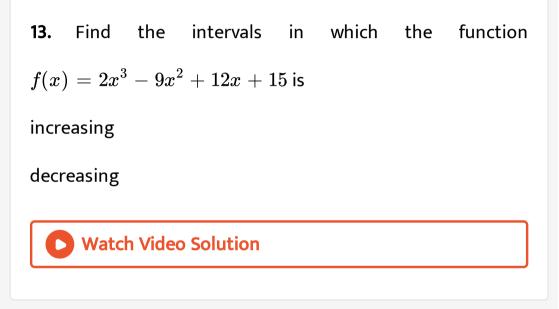▶ **Watch Video Solution**

11. Answer any three questions

If $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, then find $\dfrac{d^2y}{dx^2}$.

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12. Find the value of k in the following function f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{1 - \cos kx}{x \sin x}, & \text{if } x \neq 0 \\ \frac{1}{2}, & \text{if } x = 0 \end{cases} \text{ at } x = 0.$$

13. Find the intervals in which the function

$f(x) = 2x^3 - 9x^2 + 12x + 15$ is

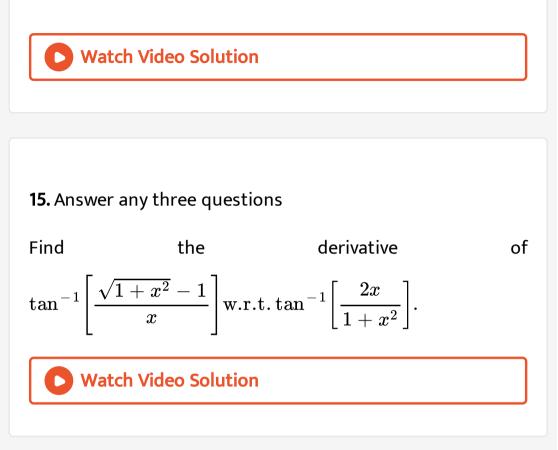increasing

decreasing

14. Answer any three questions

Use Lagrange's mean value theorem to determine a point P

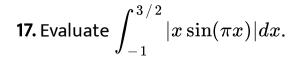on the curve $f(x) = \sqrt{x - 2}$ defined in the interval $[2, 3]$,

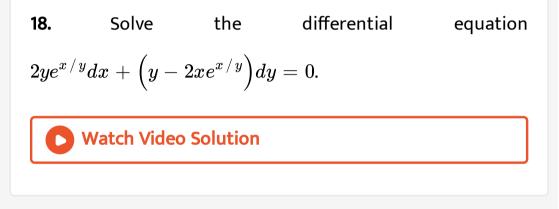where the tangent is parallel to the chord joining the end point on the curve.

15. Answer any three questions
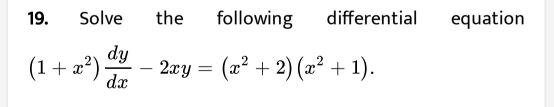
Find the derivative of $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$ w.r.t. $\tan^{-1}\left[\dfrac{2x}{1+x^2}\right]$.

16. Answer any three questions

Evaluate $\displaystyle\int e^x\left(\dfrac{1+\sin x}{1+\cos x}\right)dx$.

17. Evaluate $\displaystyle\int_{-1}^{3/2} |x\sin(\pi x)|\,dx.$

18. Solve the differential equation $2ye^{x/y}dx + \left(y - 2xe^{x/y}\right)dy = 0.$

19. Solve the following differential equation $\left(1 + x^2\right)\dfrac{dy}{dx} - 2xy = \left(x^2 + 2\right)\left(x^2 + 1\right).$

20. Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x + 3| dx$.

21. Answer any three questions

if $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a} + \hat{b} + \hat{c} = 0$, then find the value of $\hat{a}.\,\hat{b} + \hat{b}.\,\hat{c} + \hat{c}.\,\hat{a}$.

22. Answer any three questions

Find the coordinates of the point, where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY-plane.

23. Answer any three questions

If $a = \hat{i} + 2\hat{j} + 3\hat{k}$ and $b = 2\hat{i} + 4\hat{j} - 5\hat{k}$ represent two adjacent sides of a parallelogram, then find unit vectors parallel to the diagonals of the parallelogram.

24. Answer any three questions

If the vertices A, B and C of $\triangle ABC$ are $(1, 2, 3), (-1, 0, 0)$ and $(0, 1, 2)$ respectively, then find $\angle ABC$.

25.

Find the value of p, so that the lines $\dfrac{1-x}{3} = \dfrac{7y-14}{2p} = \dfrac{z-3}{2}$ and $\dfrac{7-7x}{3p} = \dfrac{y-5}{1} = \dfrac{6-z}{5}$ are at right angle.

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Long Answer Type Questions

1. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$ and $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, then find the area of the parallelogram having diagonals $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{b} + \vec{c}\right)$.

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2.

Find the image of the line $\dfrac{x-1}{3} = \dfrac{y-3}{1} = \dfrac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$.

3. Answer any one question

A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is almost half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by almost 600 units, It the company makes profit of Rs. 12 and Rs. 16 per doll, respectively on dolls A and B,

how many of each should be produced weekly in order to maximize the profit.

4.

Let $A = N \times N$ and $*$ be the binary operation on A defined by $a * b = (a + b)$

Show that $*$ is commutative and associative.

5.

Solve the equation $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right] = 0$.

6.
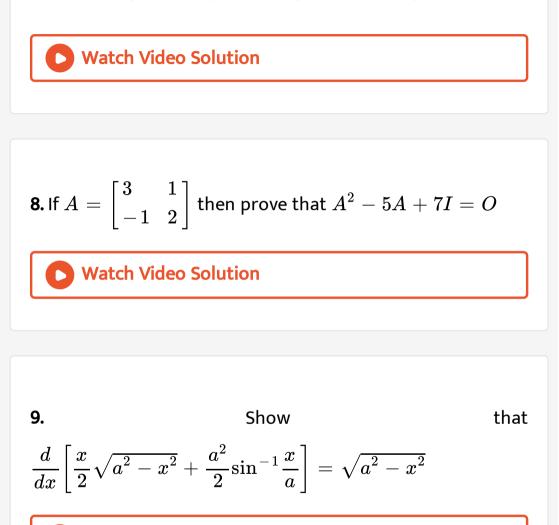
Let $f: R \to R$ be defined by $f(x) = 2x + 7$ Show that f is bijective.∴.

7. Answer any one question :

Suppose that, the reliability of a HIV test is specified as follows of people having HIV 90% of the test detect the disease but 10% go undetected of people free of HIV, 99% of the test are judged HIV negative but 1% are diagnosed as showing HIV positive. From a large population of which only 0.1% have HIV, one person is selected at random given the

HIV test and the pathologist reports him/her as HIV positive.

What is the probability that the person actually has HIV?

8. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = O$

9. Show that

$$\frac{d}{dx}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} \right] = \sqrt{a^2 - x^2}$$

10.

A window is in the form of a rectangle above in which there is a semi-circle. If the perimeter of the window is P cm. Show that the window will allow the maximum possible light only when the radius of the semi-circle is $\dfrac{P}{\pi + 4}$ cm.

11.

Let $f: R \to R$ be defined by $f(x) = 3x$ Show that f is bijective.∴

12. Answer the question :

Evaluate $\int (2x^3 + 3x + 5)\, dx$.

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13.

Solve the following differential equation

$\left(1 + e^{x/y}\right) dx + e^{x/y}\left(1 - \dfrac{x}{y}\right) dy = 0.$

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