MATHS

BOOKS - ARIHANT PUBLICATION

SAMPLE PAPER 4

Very Short Answer Type Questions

1. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then find the value of $\cos^{-1} x + \cos^{-1} y$.

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2. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

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3. Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$.

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4. If the binary operation $*$ on the set Q of rational numbers is defined as $a * b = 2a + b - ab$, for all $a, b \in Q$, find the value of $3 * 4$.

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5. $f(x) = \int_0^x t \sin t dt$ then $f'(x) =$ _ _

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6. If α, β, γ are the angles that a line makes with X, Y and Z-axes respectively, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

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7. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ and find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

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8. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

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9. Write an integrating factor of the differential equation $\left(1+y^2\right)dx = \left(\tan^{-1} y - x\right)dy.$

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10. Find the point on the curve $y = x^2$, where the rate of change of x-coordinate is equal to the rate of change of y-coordinate.

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Short Answer Type Questions

1. Solve graphically: Maximize Z=5x+6y subject to $2x + 3y \leq 6, x, y \geq 0$

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2. Directions (Q. Nos. 16-25) Prove the following

$$\sin^{-1}\left(\frac{5}{13}\right)+\cos^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{63}{16}\right).$$

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3. If functions f and g are given by

$f=\{(1,2),(3,5),(4,6)\}$ and

$g=\{(2,3),(5,1),(6,3)\}$. write down the functions fog and gof.

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4. Show that the relation R in the set of real numbers, defined as $R=\left\{(a,b):a\leq b^2\right\}$ is neither reflexive nor

symmetric nor transitive.

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5. Solve

$$\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\sin^{-1}1.$$

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6. Using properties of determinants, prove the following

$$\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2.$$

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7. Solve the matrix $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O.$

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8. A coin is tossed once, if it shows head it is tossed again. but if it shows a tail, a die is tossed. If 8 possible outcomes are equally likely, find the probability that the die shows a number greater than 4, if it is known that first throw of the coin results in tail.

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9. If A = $\begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 1 & 3 \end{bmatrix}$, then verify that A (adj A) = $|A|$.

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10. If A and B are two independent events such that $P(\overline{A} \cap B) = \frac{2}{15}$ and $P(A \cap \overline{B}) = \frac{1}{6}$ then find P(A) and P(B)

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11. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

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12. Show that the function

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, \text{ if } & x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$.

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13. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$.

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14. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at the right angles, if $c^4 = 32a^4$.

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15. Using differential find the approximate value of $\sqrt{0.0037}$.

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16. Solve the differential equation $\left(x^2 - y^2\right)dx + 2xydy = 0$.

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17. Find the area of region included between the parabola $4y = 3x^2$ and the line $3x - 2y + 12 = 0$.

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18. Evaluate $\int_0^{\pi/3} \frac{\sec x . \tan x}{1+\sec^2 x} dx.$

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19. Find $\int \frac{dx}{(x-1)(x+1)}$

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20. Form the differential equation representing the family of ellipses having foci on the X-axis and centre at the origin.

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21. If $a \times b = c \times d$ and $a \times c = b \times d$, show that $(a - d)$ is parallel to $(b - c)$, it being given that $a \neq b$ and $b \neq c$.

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22. Show that the line $\frac{5 - x}{-4} = \frac{y - 7}{4} = \frac{z + 3}{-5}$ and $\frac{x - 8}{7} = \frac{2y - 8}{2} = \frac{z - 5}{3}$ are coplanar.

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23. Find the vector equation of the plane through the points (2,1, -1) and (-1, 3, 4) and .. perpendicular to the plane $x - 2y + 4z = 10$.

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24. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

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25. If $\vec{\alpha} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 4\hat{k}$, then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

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Long Answer Type Questions

1. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$.

Interpret this result geometrically.

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2. Find the shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$.

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3. If a young man rides his motorcycle at 25 km/h, he had to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increases to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel in one hour. Express this as LPP and solve it graphically.

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4. Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}$

.

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5. Show that the function f : R $\rightarrow \{x \in R: -1 < x < 1\}$ defined by f (x) = $\frac{x}{1+|x|}, x \in R$ is one - one and onto function.

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6. Given $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Verify that BA = 6I.

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7. Obtain the inverse of the following matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

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8. Find the probability distribution of number of doublets in four throws of a pair of dice. Find also the mean and the variance of the number of doublets.

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9. An open box with a square base is to be made of a given quantity of metal sheet of area c^2. Show that the

maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cu units.

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10. If $f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \frac{\pi}{4}$. Then, find value of $f\left(\frac{\pi}{4}\right)$, so that f(x) becomes continuous at $x = \frac{\pi}{4}$.

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11. Evaluate $\int_1^3 \left(3x^2 + 2x\right) dx.$

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12. Evaluate

$$\int \frac{\sin^2 x.\cos^2 x}{\left(\sin^5 x+\cos^3 x.\sin^2 x+\sin^3 x.\cos^2 x+\cos^5 x\right)^2}dx.$$

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13. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

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