MATHS

BOOKS - ARIHANT PUBLICATION

SAMPLE PAPER 5

Very Short Answer Type Question

1. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, find the value of x.

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2. Find the value of $\sin\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$.

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3. Let $f: R \to R$ be the function defined by $f(x) = \frac{1}{2 - \cos x} \forall x \in R.$

Then, find the range of f.

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4. Let $f(x) = x|x|, \forall x \in R.$ Discuss the derivability of $f(x)$ at $x = 0.$

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5. Find the order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}.$

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6. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

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7. The x-coordinate of a point on the line joining the points P(2,2,1) and Q(5,1,−2) is 4. Find its z-coordinate.

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8. Evaluate $\int \frac{dx}{\sin^2 x \cdot \cos^2 x}$

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9. If the area of circle increasing at a uniform rate, then prove that perimetre varies inversely as the radius.

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10. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3, then evaluate

$P(\overline{A}) + P(\overline{B})$.

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Short Answer Type Question

1. Directions (Q. Nos. 16-25) Prove the following

$$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right).$$

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2. Show that the relation R on the set A = {1,2,3,4,5) given by R = {(a,b): la - bl is even)} is an equivalence relation. Also, show that all elements of (1, 3, 5) are related to each other and all the elements of (2, 4) are related to each other, but no element of (1, 3, 5) is related to any element of (2, 4).

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3. Evaluate $\int (x^2 + 2x)\,dx$.

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4. Show that the function $f: R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$ is neither one-one nor onto.

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5. Solve the equation $\sin^{-1} 6x + 2\sin^{-1} 6\sqrt{3}x = \frac{-\pi}{2}$.

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6. Find the inverse of the following matrix $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

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7. A bag contains 5 red marbles and 3 black marbles. Three , marbles are drawn one-by one without replacement. What is the probability that at least one of the three marbles drawn be black if the first marble is red ?

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8. Using properties of determinants prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

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9. A and B throw a pair of dice alternately. A wins the game, if he gets a total of 6 and B wins, if she gets a total of 7. If A starts the game, then find the probability of winning the game by A in third throw of the pair of dice.

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10. If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, find a matrix C such that $3A + 5B + 2C$ is a null matrix.

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11. Find the derivative of $\cos^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right), \frac{-\pi}{4} < x < \frac{\pi}{4}$.

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12. If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

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13. Find the condition that curves $2x = y^2$ and $2xy = k$ intersect orthogonally.

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14. Evaluate $\int_1^3 (3x^2 + 2x + 1)dx$.

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15. Examine the differentiability of the function f defined by

$$f(x) = \begin{cases} 2x + 3, \text{ if } -3 \leq x < -2 \\ x + 1, \text{ if } -2 \leq x < 0 \\ x + 2, \text{ if } 0 \leq x \leq 1 \end{cases}$$

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16. Evaluate $\int \sqrt{\frac{1+x}{1-x}} dx, x \neq 1$

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17. Solve the following differential equation

$$x^2 . \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0.$$

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18. Find the area of region enclosed by the curve $x = 3\cos t, y = 2\sin t.$

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19. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x dx}{\sin x + \cos x}$

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20. Form the differential equation having $y = \left(\sin^{-1} x\right)^2 + A\cos^{-1} x + B,$ where A and B are arbitrary constants, as its general solution.

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21. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j} + 4\hat{k}$ represents the two sides AB and AC respectively of a ΔABC, find the length of median through A.

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22. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} . \vec{c} = 18$.

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23. Find the co-ordinates of the point where the line joining $(3, 4, -5)$ and $(2, -3, 1)$ meets the plane $2x + y + z - 7 = 0$.

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24. The vectors from origin to the points A and B are $A = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $B = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively. Find the area of ΔOAB.

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25. For any vector a , show that

$a = \left(a \cdot \hat{i}\right)\hat{i} + \left(a \cdot \hat{j}\right)\hat{j} + \left(a \cdot \hat{k}\right)\hat{k}.$

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Long Answer Type Question

1. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{k}$. Find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3.$

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2. Find the foot of perpendicular from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}.$

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3. If $\cos^{-1}\left(\frac{x}{y}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$, prove that $9x^2 - 12xy\cos\theta + 4y^2 = 36\sin^2\theta$.

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4. Find graphically the maximum value of $Z = 2x + 5y$, subject to constraints given below $2x + 4y \leq 8, 3x + y \leq 6, y \leq 4, x \geq 0, y \geq 0$

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5. If the function'f : R $\rightarrow$ R is given by f(x) = $x^2 + 2$ and g:R $\rightarrow$ R is given by g(x) = $\frac{x}{x-1}$, $x \neq 1$ then find fog and gof and hence find fog (2) and gof (-3) .

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6. Evaluate the following determinant $\begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$.

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7. Three cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of spades. Also, find the mean of the distribution.

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8. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 6I$.

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9. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha + b^2\sin^2\alpha = p^2$.

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10. If $y = x \log\left(\frac{x}{a+bx}\right)$, prove that $\frac{d^2y}{dx^2} = \frac{1}{x}\left(\frac{a}{a+bx}\right)^2$.

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11. Find the area of region above the X-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

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12. Evaluate the following integrals :

$$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x} dx.$$

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13. Find the particular solution of the differential equation $\left(\tan^{-1} y - x\right)dy = \left(1 + y^2\right)dx$, given that x= 1, when y=0.

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