MATHS

BOOKS - ARIHANT PRAKASHAN

APPLICATION OF DERIVATIVES

Topic 1 Practice Questions 1 Mark Questions

1. The radius of a spherical soap bubble is increasing at the rate of 0.2 cm / sec. Find the rate of increase of its surface area, when the radius is 7 cm. (π = 3.141 approx)

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2. Write the equation of the tangent to the curve y=[x] at the point (-2,2).

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3. Are there two points on the curve y^2 = x, where the tangents are parallel to each other?

Give reasons to your answer.

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4. What is the acceleration at the end of 2 s of the particle that moves with rule $s = \sqrt{t} + 1$?

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5. Determine the point on the curve y = ln x, at which the tangent will be parallel to the chord joining the points P(1,0) and Q(e, 1).

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6. What is the acceleration at the end of 3 s of the particle that moves with the rule

$s = 3t^2 + 5t + 2.$

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7. Find the point on the curves x = a(θ - sin θ) and y=a(1 - cosθ), at which the tangent is parallel to X-axis.

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8. Find the slope of the tangent to the curve

$x = a\left(\frac{1-t^2}{1+t^2}\right)$ and y=$\frac{2at}{1+t^2}$ at t=$\frac{1}{\sqrt{3}}$

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9. Write the slope of the tangent to the curve

y=$\sqrt{3}$sinx+cosx at $\left(\frac{\pi}{2}, \sqrt{3}\right)$.

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10. What is the slope of the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 20$ at the point $(8, 64)$?

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11. Find the velocity at the end of 2 s of the particle moving according to the equation

$s = t^2 - 6t^2 + 15t + 12$

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12. Find the slope of the tangent to x = t^2 and y = 2t at t = 1.

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Topic 1 Practice Questions 4 Mark Questions

1. Find the point on the curve $x^2 + y^2 - 4xy + 2 = 0$ where the normal is paralell to the x-asis.

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2. A balloon is pumped at the rate of 2 $cm^3/$ min. .Write the rate of increase of the surface area, when the radius is 0.5 cm.

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3. Show that no two normals to a parabola are parallel.

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4. Find the slope of the tangent to the curve

x=2(t-sin t) and y = 2(1-cost) at t=$\frac{\pi}{4}$

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5. Find the equation of tangent to the curve

$x = y^2 - 2$ at the points where slope of the

normal equal to (-2).

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6. Find the equation of the normal to the curve

$5x^2 + 3y^2 = 23$ at (2,-1)

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7. Show that the line y = mx + c touches the

ellips

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$.

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8. Show that the length of the portion of the tangent to `x^(2/3) + y^(2/3) = a^(2/3)

intercepted between the axes is constant.

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9. Find the equation of normal to the curve

$3y^2 = 16x$ at (3,4).

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Topic 1 Practice Questions 6 Mark Questions

1. Show that the sum of the intercepts on the coordinate axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is constant.

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2. Find the tangent to the curve

$y = \cos(x + y), 0 < x < 2\pi$

which is parallel to the line $x + 2y = 0$.

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3. If $x\cos\alpha + y\sin\alpha = p$ is a tangent to the curve

$\left(\frac{x}{a}\right)^{\frac{n}{n}-1} + \left(\frac{y}{b}\right)^{\frac{n}{n}-1} = 1$then so that

$(a\cos\alpha)^n + (b\sin\alpha)^n = p^n.$

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4. Prove that the sum of the cubes of the intercepts on the coordinate axes of any tangent to the curve $x^{\frac{3}{4}} + y^{\frac{3}{4}} = a^{\frac{3}{4}}$ is a constant.

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5. (i) If the line y=mx+ c touches the curve y^2 = 4ax, then . prove that mc= a.

(ii) Find the equation of normal to the curve given by $x = \cos^3 \theta$ and $y = \sin^3 \theta$ at $\theta = \frac{\pi}{4}$.

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6. Water is leaking from a conical funnel at the rate of 5 cm^3 / s. If the radius of the base of funnel is 5 cm and height 10 cm, then find the rate at which the water level is dropping when it is 2.5 cm from the top.

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7. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

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Topic Test 1

1. Find the rate at which the volume of a spherical balloon will increase when its radius is 2 meters if the rate of increase of its redius is 0.3 m/min.

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2. For the curve y=5x-$2x^3$, if x increase at the rate of 2 units / s, then how fast is the slope of curve changing when X =3?

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3. The radius of a circle-is increasing uniformly at the rate of 3cm/s. Find the rate at which, the area of the circle is increasing, when the radius is 10 cm.

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4. The total revenue in rupees received from the sale of x units of a product is given by R(x)= $3x^2$ + 6x +5. Find the marginal revenue, when x =5, where marginal revenue means rate of change of total revenue at any level of ouput.

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5. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand from a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm.

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6. Find the velocity and acceleration at the end of 2 s of the particle moving according to the rule $s = \dfrac{3}{2t+1}$

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7. Find the acceleration at the end of 2s of the particlemoving according to the equation $s = \dfrac{2-t}{2+t}$.

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8. Find the equations of the tangent and normal to the curve y=$(\log x)^2$ at point x = $\frac{1}{e}$.

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9. Find the point on the curve

$y^2 - x^2 + 2x - 1 = 0$

where the tangent is parallel to the x - axis.

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10. Find the equation of the normal to the curve given by $x = 3\cos\theta - \cos^3\theta$ and $y = 3\sin\theta - \sin^3\theta$ at $\theta = \frac{\pi}{4}$

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11. Find the points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$, at which the tangents are parallel to X-axis,

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12. Find a point on the curve f(x)=$(x-3)^2$, where the tangent is parallel to the chord joining the points (3, 0) and (4, 1).

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13. Find the points on the curve $y = x^3 - 3x^2 + 2x$ at which the tangents to the curve is parallel to the line y-2x+3 = 0.

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14. Show that the curves $y = 2^{x}$ and $y = 5^{x}$ intersect at an angle $\tan^{-1}\left|\frac{1n\left(\frac{5}{2}\right)}{1 + 1n21n5}\right|$.

Note Angle between two curves is the angle between their tangents at the point of intersection.

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15. Find the angle of intersection of the curves $y^{2} = 4x$ and $x^{2} = 4y$ at (4, 4).

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Topic 2 Practice Questions 1 Mark Questions

1. If ϕ(x)=f(x)+f(1-x),f..(x)=0n for $0 \leq x \leq 1$, then $x = \frac{1}{2}$ is a point of maxima or minima of ϕ(x)?

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2. Write the interval in which the function $\sin^2 x - x$ is increasing.

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3. Write the set of values of x for which the function $f(x) = \sin x - x$ is increasing.

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4. Write a function which has both relative and absolute maximum at the point (1, 2).

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5. Write the maximum value of the function $y = x^5$ in the interval [1, 5].

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6. Mention the values of x for which the function $f(x) = x^2 - 12x$ is increasing.

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7. Write the value of df, if f(x) = ln (1 + x), x = 1 and &x=0.04.

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8. Write the set of points, where the function $f(x) = x^3$ has relative (local) extreme.

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9. Answer with reasons, whether the following function has a relative (local) maximum at x = 2 or not.

$$f(x) = \begin{cases} x & 0 \leq x < 1 \\ 1 & 1 \leq x \leq 2 \\ 3 - x & 2 < x \leq 3 \end{cases}$$

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10. Find an approximate value of $\sqrt{16.04}$ using differential.

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11. In which sub-interval of $\left(0, \frac{\pi}{2}\right)$ is x + 2 cos x increasing?

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12. Write df, when $f(x) = \sqrt{x}, x = 4$ and $\Delta x = 0.2.$

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13. Find approximately the difference between the volumes of two cubes of sides 3cm and 3.04 cm.

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14. Determine the interval in which $g(x) = \frac{x^2 + 3x + 3}{x + 1}$ is decreaing.

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15. Determine the sub-interval of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,in which f(x)= tan x - 4x is increasing.

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16. Using differential, find approximately the difference between the volumes of two cubes of sides 2 cm and 2.01 cm.

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17. Prove that, if $f(x) = a_0 + a_1 x^2 + a_2 x^4$ and $0 < a_0 < a_1 < a_2$, then f(x) has only one minima at x = 0.

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18. Find the open interval in which $f(x) = x^{\frac{1}{x}}, x > 0$ is decreasing.

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19. Find the intervals where function is increasing function y = cos x + sin x,x $\varepsilon[0, 2\pi]$

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20. Write the subinterval of $(0, \pi)$ in which sin $\left(x + \frac{\pi}{4}\right)$ is increasing.

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21. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values. $y = x + \frac{1}{x}$

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22. For what value of x, is the function $f(x) = 3 - 2x^2$ the maximum?

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23. What is the interval, in which $y = x^2 + 2x + 3 \quad x \varepsilon R$ is decreasing?

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24. Write the point, where f(x) = x log x attains minimum value.

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25. Find δf and df when $f(x) = 2x^2 - 1, x = 1, \delta x = 0 \cdot 02$

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Topic 2 Practice Questions 4 Mark Questions

1. Find the intervals in which the function $y = \frac{\ln x}{x}$ is increasing and decreasing.

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2. Find the intervals where function is increasing function y = cos x + sin x,x $\varepsilon[0, 2\pi]$

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3. Show that 2sinx+tanx $\geq$ 3x for all x $\varepsilon\left(0, \frac{\pi}{2}\right)$.

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4. Find approximate values of the following :

$\sqrt[6]{63}$

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5. Find the extreme values of the function $y = X + \frac{1}{x}$.

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6. Show that the function $\frac{e^x}{x^p}$ is strictly increasing for $x > p > 0$.

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7. Find the interval where y=sinx-cosx $x \in [0, 2\pi]$ is increasing.

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8. Find the maximum value of

y=(1+cosx) sinx,x$\varepsilon \left[0, \frac{3\pi}{4}\right]$

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9. Determine the interval in which the function f(x) = $x^3 - 5x^2 + 3x + 97$ is decreasing and that in which it is increasing.

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10. Find the approximate value of $(26.9)^{\frac{1}{3}}$

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11. Find the intervals in which the function $f(x) = 2x^3 + 9x^2 + 12x + 20$ is increasing and decreasing.

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12. If f(x) = a In $x + bx^2 + x$ has extreme values at $x = -1$ and $x = 2$ then find a and b.

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Topic 2 Practice Questions 6 Mark Questions

1. Show that the shrtest distance of the point (0, 8a) from the curve $ax^2 = y^3$ is $2a\sqrt{11}$.

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2. Find the coordinates of the point on the curve $x^2y - x + y = 0$

where the slope of the tangent is maximum.

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3. A cylindrical open water tank with a circular base is to be made out of 30 sq metres of metal sheet. Find the dimensions so that it can hold maximum water. (Neglect thickness of sheet).

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4. Shows that the triangle of greatest area that can be inscribed in a circle is equilateral.

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5. Find the minimum distance of a point on the curve $\frac{2}{x^2} + \frac{1}{y^2} = 1$ from the origin.

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6. Use the function $f(x) = x^{1/x}, x > 0$ to show that $e^{\pi} > \pi^{e}$.

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7. Determine the points of extreme values on the following curve.

$y = (x - 1)^2(x + 2)$

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8. Discuss the extreme value of the function

$y = (x + 2)^4(x - 1)^5$

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9. Show that the rectangle of maximum area that can be inscribed in a given circle is a square.

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10. Find the points on the curve $y = x^2 + 1$ which are nearest to the point (0,2).

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11. Show that the semivertical angle of a cone of given slant height is $\tan^{1}\sqrt{2}$ when its volume is maximum.

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12. Find two numbers x and y whose sum is 15 such that xy^2 is maximum.

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13. Find the altitude of a right circular cylinder of maximum volume inscribed in a sphere of radius r.

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14. Use differential to approximate $(255)^{\frac{1}{4}}$

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Topic Test 2

1. Find the interval for the function

$y = 2x^3 + 3x^2 - 36x - 7$

(i) increasing (ii) decreasing.

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2. Find the intervals where the following functions are (a) increasing and (b) decreasing.

$y = \tan x - 4(x - 2), x \in$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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3. Find the interval for which the function f(x) = x^2e^{-x} is increasing and decreasing.

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4. Prove that In(1+x) $<$ x for x $>$ 0.

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5. Prove that the function f given by f(x) = logsin x is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$

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6. Find extreme value of $f(x). = \begin{cases} \frac{x}{1-x^2} & -1 < x < 0 \\ x^3 - x & 0 \le x < 2 \end{cases}$ on (-1,2)

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7. If $f(x) = \begin{cases} 3x+2 & x \le 0 \\ 2-3x & x > 0 \end{cases}$

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8. Shows that the following functions do not possess maximum or minimum. x^5

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9. Determine a rectangle of area 25sq. Units which has minimum perimeter.

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10. Find the extreme points of the following functions. Specify if the extremum is a maximum or minimum. Find the extreme values.

$y = 60 / (x^4 - x^2 + 25)$

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11. Let f(x) = $\frac{a}{x} + x^2$. If it has a maximum at x =-3, then find the value of a.

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12. A window is in the form of a rectangle surmounted by a semi-circular opening. The total perimeter of the window is 10m.

Find the dimensions of the window to admit maximum light through the whole opening.

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13. Show that the height of a closed right circular cylinder of given surface and maximum volume is equal to diameter of base.

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14. Find δf and df when $f(x) = \sqrt{x}, x = 16, \delta x = 0.3$

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15. Find the approximate value of $\sqrt[5]{63}$

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16. Find the approximate value of $\sqrt{0.24}$.

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17. Find the approximate value of f(3.02), where

f(x)=$3x^2 + 5x + 3.$

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Chapter Test 1 Mark Questions

1. Find the slope of the normal for the curve x= t^2 and y= 2t at t = 1.

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2. Write the subinterval of $(0, \pi)$ in which sin $\left(x+\frac{\pi}{4}\right)$ is increasing.

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3. What is the value of δy, if y = x^2-1, x=1 and δ x = 0.02?

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4. What is the point of inflexion of the function

f(x)=x^3?

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5. Find the velocity and acceleration at the end of 2 seconds of the particle moving according to the following rules. $s = 2t^2 + 3t + 1$

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6. Find the slope of tangent to the curve Y= $\sqrt{x}+2x+6$ at x=4

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7. Find the intervals where the curve $y=a^x, a>0, x\varepsilon$ R, is increasing.

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8. Shows that the following functions do not possess maximum or minimum. x^3

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9. Evaluate : δy, if =$2x^2 + x - 1, x = 2$ and $\delta x = 0.04$.

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Chapter Test 4 Mark Questions

1. Show that the tangents to the curve $y = 7x^3 + 11$ at the points, where x=2 and x=-2 are parallel.

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2. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in R.

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3. Determine a rectangle of area 25sq. Units which has minimum perimeter.

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4. Find the coordinates of the point on the ellipse $16x^2 + 9y^2 = 400$ where the ordinate decreases at the same rate at which the abscissa increases.

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5. Find the equations of tangent and normal to the curve y= e^x at x= 0.

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Chapter Test 6 Mark Questions

1. Prove that all normals to the curve $x = a\cos t + at\sin t, y = a\sin t - at\cos t$ are at a distance a from the origin.

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2. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate 3 cm/s. How fast is the area decreasing, where the equal sides are equal to the base.

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3. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is 8 m^3. If building of tank costs 70 per sq m for the base and 45 per sq m for sides, then what is the cost of least expensive tank?

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4. Find the intervals in which the function f is given by f(x) = $\frac{4\sin x - 2x - x\cos x}{2+\cos x}$, if (i)increasing (ii) decreasing

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5. Find the equasion of the tangents drawn from the point (1,2) to the curve.

$y^2 - 2x^3 - 4y + 8 = 0$

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6. Show that the vertical angle of a right circular cone of minimum curved surface that circumscribes a given sphere is 2 $\sin^{-1}(\sqrt{2} - 1)$

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7. Use differential to approximate $(82)^{1/4}$.

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