



MATHS

BOOKS - ARIHANT PRAKASHAN

CONTINUITY AND DIFFERENTIABILITY

Topic 1 Practice Question 1 Mark Question

1. If $f'(2^+) = 0$ and $f'(2^-) = 0$, then is $f(x)$ continuous at $x = 2$?



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2. Write the interval in which the function $f(x) = \sin^{-1}(2 - x)$ is differentiable.



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3. Examine the continuity of function $f(x) = 5x - 3$ at $x = -3$



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1. Test the continuity of the following function

at $x = 0$

$$f(x) = \begin{cases} \frac{1-e^{-x}}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$



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2. If $f(x) = \begin{cases} ax^2 + b & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ 2ax - b & \text{if } x > 1 \end{cases}$

is continuous at $x=1$, then find a and b .



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3. Find the value of a such that the function f

defined by $f(x) = \begin{cases} \frac{\sin ax}{\sin x} & \text{if } x \neq 0 \\ \frac{1}{a} & \text{if } x = 0 \end{cases}$ is

continuous at $x = 0$



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Topic 1 Topic Test 1

1. Examine the continuity of the following functions at indicated points.

$$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$



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2. Examine the continuity of given function

$$f(x) = \begin{cases} 2x + 1 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases} \text{ at } x = 0, 1$$



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3. Discuss the continuity of the function

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \text{ at } x=0$$



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4. Show that $\sin x$ is continuous for every real x .



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5. Show that the function f defined by

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

is discontinuous $\forall \epsilon \neq 0 \in \alpha$



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6. Show that the function f defined by

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ -x & \text{if } x \text{ is irrational} \end{cases}$$

is continuous at $x=0 \forall x \neq 0 \in \mathbb{R}$



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7. Test differentiability and continuity of the following functions.

$$\left| 1 - \frac{1}{x} \right| \text{ at } x = 1$$



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8. Test differentiability and continuity of the following functions.

$$x^2|x| \text{ at } x = 0$$



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9. Test differentiability and continuity of the following functions.

$$f(x) = \tan x \text{ at } x = \frac{\pi}{2}$$



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10. Test differentiability and continuity of the following functions.

$$f(x) = \cot x \text{ at } x = \frac{\pi}{2}$$



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11. Test differentiability and continuity of the following functions.

$$f(x) = |\sin x| \text{ at } x = \pi$$



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12. Test differentiability and continuity of the following functions.

$$f(x) = \frac{x}{1 + |x|} \text{ at } x = 0$$



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Topic 2 Practice Question 1 Mark Question

1. Write a logarithmic function which is differentiable only in the open interval $(-1, 1)$



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2. Write the derivative of $e^{3 \log x}$ w.r.t. x^2



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3. Differentiate $a^{\log x}$ w.r.t. x





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4. Differentiate $\sin^{-1}(\cos x)$ w.r.t. x



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5. Find $\frac{dy}{dx}$ if $y = \log_e 3x$



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6. Find $\frac{dy}{dx}$ if $y = (\log_e \sin x)$





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7. What is the derivative of $\sec^{-1} x$ w.r.t. x , if $x < -1$?



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8. Differentiate $\cos^{-1}(\sin x)$ w.r.t. x .



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9. What is the derivative of $\sin^{-1}(3x - 4x^3)$
w.r.t $\sin^{-1} x$?



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10. Write the derivative of $|x|$, when $x = 3$



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11. Find $\frac{dy}{dx}$ when $\sin xy + x^2y = e^x$



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12. What is the derivative of $\sin x^\circ$ w.r.t. x ?



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13. What is the value of x , if $\frac{d}{dx} e^{x^2} = e^{x^2}$?



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14. If $f(x) = \log(\log x)$, then find $f(e)$.



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15. Find dy/dx if

$$x^y = c$$



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Topic 2 Practice Question 4 Mark Question

1. If $y = e^{x^{e^x}}$, find $\frac{dy}{dx}$.



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2.

Differentiate

$$y = \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$$



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3. Differentiate $y = (\sin y)^{\sin 2x}$



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4. Find $\frac{dy}{dt}$, when $y = \sin^{-1} \left(2 \frac{\sqrt{t^2 - 1}}{t^2} \right)$



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5. Find dy/dx if

$$x^m y^n = \left(\frac{x}{y} \right)^{m+n}$$



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6. Find $\frac{dy}{dx}$, when $y^x = x^{\sin y}$



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7. Find the dy/dx when

$$\cos x = \sqrt{\frac{1}{1+t^2}}, \sin y = \frac{2t}{1+t^2}$$



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8. If $y = \operatorname{cosec}^{-1}x$, then find $\frac{dy}{dx}$ and determine its value at $x = -2$.



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9. Examine the differentiability to $\log|x|$ for all real values of x .



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10. Find the derivative of $x^{\sin x}$ w.r.t.x.



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11. Differentiate.

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right) \text{ w. r. t. } \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

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12. If $\cos y = x \cos(a+y)$ then prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

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13. Prove that , if $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then

$$\frac{dy}{dx} = \sec x$$

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14.

Differentiate

$$y = 2^{x^2} + \tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right) \text{ w.r.t. } x.$$



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15. Write, why the function $\sin^{-1} \left(\frac{1}{\sqrt{1-x^2}} \right)$

cannot be differentiated anywhere ?



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16. Prove that $y = \tan(x + y)$, then

$$\frac{dy}{dx} = \frac{1 + y^2}{-y^2}$$



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17. Find the derivative of $\tan^{-1}(1 + x^2)$ with respect to $\log(x^2 + 1)$



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18. If $y = \sin^{-1}\left(\frac{2\sqrt{x^2 - 1}}{x^2}\right)$, then find $\frac{dy}{dx}$.



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19. Find $\frac{dy}{dx}$,

$$\sin x = \frac{2t}{1+t^2}, \tan y = \frac{2t}{1-t^2}$$



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20. If $y = \frac{x}{x+2}$ then prove that

$$x \frac{dy}{dx} = (1-y)y.$$



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21. Find the derivative of $(\sin x)^{\cos x}$ w.r.t.x.



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22. If $f(x) = x \cos x + e^x$, then find $f'(0)$.



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23. Differentiate $\frac{x}{\sin x}$ with respect to $\sin x$.



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Topic 2 Practice Question 6 Mark Question

1. If $x = \frac{1 - \cos^2 \theta}{\cos \theta}$, $y = \frac{1 - \cos^{2n} \theta}{\cos^n \theta}$ then
show that $\left(\frac{dy}{dx}\right)^2 = n^2 \left(\frac{y^2 + 4}{x^2 + 4}\right)$



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2. Find $\frac{dy}{dx}$ if $y = \cot^{-1}(\log \cos e^{-x})$



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3. Prove that

$$(i) y = \tan^{-1} \sqrt{\frac{1 + \sin x}{1 - \sin x}} \Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

$$(ii) x^p y^q = (x + y)^{p+q} \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$



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4. Find $\frac{dy}{dx}$ if $x^y = y^{\cos x}$



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5. Differentiate following function with respect to

$$x \frac{x^{\frac{1}{2}}(1-2x)^{\frac{2}{3}}}{(x-1)^2(x-4)^3} + \tan^{-1} \left(\frac{\cos x - \sin x}{\cos x + \sin x} \right)$$



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6. Find $\frac{dy}{dx}$, if $y = (\sin x)^x + (\tan^{-1} x)^{\log x}$



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7. If $y = x^{\sin x - \cos x} + \frac{x^2 - 1}{x^2 + 1}$, then find $\frac{dy}{dx}$



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8. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, then show that $\frac{dy}{dx} = -\frac{y}{x}$.



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Topic 2 Topic Test 2

1. Differentiate the following $\cot^{-1} \frac{\sqrt{1-x^2}}{x}$



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2. Differentiate with respect to x

(i) $\frac{x}{\sin x}$

(ii) $\tan^{-1} \sqrt{x}$



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3. If $y = \sin^2(2x + 3)$ then find the derivative.



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4. If $y = \sin^{-1}(3x + 2)$, then find $\frac{dy}{dx}$



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5. If $\sin y = x \cos(a + y)$, then find $\frac{dy}{dx}$



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6. If $\sqrt{1 - x^2} + \sqrt{1 - y^2} = a(x - y)$, then

prove that $\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$



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7. If $y = (\sec x + \tan x)^{\cot x}$ then find $\frac{dy}{dx}$



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8. Differentiate $(\log x)^x$ with respect to $\log x$.



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9. If $x = 2 \cos \theta - \cos 2\theta$ and

$y = 2 \sin \theta - \sin 2\theta$, then prove that

$$\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$



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Topic 3 Practice Question 1 Mark Question

1. If $f(x) = \sin x$ and $g(x) = x^3$, then find the value of $[f(x) \cdot g(x)]$...at $x = \frac{\pi}{2}$



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2. $\frac{\frac{d^m}{dx^m}(x^m)}{\frac{d^{m-1}}{dx^{m-1}}(x^{m-1})}$ is equal to

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3. If $y = \frac{1}{\sqrt{x}}$ then what is the value of y_2 ?

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Topic 3 Practice Question 4 Mark Question

1. Find $\frac{d^2y}{dx^2}$ if $x = a \cos \theta$, $y = b \sin \theta$.



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2. If $x = a \sec \theta$, $y = b \tan \theta$, then prove that

$$\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$$



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3. If

$$g(x) = x^3 f(x), g'(1) = 20, f(1) = 2, f'(1) = 1$$

and $f'(1) = 0$ then find the value of $f''(1)$.



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4. If $2z = x \left(2 + \frac{dz}{dx} \right)$, then prove that $\frac{d^2z}{dx^2}$ is

a constant.



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5. If $2y = x \left(1 + \frac{dy}{dx} \right)$, then show that y_2 is a

constant.



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6. If $x = \cos \theta$ and $y = \sin 5\theta$, then prove that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 25y = 0$$



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7. If $y = e^{m \cos^{-1} x} (1 - x^2) y_2 - x y_1 = m^2 y$



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8. If $y = Ae^{mx} + Be^{nx}$ then show that

$$\frac{d^2y}{dx^2} - (m + n)\frac{dy}{dx} + mny = 0$$



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9. If $y = 2\sin x + 3\cos x$, show that

$$\frac{d^2y}{dx^2} + y = 0.$$



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Topic 3 Practice Question 6 Mark Question

1. If $e^{y/x} = \frac{x}{a + bx}$ then show that

$$x^3 \frac{d}{dx} \left(\frac{dy}{dx} \right) = \left(x \frac{dy}{dx} - y \right)^2$$



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Topic 3 Topic Test 3

1. If $2y = x \left(1 + \frac{dy}{dx} \right)$, then show that y_2 is a constant.



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2. If $y = ax \sin x$ show that

$$x^2 y_2 - 2xy_1 + (x^2 + 2)y = 0$$



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3. If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)y_2 - xy_1 - 2 = 0$$



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4. If $x = 4z^2 + 5$ and $y = 6z^2 + 7z + 3$, then

find $\frac{d^2y}{dx^2}$



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5. If $x = \sin t$, $y = \sin pt$, prove that

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0.$$



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6. If $y^{1/m} + y^{-1/m} = 2x$, then prove that

$$(x^2 - 1)y_2 + xy_1 = m^2y^2$$



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Topic 4 Practice Question 1 Mark Question

1. Write that condition of Rolle's theorem which is violated by the function $f(x) = |x - 1|$ in $[0, 2]$.



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2. Which condition of Rolle's theorem is violated by the function $f(x) = \sin x$ in $\left[0, \frac{3\pi}{4}\right]$



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3. Which condition of Rolle's theorem is violated by the function $f(x) = |x|$ in $[-1,1]$



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4. Interpret Lagrange's mean value theorem geometrically.



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5. State Rolle's theorem.



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6. Find the value of c for which the Rolle's theorem is applicable for the function

$f(x) = x^2 - 5x + 4$ on the interval $[1, 4]$



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Topic 4 Practice Question 4 Mark Question

1. Verify Lagrange's Mean-Value theorem for

$F(x) = x^3 - 2x^2 - x + 3$ on $[1, 2]$



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2. Interpret Lagrange's mean value theorem geometrically.



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3. Verify Rolle's theorem for the function

$f(x) = x^2 + 3x - 10$ in the interval $[-5, 2]$



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4. Verify Rolle's theorem for the function

$$f(x) = x(x - 2)^2, 0 \leq x \leq 2$$



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5. Examine if Rolle's theorem is applicable to the following functions :

(i) $f(x) = |x|$ on $[-1, 1]$

(ii) $f(x) = [x]$ on $[-1, 1]$

(iii) $f(x) = \sin x$ on $[0, \pi]$

(iv) $f(x) = \cot x$ on $[0, \pi]$



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6. Verify Lagrange's Mean-Value theorem for

$$F(x) = x^3 - 2x^2 - x + 3 \text{ on } [1,2]$$



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Topic 4 Practice Question 6 Mark Question

1. Verify Rolle's theorem for the function

$$f(x) = x^2 - 3x + 2 \text{ in the interval } [1, 2]$$



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2. Using mean value theorem, prove that there is a point on the curve $y = 2x^2 - 5x + 3$ between the points $A(1, 0)$ and $B(2, 1)$ where tangents are parallel to the chord. Also, find that point.



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Topic 4 Topic Test 4

1. Verify Rolle's theorem for the function

$$f(x) = x(x - 2)^2, 0 \leq x \leq 2$$



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2. Examine if Rolle's theorem is applicable to the following functions :

(i) $f(x) = |x|$ on $[-1, 1]$

(ii) $f(x) = [x]$ on $[-1, 1]$

(iii) $f(x) = \sin x$ on $[0, \pi]$

(iv) $f(x) = \cot x$ on $[0, \pi]$



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3. Verify Lagrange's Mean-Value theorem for

$$F(x) = x^3 - 2x^2 - x + 3 \text{ on } [1,2]$$



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Chapter Test 1 Mark Question

1. Is the function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases} \text{ continuous at } x=0, \text{ at}$$

$x=1$ and at $x=2$?



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2. It $f(x) = |\cos x|$, then find $f. \left(\frac{3\pi}{4} \right)$



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3. If $y = \frac{\log x}{x}$, then find $\frac{dy}{dx}$



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4. Differentiate $e^{\sin^{-1} x}$ w.r.t.x.



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5. Find the derivative of function $(x^2 + 5)^8$



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6. Differentiate the function $\tan(x^2 + 5)$



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7. Differentiate the following functions by proper substitution.

$$\tan^{-1} \left[\frac{2x}{1-x^2} \right]$$



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8. If $f(x) = \cos^{-1}(\sin x)$, then find $f'(x)$.



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9. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$



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10. If $x = at^2$, $y = 2at$ then find $\frac{dy}{dx}$ at $t = \frac{1}{2}$



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Chapter Test 4 Mark Question

1. Determine $f(0)$ so that the function $f(x)$

defined by $f(x) = \frac{(4^x - 1)^3}{\sin \frac{x}{4} \log\left(1 + \frac{x^2}{3}\right)}$

becomes continuous at $x = 0$.



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2. Find the derivative of

$$y = (x + 3)^2(x + 4)^3(x + 5)^4$$



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3. Differentiate

$$y = \tan^{-1} \cdot \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$$



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4. If $y = \cos^{-1} \left[\frac{2x - 3\sqrt{1-x^2}}{13} \right]$ then find

$$\frac{dy}{dx}$$



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5. Find derivative of the function

$$\sin^{-1} \sqrt{\frac{1-x}{1+x}}$$



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6. Differentiate the function $\log_x 5$



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7. Verify mean value theorem for the function

$$f(x) = \sin x - \sin 2x \text{ in } [0, \pi]$$



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Chapter Test 6 Mark Question

1. Find the values of a and b such that the
function f defined by

$$f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & \text{if } x < 4 \\ a + b, & \text{if } x = 4 \\ \frac{x-4}{|x-4|} + b, & \text{if } x > 4 \end{cases}$$

continuous function at $x = 4$.



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2. If $f(x) = |x|^3$, then show that $f''(x)$ exists for all real x and find it.



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3. Differentiate $\tan^{-1} \left(\frac{x \sin \alpha}{1 - x \cos \alpha} \right)$



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4. prove that

$$\frac{d}{dx} \left[\frac{1 - \tan x}{1 + \tan x} \right]^{\frac{1}{2}} = \frac{-1}{(\sqrt{\cos 2x}) \cos x}$$



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5. If $y = \sin^3 \sqrt{ax^2 + bx + c}$ then find $\frac{dy}{dx}$



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6. Differentiate from definition $\sqrt{\cos x}$



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7. If $\sqrt{1-x^4} + \sqrt{1-y^4} = k(x^2 - y^2)$ then

show that

$$\frac{dy}{dx} = \frac{x\sqrt{1-y^4}}{y\sqrt{1-x^4}}$$



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8. If $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$, then find

$$\frac{dy}{dx}.$$



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