MATHS

BOOKS - ARIHANT PRAKASHAN

RELATIONS AND FUNCTIONS

Topic 01 Practice Question Exam Textbook S Other Imp Questions 1 Mark Questions

1. A R is a relation on set A such that $R = R^{-1}$, then write the type of the relation

R.

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2. Sets A and B have respectively m and n elements. The total number of relations from set A to set B is 64. If $m < n$ and $m \neq 1$, write the values of m and n, respectively.

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3. If $R = \{(a, a^3)$: a is prime number less than 5} be a relation. Find the range of R.

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4. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, then write the range of R.

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5. State the reason for the relation R in the set {1, 2, 3} given by R ={(1, 2), (2, 1)} not to be transitive.

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6. Let R is the equivalence in the set A = {0, 1, 2, 3, 4, 5} given by R = {(a, b) : 2 divides (a - b)}. Write the equivalence class [0].

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7. Find the least positive integer .r. such that $185 \in [r]_7$.

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8. If X and Y are sets containing m and n elements respectively then what is the total number of function from X to Y ?

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9. Show that the function $f : R \to R$ defined by $f(x) = \frac{x}{x^2 + 1}$ is neither one-one nor onto.

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10. If A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} is a function from A to B. State whether f is one-one or not.

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11. What is the range of the function

$$f(x) = \frac{|x-1|}{x-1}, x \neq 1?$$

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12. Write fog, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

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13. If $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \sin x$ and $g(x) = x^5$, then find gof(x).

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14. If $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$ and $g: \{1, 2, 5\} \rightarrow \{1, 3\}$ given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

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15. Show that f(x)=sin x on [0, pi/2] functions are injective.

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16. Let f = {(1,3), (2,4), (3,7)} and g = {(3,2), (4,3), (7,1)} determine gof?

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1. Let R be the relation on the set R of real numbers such that aRb iff a-b is and integer. Test whether R is an equivalence relation. If so find the equivalence class of $1 \text{and} \frac{1}{2}$ wrt. This equivalence relation.

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2. Show that the relation R on the set A of real numbers defined as R = {(a,b): a $\leq b$).is reflexive. and transitive but not symmetric.

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3. If Z is the set of all integers and R is the relation on Z defined as $R = \{(a, b) : a, b \in Z$ and $a - b$ is divisible by 3. Prove that R is an equivalence relation.

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4. If $f : X \to Y$ is a function. Define a relation R on X given by R={(a, b): f(a)=f(b)}. Show that R is an equivalence relation on X.

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5. Show that the relation R on IR defined as $R = \{(a, b) : (a \leq b)\}$, is reflexive and transitive but not symmetric.

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6. If A = {1,2,3,...,9} and R is the relation in A $\times$ A defined by (a, b) R (c, d), if $a + d = b + c$ for (a, b), (c, d) in $A \times A$. Prove that R is an equivalence relation.

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7. Check whether the relation R defined on the set A = {1,2,3,4,5,6} as R = {(a,b) : b = a + 1} is reflexive, symmetric or transitive.

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8. Let R be the set of all non -zero real numbers. Then show that f: R $\rightarrow$ R given by f(x) = $\frac{1}{x}$ is one- one and onto.

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9. Show that a function $f\colon R \rightarrow R$ given by $f(x) = 3x + 5$ is a bijective.

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10. If $f: N \to N$ is defined by.

$$f(n) = \begin{cases} \frac{n+1}{2}, \text{if n is odd} \\ \frac{n}{2}, \text{if n is even} \end{cases}$$ for all $n \in N$.

Find whether the function f is bijective.

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11. If the function $f: R \to R$ is given by $f(x) = \dfrac{x+3}{3}$ and $g: R \to R$ is given $g(x) = 2x - 3$, then find

(i) fog (ii) gof.

Is $f^{-1} = g$?

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12. If A = R -{3} and B = R -{1}. Consider the function $f: A \to B$ defined by $f(x) = \frac{x-2}{x-3}$, for all $x \in A$. Then, show that f is bijective. Find $f^{-1}(x)$.

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Topic 01 Practice Question Exam Textbook S Other Imp Questions 6 Marks Questions

1. Prove that $f: X \to Y$ is injective iff for all subsets A, B of X, $f(A \cap B) = f(A) \cap f(B)$.

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2. Let $f: X \to Y$ and $g: Y \to Z$. Prove that gof is bijective if both f and g are bijective. Also prove that $(gof)^{-1} = f^{-1} og^{-1}$.

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3. If N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

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4. Consider $f: R - \left\{-\frac{4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x + 3}{3x + 4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

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5. Consider $f: R_{+} \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$. Hence. Find

$(i) f^{-1}(10)$ $\quad$ $(ii) y \text{ if } f^{-1}(y) = \frac{4}{3}$

where R_{+} is the set of all non-negative real numbers.

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Topic 01 Topic Test 1

1. For real numbers x and y, define x R y if and only if $x - y + \sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

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2. If the relation R is defined on the set A={1,2,3,4,5} by R={a,b} : $\left|a^2 - b^2\right| < 8$. Then, find the relation R.

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3. Find least positive integer x, satisfying

$276x + 128 = 4$ (mod 7).

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4. If the mappings f and g are given by

f={(1,2),(3,5),(4,1) and g={(2,3),(5,1),(1,3)}, then

write fog.

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5. Let A={1,2,3}, B={4,5,6,7} and let f={(1,4),(2,5),(3,6)} be a function from A to B. State whether f is one-one or not.

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6. Let X and Y be set containing m and n elements, respectively. How many functions from X to Y are one-one according to $m < n$.

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7. Show that the function $f(x) = a^x, x \in R$ is injective, where $(a > 0 \text{ and } a \neq 1)$.

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8. Show that the relation S in set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1.

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9. Show that the relation S defined on set $N \times N$ by $(a, b)S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation.

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10. Show that $f: N \to N$, given by

$$f(x) = \begin{cases} x + 1, \text{if x is odd} \\ x - 1, \text{if x is even} \end{cases}$$

is bijective (both one-one and onto).

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11. If $f: R \to R$ is defined as $f(x) = 10x + 7$. Find the function $g: R \to R$, such that $gof = fog = I_R$.

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12. If the function'f : R $\to$ R is given by f(x) = $x^2 + 2$ and g:R $\to$ R is given by g(x) = $\frac{x}{x-1}, x \neq 1$ then find fog and gof and hence find fog (2) and gof (-3) .

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13. Show that the relation R is in the set $A = \{1, 2, 3, 4, 5\}$ given by R={(a, b): |a-b| is divisible by 2}, is an equivalence relation. Write all the equivalence classes of R.

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14. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where S is the range of f, is invertible. Also, find the inverse of f.

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15. Prove that f:$X \to Y$ is surjective iff for all $A \subseteq X, (f(A))' \subseteq f(A')$, where A' denotes the complement of A in X.

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Topic 02 Practice Question Exam Textbook S Other Imp Questions 1 Mark Questions

1. Let * is a binary operation on N given by $a * b = LCM$ (a, b) for all $a, b \in N$. Find $5 * 7$

.

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2. Let * is a binary operation on set of integers I defined by $a * b = 3a + 4b - 2$, then find the value of $4 * 5$.

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3. Let * is the binary operation on N given by $a * b = HCF$ (a, b), where $a, b \in N$. Write the value of $22 * 4$.

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4. If * is binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{5}$. Write the identity for *, if any.

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5. Find the number of binary operations on the set {a, b}.

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6. Is the binary operation * defined on Z (set of integers) by

$m * n = m - n + mn, \forall m, n \in Z$

commutative?

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7. Is * defined on the set S={0,1,2,3...,10} by $a * b = LCM$(a, b) for all $a, b \in S$.

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8. Let * be a binary operation on N given by $a * b = GCD$ (a, b) for $a, b \in N$. Check the commutativity and associativity of * on N.

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9. Let * be a binary operation on the set S of all non-negative real numbers defined by $a * b = \sqrt{a^2 + b^2}$. Find the identity elements in S with respect to *.

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Topic 02 Practice Question Exam Textbook S Other Imp Questions 4 Marks Questions

1. Construct the multiplication table $\times_7$ on the set {1,2,3,4,5,6}. Also find the inverse element of 4 if it exists.

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2. Let * be a binary operation on N given by

$a * b = LCM(a, b)$ for all $a, b \in N$.

(i) Is * commutative.

(ii) Is * associative.

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3. If S is the set of all rational numbers except 1 and * be defined on S by $a * b = a + b - ab$, for all $a, b \in S$.

Prove that

(i) * is a binary operation on S.

(ii) * is commutative as well as associative.

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4. Consider the binary operation * on the set {1,2,3,4,5} defined by $a * b$= min{a,b}. Write operation table of operation *.

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5. Consider the binary operation $*: R \times R \to R$ and $o: R \times R \to R$ defined as $a * b = |a - b|$ and $aob = a$. For all $a, b \in R$. Show that * is commutative but not associative, .o. is associative but not commutative.

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1. Constract the composition table/multiplication table for the binary operation * defined on {0,1,2,3,4}by $a * b = a \times b(\text{mod} = 5)$. Find the identity element if any. Also find the inverse elements of 2 and 4.

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2. A binary operation * is defined on the set

$X = R - \{-1\}$ by

$x * y = x + y + xy, \forall x, y \in X.$

Check whether * is commutative and associative. Find its identity element and also find the inverse of each element of X.

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Topic 02 Topic Test 2

1. Determine whether $a * b = a + b(\bmod 7)\text{on}\{0, 1, 2, 3, 4, 5, 6\}$ operations as defined by * are binary operations on the sets specified in each case. Give reasons if it is not a binary operation.

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2. Determine whether $a * b = \sqrt{a^2 + b^2} on Q_+$ operations as defined by * are binary operations on the sets

specified in each case. Give reasons if it is not a binary operation.

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3. Let * is a binary operation on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all $c, b \in R - (0)$. Find the value of x, given that $2 * (x * 5) = 10$.

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4. For binary operation * defined on 8 -{1}, such that $a * b = a + b - ab.$ Determine the identity element.

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5. Let $A = N \cup \{0\} \times N \cup \{0\}$ and Let * be a binary operation on a defined by $(a, b) * (c, d) = (a + c, b + d)$ for (a, b), (c, d) $\in$ N. Show that.

(i) B commutative as A

(ii) * is associative on A

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6. Let * be a binary operation on Q defined by $a * b = ab + 1$. Determine whether * is commutative but not associative.

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7. Construct the composition table/multiplication table for the binary operation * defined on {0, 1, 2, 3, 4, 5} given by $a * b = ab$ (mod 6). Find the identity element if any. Also, find the inverse of elements 1 and 5.

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8. Given a non-empty set X, Let *: $P(x) \times P(x)$ be defined as

$A * B = (A - B) \cup (B - A), \forall A, B \in P(x)$

. Show that the empty set ϕ is the identity for the operation * and all the elements A of p(x) are invertible with $A^{-1} = A$.

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Chapter Test 1 Mark Questions

1. Let A = {1,2,3,4,...., 15,16} and let R be a relation in A given by $R = \{(a, b)\} : b = a^2\}$, then find domain and range of relation R.

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2. Let $f: N \rightarrow N$ be defined by $f(x) = x + 2$. Then, find whether f is injective.

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3. If $f(x) = 27x^3$ and $g(x) = x^{1/3}$. Then, find gof(x).

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4. If the mappings f and g are given by f={(1,2),(3,5),(4,1) and g={(2,3),(5,1),(1,3)}, then write fog.

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5. If A = {a,b,c,d} and the function f = {(a,b),(b,d),(c,a),(d,c)}, then write f^{-1}.

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6. If $f: R \rightarrow R$ defined by $f(x) = \frac{3x+5}{2}$ is an invertible function, then find $f^{-1}(x)$.

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7. If * defined on the set A ={1,2,3,4,5} by $a * b = LCM$ of a and b a binary operation ? Justify your answer.

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8. Let * be the binary operation defined on set 8 of rational numbers as $a * b = a^2 + b^2, \forall a, b \in \theta$, then find $\sqrt{8 * 6}$.

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9. Let * be a binary operation on set of integer. I defined by $a * b = 2a + b - 3$. Find the value of $3 * 4$.

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10. Let * is binary operation on set Q of rational number defined as $a * b = \frac{ab}{2}$. Write the identity for *, if any.

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Chapter Test 4 Marks Questions

1. Show that the relation S in set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, b) : a, b \in A, |a - b|$ is divisible by 4$\}$

is an equivalence relation. Find the set of all elements related to 1.

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2. Show that the relation S defined on set $N \times N$ by $(a, b)S(c, d) \Rightarrow a + d = b + c$ is an equivalence relation.

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3. Show that the relation R on the set A of real numbers defined as R = {(a,b): a $\leq b$).is reflexive. and transitive but not symmetric.

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4. Show that $f: N \rightarrow N$, given by

$$f(x) = \begin{cases} x+1, \text{ if x is odd} \\ x-1, \text{ if x is even} \end{cases}$$

is bijective (both one-one and onto).

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5. If the function $f: R \rightarrow R$ is given by $f(x) = x^2 + 2$ and $g: R \rightarrow R$ is given by $g(x) = \frac{x}{x-1}, x \neq 1$, then find fog and gof, and hence find fog(2) and gof(-3).

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6. If $f: R \rightarrow R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijection.

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7. If $A = N \times N$ and * is a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Also, find identity element for * on A, if any.

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Chapter Test 6 Marks Questions

1. Show that the relation R is in the set $A = \{1, 2, 3, 4, 5\}$ given by R={(a, b): |a-b| is divisible by 2}, is an equivalence relation. Write all the equivalence classes of R.

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2. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where S is the range of f, is invertible. Also, find the inverse of f.

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3. Discuss the commutativity and associativity of binary operation .*. defined on A = Q - {1} by the rule $a * b = a - b + ab$ for all $a, b \in A$. Also, find the identity element of * in A and hence find the invertible elements of A.

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