MATHS

BOOKS - ARIHANT PRAKASHAN

VECTORS

Topic 1 Practice Questions 1 Mark Questions

1. Write the unit vectors in R^3, which makes angles 45° and 60° with positive directions of X-axis and Y-axis, respectively.

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2. Is $\overrightarrow{0}$ unique

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3. Write the value of α, if the vector $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = \alpha\hat{i} - \hat{j} + 2\hat{k}$ are parallel.

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4. If $\left|k\vec{a}\right|$ = 1`, then

A. $\vec{a} = \frac{1}{k}$

B. $\vec{a} = \frac{1}{|k|}$

C. $k = \frac{1}{\left|\vec{a}\right|}$

D. $k = \pm\frac{1}{\left|\vec{a}\right|}$

Answer: D

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5. Write the values of m and n for which the vectors $(m-1)\hat{i}+(n+2)\hat{j}+4\hat{k}$ and $(m+1)\hat{i}+(n-2)\hat{j}+8\hat{k}$ will be parallel.

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6. How many directions a null vector has ?

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7. If the position vectors of the points A, B, C are $2\hat{i}+\hat{j}-\hat{k}, 3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively, then prove that A, B, C are collinear.

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8. If A, B, C, D, E are the.vertices of a regular pentagon, find the vector sum $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$.

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9. Write the unit vector along $\overrightarrow{PQ}$ joining the points $P(7, -4, 5)$ to $Q(7, 1, 5)$.

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10. What is the unit vector in the direction of the vector $3\hat{i} + 4\hat{j}$?

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11. If $\overrightarrow{OP_1} = 4\hat{i} + 3\hat{j}$ and $\overrightarrow{OP_2} = 8\hat{j} - 5\hat{j}$, then what is $\overrightarrow{P_1P_2}$?

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12. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} + \hat{k}$, then

A. $\vec{a}$ and $\vec{b}$ have the same directions

B. $\vec{a}$ and $\vec{c}$ have opposite directions

C. $\vec{b}$ and $\vec{c}$ have opposite directions

D. no pair of vectors have same directions

Answer: D

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13. The direction cosines of the vectors $\overrightarrow{PQ}$ where $\overrightarrow{PQ}$ = (1, 0, -2) and $\overrightarrow{OQ}$ = (3, -2, 0) are

A. 2,-2,2

B. 4,-2,-2

C. $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

D. $\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}$

Answer: C

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14. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then find the value of x + y +z.

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15. Find the vector from origin to the mid-point of the vector $\overrightarrow{P_1P_2}$ joining the points P_1 (4,3) and P_2(8, -5).

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16. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units.

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17. If P(1, 5, 4) and Q(4, 1, -2), then find the direction ratios of $\overrightarrow{PQ}$.

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Topic 1 Practice Questions 4 Mark Questions

1. Prove that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}, 3\hat{i} - 4\hat{j} - 4\hat{k}$ are the sides of a right angled triangle.

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2. The projection of a line segment $\overline{OP}$, through origin O, on the co-ordinate axes are 6, 2, 3. Find the length of the line segment OP and its direction cosines.

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3. Prove that

$|a + b| \leq |a| + |b|$

State when equality will hold,

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4. Prove that the lines joining the midpoints of consecutive sides of a quadrilateral form a parallelogram using vector method.

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5. ABCD is a parallelogram. Using vector method prove that line joing A and the mid -point of BC intersects the diagonal BD in the ratio $1:2$.

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6. If $\vec{a} = (2, -2, 1)$, $\vec{b} = (2, 3, 6)$ and $\vec{c} = (-1, 0, 2)$,Find the magnitude and direction of $\vec{a} + \vec{b} - \vec{c}$.

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7. Show that the point (3, -2, 4), (1, 1, 1) and (-1, 4,-2)are collinear.

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8. Show that the vectors $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$ form a right angled triangle.

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9. If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, the show that the point P,Q and R are collinear.

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10. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

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11. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then find the value of $\vec{r}$.

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12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$ and find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

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Topic Test 1

1. Compute the magnitude of the following vectors

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$

and $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$.

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2. Find the value of x and y, so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

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3. find the unit vector in the direction of 'PQ' where P and Q are the points (1,2,3) and (4,5,6).

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4. Find the unit vector in the direction of the vector $r_1 - r_2$, where $r_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $r_2 = 3\hat{i} + \hat{j} - 5\hat{k}$.

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5. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

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6. If $\vec{a} + \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{a} - \vec{b} = 2\hat{i} + 4\hat{j} + 2\hat{k}$, then find the scalar components of $\vec{a}$ and $\vec{b}$

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7. Prove that the vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}, 3\hat{i} - 4\hat{j} - 4\hat{k}$ are the sides of a right angled triangle.

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8. Find the unit vector parallel to the sum of the vectors $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$. Also, find its direction cosines.

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9. If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4p\hat{j} + q\hat{k}$ are parallel, then find the values of p and q.

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10. If the points with position vector $10\hat{i}+3\hat{j}, 12\hat{i}-\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear, find the value of a.

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11. If O is origin and P_3 is the mid - point of the line joining $P_1(2, -1)$ and $P_2(-4, 5)$ then find $\overrightarrow{OP_3}$.

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12. Find the vectors from the origin to the points of trisection the vector $\overrightarrow{P_1P_2}$ joining $P_1(-4, 3)$ and P_2(5, -12).

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13. If $\vec{a} = (2, -2, 1)$, $\vec{b} = (2, 3, 6)$ and $\vec{c} = (-1, 0, 2)$,Find the magnitude and direction of $\vec{a} + \vec{b} - \vec{c}$.

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14. Find the vectors from the origin to the intersection of the medians of the triangle whose vertices are A(5,2,1), B(-4,7,0) and C(5, -3,5)

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15. Let the position vectors of A and B be $3\hat{i} - \hat{j} + \hat{k}$ and $-\hat{i} + 2\hat{j} + 3\hat{k}$. Find the vector $\overrightarrow{AB}$ and its magnitude Also, determine the unit vector in the direction of $\overrightarrow{AB}$.

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16. ABCD is a quadrilateral. If M and N are the mid points of the sides $\overrightarrow{BD}$ and $\overrightarrow{AC}$, respectively. Show that $\overrightarrow{AB}+\overrightarrow{AD}+\overrightarrow{CB}+\overrightarrow{CD}=4\overrightarrow{NM}$

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Topic 2 Practice Questions 1 Mark Questions

1. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2=144$, write the value of ab.

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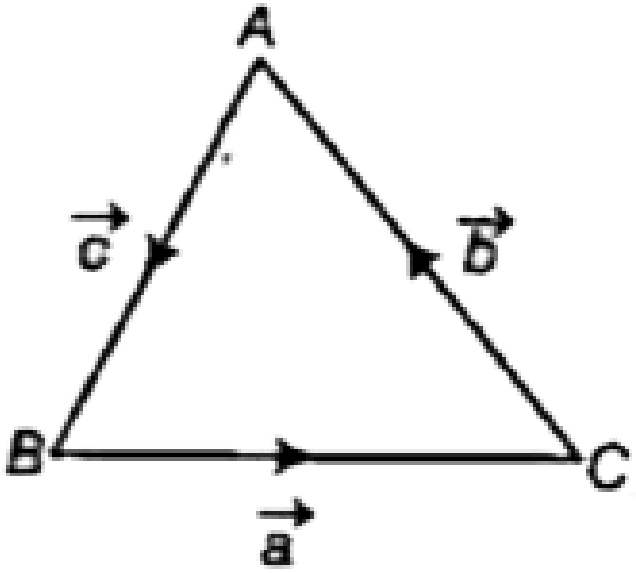
2. It the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the sides $\overrightarrow{BC}$, $\overrightarrow{CA}$ and $\overrightarrow{AB}$ respectively of a triangle ABC, then write the value of

$\vec{a} \times \vec{c} + \vec{b} \times \vec{c}$.



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3. If $a = 3\hat{i} + \hat{j} + 2\hat{k}, b = 2\hat{i} - 3\hat{j} + 4\hat{k}$, then verify that $a \times b$ is perpendicular to both a and b.

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4. If $\left|\vec{a}\right| = 3, \left|\vec{b}\right| = 2$ and $\vec{a} \cdot \vec{b} = 0$, then write the value of $\left|\vec{a} \times \vec{b}\right|$.

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5. Find the component of the vector $\vec{b} = 8\hat{i} + \hat{j}$ in the direction of the vector $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$.

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6. Write a vector normal to $\left(\hat{i} + \hat{k}\right)$ and $\left(\hat{i} + \hat{j}\right)$.

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7. Find the area of the parallelogram whose diagonals are the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$?

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8. Determine the value of m, for which the following vectors are orthogonal.

$(m+1)\hat{i}+m^2\hat{j}-m\hat{k}, (m^2-m+1)\hat{i}-m\hat{j}+\hat{k}$

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9. Find the value of λ such that the following vectors are coplanar:

$-\hat{i}+\lambda\hat{j}-\lambda\hat{k}, 2\hat{i}+4\hat{j}+5\hat{k}, -2\hat{i}+4\hat{j}-4\hat{k}.$

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10. For what value of λ, the vectors $\lambda\hat{i} + 3\hat{j} + \lambda\hat{k}$ and $\lambda\hat{i} - 2\hat{j} + \hat{k}$ are perpendicular to each others.

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11. Find the scalar projection of the vector $\vec{a} = 3\hat{i} + 6\hat{j} + 9\hat{k}$ on $\vec{b} = 2\hat{i} + 2\hat{j} - \hat{k}$.

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12. Determine μ, for which the vector $\vec{a} = \mu\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ will be of unit length.

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13. Show that the vectors $2\hat{i}+3\hat{j}, 5\hat{i}-5\hat{k}$ and $6\hat{j}+4\hat{k}$ are coplanar.

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14. If $\vec{a}.\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{0}$, then draw the conclusion.

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15. If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}\neq\vec{0}$, then prove that $\vec{a}+\vec{c}=m\vec{b}$, where m is a scalar

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16. What is the angle between the vectors $2\hat{i} - \hat{j} - \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$?

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17. Determine the area of the parallelogram whose sides are the vectors $2\hat{i} + 2\hat{j}$ and $\hat{i} - \hat{k}$

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18. Using vector method find the area of the triangle with vertices (1, 0, 0) (0, 1, 0) and (0, 0, 1)

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19. Write the angle between $\vec{a}$ and $\vec{c}$, if $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{c}$.

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20. What is $\left(\hat{i} - \hat{j}\right) \cdot \left(\hat{j} - \hat{i}\right)$?

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21. $\left(2\hat{i} - 4\hat{j}\right) \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = \ldots.$

A. -3

B. $+2$

C. -1

D. -2

Answer: D

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22. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} + \hat{j} + 2\hat{k}, \vec{c} = 2\hat{i} - \hat{j} - 2\hat{k},$ then what is

A. $\vec{a} \perp \vec{b}$

B. $\vec{b} \perp \vec{c}$

C. $\vec{a} \perp \vec{c}$

D. no pair of vectors are perpendicular

Answer: C

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23. Each question given below have four possible answers, out of which only one is correct. Choose the correct one.

$(-3, \lambda, 1) \perp (1, 0, -3) \Rightarrow \lambda =$ .....

A. 0

B. 1

C. impossible to find

D. any real number

Answer: C

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24. Each question given below have four possible answers, out of which only one is correct. Choose the correct one.

A vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{i} + \hat{k}$ is .....

A. $\hat{i} - \hat{j} - \hat{k}$

B. $\hat{j} - \hat{k} + \hat{i}$

C. $\hat{k} - \hat{j} - \hat{i}$

D. $\hat{j} + \hat{k} + \hat{i}$

Answer: A

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25. Each question given below have four possible answers, out of which only one is correct. Choose the correct one.

If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\hat{a} \times \hat{b}$ is a unit vector, then the angle between $\hat{a}$ and $\hat{b}$ is .....

A. of any measure

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: C

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26. $\left(-\vec{a}\right)\cdot\vec{b}\times\left(-\vec{c}\right)=....$

A. $\vec{a}\times\vec{b}\cdot\vec{c}$

B. $-\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

C. $\vec{a}\times\vec{c}\cdot\vec{b}$

D. $\vec{a}\cdot\left(\vec{c}\times\vec{b}\right)$

Answer: A

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27. For the non-zero vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$, if

A. $\vec{b} \perp \vec{c}$

B. $\vec{a} \perp \vec{b}$

C. $\vec{a} \mid\mid \vec{c}$

D. $\vec{a} \perp \vec{c}$

Answer: C

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28. What is the projection of $\hat{i} + \hat{j} - \hat{k}$ upon the vector $\hat{i}$?

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29. If $\hat{a}$ is a unit vector and $\left(\vec{x}-\hat{a}\right).\left(\vec{x}+\hat{a}\right)=8$, then find $\left|\vec{x}\right|$

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30. If $\left|\vec{a}\right|=\sqrt{3}, \left|\vec{b}\right|=2$ and $\vec{a}\cdot\vec{b}=3$, then find the angle between $\vec{a}$ and $\vec{b}$.

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31. Find the angle between the vectors $\vec{a}$ and $\vec{b}$, if $\left|\vec{a}\times\vec{b}\right|=\sqrt{3}, \left|\vec{a}\right|=2$ and $\left|\vec{b}\right|=1$.

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32. Find $\vec{a}.\left(\vec{b}\times\vec{c}\right)$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$.

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33. Show that the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, if $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

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34. Evaluate $\left[\hat{i}\hat{k}\hat{j}\right]+\left[\hat{i}\hat{j}\hat{k}\right]$.

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Topic 2 Practice Questions 4 Mark Questions

1. Find a unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$.

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2. Prove that $\left(\vec{a}\times\vec{b}\right)^2=a^2b^2-\left(\vec{a}.\vec{b}\right)^2$.

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3. Prove that for any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\left[\vec{a}+\vec{b}\,\vec{b}+\vec{c}\,\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

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4. Show that $\hat{i} \times \left(\vec{a} \times \hat{i}\right) + \hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$.

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5. Find the area of the triangle ABC with vertices A(1,2,4), B(3,1,-2) and C(4,3,1) by vector method.

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6. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitude show that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$, $\vec{c}$.

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7. Prove that $\left[\vec{a} \times \vec{b}\,\vec{b} \times \vec{c}\,\vec{c} \times \vec{a}\right] = \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$.

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8. Prove that the following vectors can never be coplanar for any real value of λ.

$(\lambda+1)\hat{i}+2\hat{j}+\hat{k},\ -\hat{i}+\lambda\hat{j}+\hat{k}, \lambda\hat{i}+\hat{j}+3\hat{k}$

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9. Prove the following by vector method. An angle inscribed in a semi-circle is a right angle.

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10. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular vectors, then prove that $\left[\vec{a} \cdot \left(\vec{b} \times \vec{c}\right)\right]^2 = a^2b^2c^2$.

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11. Find a vector $\vec{b}$ such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} . \vec{b} = 3$, where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$.

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12. Prove by vector method that in a ΔABC, $c^2 = a^2 + b^2 - 2ab\cos C$.

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13. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{b} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

Show that the parallelogram is a rhombus. Determine the area of the ehombus and the length of each side.

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14. Resolve the vector $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ into vectors parallel and perpendicular to the vector $\vec{a} = \hat{i} + \hat{j}$.

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15. Prove that the following vectors are coplanar $-4\hat{i} + 4\hat{j} + 4\hat{k}, 4\hat{i} + 5\hat{j} + \hat{k} - \hat{j} - \hat{k}, 3\hat{i} + 9\hat{j} + 4\hat{k}$.

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16. If the magnitude of the difference of two unit vectors is $\sqrt{3}$ then find the magnitude of their sum.

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17. Find the value of t, such that the following vectors are perpendicular to each other.

$\vec{c} = \hat{i} - 4\hat{j} + t\hat{k}, \vec{d} = 6\hat{i} - 2\hat{j} - 3\hat{k}$

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18. Find the unit vector perpendicular to the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - \hat{k}$.

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19. Find $\left|\vec{a}\right|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right)=8$ and $\left|\vec{a}\right|=8\left|\vec{b}\right|$

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20. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$, then show that $\vec{a}-\vec{d}$ is parallel to $\vec{b}-\vec{c}$.

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21. If $\vec{a}=3\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, then calculate $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)$.

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22. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then prove that $\vec{a} \times \vec{b}=\vec{b} \times \vec{c}=\vec{c} \times \vec{a}$.

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23. Show that $\left(\vec{a}-\vec{b}\right) \times \left(\vec{a}+\vec{b}\right)=2\left(\vec{a} \times \vec{b}\right)$.

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24. If $\vec{a}=2\hat{i}+\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$, then find $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{a} \times \vec{c}\right)$.

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Topic 2 Practice Questions 6 Mark Questions

1. If $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$, then find the vector $\vec{r}$ which satisfies $\vec{r} \times \vec{b} = \vec{c}\,\vec{b}$ and $\vec{r} \cdot \vec{a} = 0$.

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2. If $\vec{p} = \frac{1}{\lambda}\left(\vec{b} \times \vec{c}\right), \vec{q} = \frac{1}{\lambda}\left(\vec{c} \times \vec{a}\right)$ and $\vec{r} = \frac{1}{\lambda}\left(\vec{a} \times \vec{b}\right)$, where $\lambda = \left[\vec{a}\,\vec{b}\,\vec{c}\right] \neq 0$, then show that $\left(\vec{a} + \vec{b} + \vec{c}\right) \cdot \left(\vec{p} + \vec{q} + \vec{r}\right) = 3$.

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3. Prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right) = 0$ and hence prove that $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$, $\vec{b} \times \left(\vec{c} \times \vec{a}\right)$ and $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are coplanar.

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4. If $\vec{a} = 2\hat{i} + \hat{j}$, $\vec{b} = -\hat{i} + 2\hat{k}$ and $\vec{c} = 2\hat{j} + \hat{k}$, then find $\vec{a} \times \left(\vec{b} \times \vec{c}\right)$ and also verify the formula $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c}$

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5. Prove that by vector methord, in any ΔABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

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6. Obtain the volume of the parallelopiped whose sides are vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$. Also find the vector $\left(\vec{a} \times \vec{b}\right) \times \vec{c}$.

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7. Prove by vector method that the medians of a triangle are concurrent.

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8. Prove the following by vector method. The diagonals of a rhombus are at right angles.

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9. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

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10. Find the altitude of a parallelopiped dtermined by the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$, if the base is taken to the parallelogram determined by $\vec{a}$ and $\vec{b}$.

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11. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

If $c_1 = 1$ and $c_2 = 2$, then find c_3, which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

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12. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

If $c_2 = -1$ and $c_3 = 1$, then show that no value of c_1 can make $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

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Topic Test 2

1. If $\vec{a} \cdot \vec{b} = \vec{c} \cdot \vec{a}$ for all vectors $\vec{a}$, then

A. $\vec{a} \perp \left(\vec{b} - \vec{c}\right)$

B. $\vec{b} - \vec{c} = \vec{0}$

C. $\vec{b} \neq \vec{c}$

D. $\vec{b} + \vec{c} = \vec{0}$

Answer: A

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2. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non - zero vectors, then $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Leftrightarrow \ldots .$

A. $\vec{a} = \vec{c}$

B. $\vec{a} \mid \mid \left(\vec{b} - \vec{c}\right)$

C. $\vec{b} \mid \mid \vec{c}$

D. $\vec{b} \perp \vec{c}$

Answer: B

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3. Find the angle between the vectors $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{b} = -2\hat{i} - 3\hat{j} + \hat{k}.$

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4. Find the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$.

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5. Find the projection of $\vec{b} + \vec{c}$ on $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

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6. Show that each of the given three vectors is a unit vector.

$$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right), \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right), \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

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7. Find $\left|\vec{x}\right|$, if for a unit vector $\hat{a}$, $\left(\vec{x} - \hat{a}\right) \cdot \left(\vec{x} + \hat{a}\right) = 12$.

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8. If $\vec{a} . \vec{a} = 0$ and $\vec{a} . \vec{b} = 0$, then what can be concluded about $\vec{b}$?

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9. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\left|\vec{a}\right| = 5, \left|\vec{b}\right| = 12, \left|\vec{c}\right| = 13$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a}$.

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10. Find the area of the triangle ABC with vertices A(1,2,4), B(3,1,-2) and C(4,3,1) by vector method.

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11. Show that the vector area of the triangle whose vertices have position vectors $\vec{a}, \vec{b}, \vec{c}$, is $\frac{1}{2}\left(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right)$.

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12. In a ΔABC, prove by vector method $b^2 = a^2 + c^2 - 2ac \cos B$.

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13. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)$.

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14. Determine the area of parallelogram whose adjacent sides are the vector (1, -3, 1), (1,1,1)

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15. Prove the following by vector method. The parallelogram whose diagonals are equal is a rectangle.

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16. Find the value of λ so that the vectors $\hat{i} - \hat{j} + \hat{k}, 2\hat{i} + \hat{j} - \hat{k}$ and $\lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar.

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17. If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$.

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18. Prove that for any three vectors $\vec{a}, \vec{b}$ and $\vec{c}, \left[\vec{a}+\vec{b}\,\vec{b}+\vec{c}\,\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

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19. If $\vec{a}=\hat{i}+2\hat{j}-2\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+3\hat{j}-\hat{k}$, then find the value of $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$.

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20. Find the value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{k}\times\hat{i}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$.

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21. If $\hat{a}$, $\hat{b}$, $\hat{c}$ are unit vectors and $\hat{a}\times\left(\hat{b}\times\hat{c}\right)=\frac{1}{2}\hat{b}$, then find the angles that $\hat{a}$ makes with $\hat{b}$ and $\hat{c}$ where $\hat{b}$, $\hat{c}$ are not parallel.

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Chapter Test 1 Mark Questions

1. If A, B and C are the vertices of a ΔABC, then what is the value of $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$?

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2. Find α, such that the vectors $(-2, \alpha, 1)$ and $(4, 3, -2)$ are parallel.

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3. Find the scalar and vector components of the vector with initial point (2,1) and terminal point $(-5, 7)$.

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4. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$ then find λ such that $\vec{a}$ is perpendicular to $\lambda \cdot \vec{b} + \vec{c}$.

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5. If $\vec{a} = (2, 1), \vec{b} = (-1, 0)$, then find $3\vec{a} + 2\vec{b}$.

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6. If $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$ then show that $\left|\vec{a}\right| = \left|\vec{b}\right|$.

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7. Evaluate $\left(2\vec{a} + 3\vec{b}\right) \cdot \left(5\vec{a} + 7\vec{b}\right)$.

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8. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular.

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9. If $\left|\vec{a} \cdot \vec{b}\right| = \left|\vec{a} \times \vec{b}\right|$, then what is the angle between $\vec{a}$ and $\vec{b}$?

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10. Find $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$, if $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$.

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Chapter Test 4 Mark Questions

1. Prove the following by vector method. In a triangle AOB, $m\angle AOB = 90^\circ$. If P and Q are the points of trisection of AB,

prove that

$$OP^2 + OQ^2 = \frac{5}{9}AB^2$$

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2. The position vectors of the point A,B,C and D are $4\hat{i} + 3\hat{j} - \hat{k}, 5\hat{i} + 2\hat{j} + 2\hat{k}, 2\hat{i} - 2\hat{j} - 3\hat{k}$ and $4\hat{i} - 4\hat{j} + 3\hat{k}$, respectively. Show that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are parallel.

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3. Prove by vector method that the lines joining the mid points of consecutive sides of a quadrilateral is a parallelogram.

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4. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5$ and $|\vec{c}|=7$. Then, find the angle between $\vec{a}$ and $\vec{b}$.

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5. If $\vec{b}\times\vec{c}=\vec{c}\times\vec{a}\neq\vec{0}$, then prove that $\vec{a}+\vec{b}=\lambda\vec{c}$, where λ is a scalar.

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6. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar then show that $c^2=ab$

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7. Find the area of a triangle having the points A(1,1,1), B(1,2,3) and C(2,3,1) as its vertices.

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8. Prove that $\left|\vec{a}-\vec{b}\right| \geq \left|\vec{a}\right| - \left|\vec{b}\right|$.

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9. Simplify $\left[\vec{a}-\vec{b}\,\vec{b}-\vec{c}\,\vec{c}-\vec{a}\right]$.

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10. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors of magnitude 1,1 and 2 respectively. If $\vec{a}\times\left(\vec{a}\times\vec{c}\right)+\vec{b}=\vec{0}$, then find the acute

angle between a and c.

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Chapter Test 6 Mark Questions

1. Find the vectors from the origin to the points of trisection the vector $\overrightarrow{P_1P_2}$ joining $P_1(-4, 3)$ and P_2(5, -12).

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2. Prove the following by vector method. Measure of the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

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3. Prove by vector method that the medians of a triangle are concurrent.

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4. Determine the angle between the vectors

$\hat{i} - 3\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$

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5. Determine the sine of the angle between the vectors

$5\hat{i} - 3\hat{j}$ and $3\hat{i} - 2\hat{k}$

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6. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} . \vec{c} = 18$.

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7. Express the vector $\vec{a} = 5\hat{i} - 2\hat{j} + 5\hat{k}$ as sum of two vectors such that one is parallel to vector $\vec{b} = 3\hat{i} + \hat{k}$ and other is perpendicular to $\vec{b}$.

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8. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 4$ and $\left|\vec{c}\right| = 5$ and each one of these is perpendicular to sum of other two find $\left|\vec{a} + \vec{b} + \vec{c}\right|$.

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9. Prove that the four points with position vectors $2\vec{a}+3\vec{b}-\vec{c}$, $\vec{a}-2\vec{b}+3\vec{c}$, $3\vec{a}+4\vec{b}-2\vec{c}$ and $\vec{a}-6\vec{b}+6\vec{c}$ are coplanar.

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