MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 1

Section A

1. If $y = x \sin^{-1} x + x \cos^{-1} x$, then whtat is $\frac{dy}{dx}$?

Watch Video Solution

2. Evaluate $\int 10^x dx$.

Watch Video Solution

3. Find IF of $x\frac{dy}{dx} + y = x^3$.

Watch Video Solution

4. Write the equation of YZ-plane.

Watch Video Solution

5. Find the unit vector in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

Watch Video Solution

6. Find an angle θ where $0 < \theta < \frac{\pi}{2}$ which increase twice as fast as its sine

Watch Video Solution

7. Evaluate $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$, where ω is a cube root of unity.

Watch Video Solution

8. Find the value of $\cot\left(\frac{\pi}{2} - 2\cot^{-1}\sqrt{3}\right)$.

Watch Video Solution

9. A die is tossed thrice. Find the probability of getting an even number atleast once.

Watch Video Solution

Section B

1. Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\}=\frac{2b}{a}$

.

Watch Video Solution

2. Solve the following LPP.

Maximise: $Z=5x+3y$

Subject to: $5x + 2y \leq 10, x, y \geq 0$

Watch Video Solution

3. Let f ,g: R $\rightarrow R$ be two functions defined as f(x) = |x| +x and g(x) = |x| -x $\forall x \in R$.

Then find fog and gof .

Watch Video Solution

4. If $A = N \times N$ and * is a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that * is commutative and associative. Also, find identity element for * on A, if any.

Watch Video Solution

5. Solve for x,

$$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x.$$

Watch Video Solution

6. In bag A, there are 5 white and 8 red balls, in bag B, 7 white and 6 red balls and in bag C, 6 white and 5 red balls. One ball is taken out at random from each bag. Find the probability that all three balls are of the same colour.

Watch Video Solution

7. Prove the following :

$$\begin{bmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{bmatrix} - (5x+4)(4-x)^2$$

Watch Video Solution

8. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find $A^2 - 5A + 4I$ and hence find a matrix X, such that $A^2 - 5A + 4I + X = O$.

View Text Solution

9. If A= $\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$, then find the values of a and b .

Watch Video Solution

10. A box contains 4 orange and 4 green balls, another box contains 3 orange and 5 groen balls, one of the two box is selected at random and a ball is drawn from the box, which is found to be orange. Find the probability that the ball is drawn from the first box.

Watch Video Solution

11. If $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, then find $\frac{dy}{dx}$ and determine its value at x = 0.

Watch Video Solution

12. If $y = \left(x + \frac{1}{x + \frac{1}{x + \ldots \infty}} \right)$ find $\frac{dy}{dx}$, the rhs being a valid expression.

Watch Video Solution

13. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

Watch Video Solution

14. Find the intervals in which the following functions is increasing or decreasing.

$f(x) = -2x^3 - 9x^2 - 12x + 1$

Watch Video Solution

15. Examine the applicability of mean value theorem for the function

$f(x) = 1 - x^2$ for $x \in [1, 2]$.

View Text Solution

16. Evaluate the integral $\int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$.

Watch Video Solution

17. Evaluate $\int \log(1 + x^2) dx$

Watch Video Solution

18. Find the area of the region enclosed by the curve $y = xe^{x^2}$, the X-axis and the ordinates x=1 and x=2.

Watch Video Solution

19. Solve $\frac{dy}{dx} + 2y = 6e^x$.

Watch Video Solution

20. Find the differential equation of the family of curves given by $x^2 + y^2 = 2ax$.

Watch Video Solution

21. Prove by vector method that the diagonals of a parallelogram bisect each other.

Watch Video Solution

22. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

View Text Solution

23. Find the acute angle between the lines passing through $(-3, -1, 0), (2, -3, 1)$ and $(1, 2, 3), (-1, 4, -2)$

respectively.

Watch Video Solution

24. Find the equation of the plane passing through the intersection of the planes $3x-y+2z-4=0$ and $x+y+z-2=0$ and the point (2, 2, 1).

Watch Video Solution

25. Find the coordinates of the point, where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane $x+y+4z=6$.

Watch Video Solution

Section C

1. If $y = e^x(\sin x + \cos x)$, then find $\frac{dy}{dx}$.

Watch Video Solution

2. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet ?

Watch Video Solution

3. Evaluate the following integrals :

$$\int \cot^{-1}\left[\frac{\sqrt{1+\cos 2x}+\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}-\sqrt{1-\cos 2x}}\right] dx.$$

Watch Video Solution

4. Find the area bounded by the curve $x^2 = y - 2$ and the straight lines $y = x, x = 0$ and $x = 3$.

View Text Solution

5. Solve the following differential equation,

$$\frac{dy}{dx} = e^{x-y}$$

Watch Video Solution

6. Find the distance of the point (-2,3,-4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$

Watch Video Solution

7. Find the altitude of a parallelopiped determined by the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$, if the base is taken to the parallelogram determined by $\vec{a}$ and $\vec{b}$.

View Text Solution

8. If $\left(\tan^{-1} x\right)^2 + \left(\cot^{-1} x\right)^2 = \frac{5\pi^2}{8}$, then find x.

Watch Video Solution

9. Solve the following system of equations using matrices.

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1 \text{ and } \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

View Text Solution

10. Using elementary transformation, find the inverse of the following matrix if it exists.

$$\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & -1\\ 3 & -5 & 0\end{bmatrix}$$

View Text Solution

Section A 10 Marks

1. Show that $f(x) = \frac{\log x}{x}$ has minimum value at x=e

Watch Video Solution

2. Evaluate $\int_2^4 \frac{x}{x^2+1} dx$

Watch Video Solution

3. Write the order of the differential equation whose solution is given by $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

Watch Video Solution

4. Find the magnitude of $\vec{a}$ given by $\vec{a} = \left(\hat{i} + 3\hat{j} - 2\hat{k}\right) \times \left(-\hat{i} + 3\hat{k}\right)$.

Watch Video Solution

5. Find the distance of the point $(2, 1, 0)$ from the plane $2x + y + 2z + 5 = 0$.

Watch Video Solution

6. Show that the function $f: N \to N$ given by $f(x) = 2x$, is one-one but not onto.

Watch Video Solution

7. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$, find the value of $\sin^{-1} x + \sin^{-1} y$.

Watch Video Solution

8. Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0.$

Watch Video Solution

9. if $P(A) = 0.4$, $P(B) = P$, $P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of P

Watch Video Solution

10. Answer all questions

(j) A function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & \text{if} x \neq 3 \\ 5, & \text{if} x = 3 \end{cases}$ Show that $f(x)$ is continuous at $x = 3$.

Watch Video Solution

11. Show that $f(x) = \frac{\log x}{x}$ has minimum value at x=e

Watch Video Solution

12. Evaluate $\int_2^4 \frac{x}{x^2+1} dx$

Watch Video Solution

13. Write the order of the differential equation whose solution is given by $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

Watch Video Solution

14. Find the magnitude of $\vec{a}$ given by $\vec{a} = \left(\hat{i} + 3\hat{j} - 2\hat{k}\right) \times \left(-\hat{i} + 3\hat{k}\right)$.

Watch Video Solution

15. Find the distance of the point $(2, 1, 0)$ from the plane $2x + y + 2z + 5 = 0$.

Watch Video Solution

16. Show that the function $f: N \to N$ given by $f(x) = 2x$, is one-one but not onto.

Watch Video Solution

17. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$, find the value of $\sin^{-1} x + \sin^{-1} y$.

Watch Video Solution

18. Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0.$

Watch Video Solution

19. if $P(A) = 0.4, P(B) = P, P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of P

Watch Video Solution

20. Answer all questions

(j) A function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & \text{if} x \neq 3 \\ 5, & \text{if} x = 3 \end{cases}$ Show that $f(x)$ is continuous at $x = 3$.

Watch Video Solution

Section B 60 Marks

1. Prove that $\left(\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\sin^{-1}\left(\frac{63}{65}\right)$

Watch Video Solution

2. Answer any three questions

(b) A house wife wishes to mix together two kinds of food X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of food are given below

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs Rs. 6 and 1 kg of food Y costs Rs. 10. Find the least cost of the mixture will produce the diet.

View Text Solution

3. Answer any three questions

(c) Show that $f:[-1,1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one, find the inverse of the function $f:[-1,1] \to$ Range (f).

View Text Solution

4. Show that the relation R in the set of real numbers, defined as $R = \{(a,b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Watch Video Solution

5. If $a > b > c > 0$, then prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c.\ -a}\right) = 0$

Watch Video Solution

6. A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that these two balls are of the same colours

Watch Video Solution

7. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = O$

Watch Video Solution

8. Answer any three questions

(c) If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -1 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

View Text Solution

9. If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that $|AB| = |A||B|$.

Watch Video Solution

10. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. Find the probability that the problem will be solved.

Watch Video Solution

Watch Video Solution

11. Find the equations of all the lines of slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, x \neq 3.$

Watch Video Solution

12. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right).$

Watch Video Solution

13. If $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$, find $\frac{dy}{dx}$.

Watch Video Solution

14. If $y = (\sin y)^x$, find $\frac{dy}{dx}$.

Watch Video Solution

15. Verify Rolle.s theorem for $f(x) = \frac{\sin x}{e^x}$ on $0 \leq x \leq \pi$.

Watch Video Solution

16. Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Watch Video Solution

17. Solve $(x + 1)\frac{dy}{dx} = 2xy$.

Watch Video Solution

18. Evaluate $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$

Watch Video Solution

19. Find the area of the region bounded by the curve $y = x^3$ and the lines $y = x + 6$ and $y = 0$.

Watch Video Solution

20. Show that the points whose position vectors are $5\hat{i} + 5\hat{k}, 2\hat{i} + \hat{j} + 3\hat{k}$ and $-4\hat{i} + 3\hat{j} - \hat{k}$ are collinear.

Watch Video Solution

21. Find the vector equation of the plane passing through the points $3\hat{i}+4\hat{j}+2\hat{k}, 2\hat{i}-2\hat{j}-\hat{k}$ and $7\hat{i}+6\hat{k}$.

View Text Solution

22. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

Watch Video Solution

23. Find the equation in vector and Cartesian from of the plane passing through the point $(3, -3, 1)$ and norrmal to

the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$

Watch Video Solution

24. Prove that $\sin\left(\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\frac{63}{65}$

Watch Video Solution

25. Answer any three questions

(b) A house wife wishes to mix together two kinds of food X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of food are given below

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs Rs. 6 and 1 kg of food Y costs Rs. 10. Find the least cost of the mixture will produce the diet.

View Text Solution

26. Answer any three questions

(c) Show that $f:[-1,1] \to R$, given by $f(x) = \dfrac{x}{x+2}$ is one-one, find the inverse of the function $f:[-1,1] \to$ Range (f).

View Text Solution

27. Check if the relation R on set of real numbers, defined as $R = \{(a, b) : a \leq b^3\}$ is reflexsive, symmetric or transitive.

Watch Video Solution

28. Answer any three questions

(e) If $a > b > c > 0$, then prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c'-a}\right) = \pi$

Watch Video Solution

29. A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from

each bag, find the probability that these two balls are of the same colours

Watch Video Solution

30. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = O$

Watch Video Solution

31. Answer any three questions

(c) If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -1 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

View Text Solution

32. If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that $|AB| = |A||B|$.

Watch Video Solution

33. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. Find the probability that the problem will be solved.

Watch Video Solution

34. Find the equations of all the lines of slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, x \neq 3$.

Watch Video Solution

35. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$.

Watch Video Solution

36. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $\frac{dy}{dx}$

Watch Video Solution

37. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

Watch Video Solution

38. Verify Rolle.s theorem for $f(x) = \frac{\sin x}{e^x}$ on $0 \leq x \leq \pi$.

Watch Video Solution

39. Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Watch Video Solution

40. Solve $(x + 1)\frac{dy}{dx} = 2xy.$

Watch Video Solution

41. Evaluate $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$

Watch Video Solution

42. Find the area of the region bounded by the curve $y = x^3$ and the lines $y = x + 6$ and $y = 0$.

Watch Video Solution

43. Show that the points whose position vectors are $5\hat{i} + 5\hat{k}, 2\hat{i} + \hat{j} + 3\hat{k}$ and $-4\hat{i} + 3\hat{j} - \hat{k}$ are collinear.

Watch Video Solution

44. Answer any three questions

(b) Find the vector equation of the plane passing through

the points $3\hat{i}+4\hat{j}+2\hat{k}, 2\hat{i}-2\hat{j}-\hat{k}$ and $7\hat{i}+6\hat{k}$.

View Text Solution

45. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

Watch Video Solution

46. Find the equation in vector and Cartesian from of the plane passing through the point $(3, -3, 1)$ and norrmal to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$

Watch Video Solution

Section C 30 Marks

1. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1$ prove that $\frac{dy}{dx} = 1$.

Watch Video Solution

2. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$, which is

(i) parallel to the line $2x - y + 9 = 0$.

(ii) perpendicular to the line $5y - 15x = 13$.

Watch Video Solution

3. Find the area of the region bounded by $y = -1, y = 2, x = y^3$ and $x = 0$.

Watch Video Solution

4. Solve $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$.

Watch Video Solution

5. Evaluate $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$.

Watch Video Solution

6. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

Watch Video Solution

7. Prove that $\cos^{-1}\left[\frac{\cos\alpha + \cos\beta}{1+\cos\alpha\cos\beta}\right] = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$

Watch Video Solution

8. Answer any one question

(b) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3h of craft man.s time in its making, while a cricket bat takes 3h of machine time

and 1 h of craftman.s time. In a day, the factory has the availability of not more than 42h of machine time and 24 h of craftman.s time. If the profits on a racket and a bat are Rs. 20 and Rs. 10. respectively then find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make an LPP and solve it graphically.

View Text Solution

9. Let f : N $\rightarrow$ N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

Show that f is many one and onto function.

Watch Video Solution

10. Answer any one question

(a) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ 7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the following system of equations

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$

View Text Solution

11. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that bath balls drawn are black?

View Text Solution

12. Find the inverse of the following matrix using elementary transformation $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

View Text Solution

13. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1$ prove that $\frac{dy}{dx} = 1$.

Watch Video Solution

14. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$, which is

(i) parallel to the line $2x - y + 9 = 0$.

(ii) perpendicular to the line $5y - 15x = 13$.

Watch Video Solution

15. Find the area of the region bounded by $y = -1, y = 2, x = y^3$ and $x = 0$.

Watch Video Solution

16. Solve $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$.

Watch Video Solution

17. Evaluate $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx$.

Watch Video Solution

18. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

Watch Video Solution

19. A variable plane is at a constant distance 3r from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the ΔABC is $x^{-2} + y^{-2} + z^{-2} = r^{-2}$.

Watch Video Solution

20. Prove that

$$\cos^{-1}\left[\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right]=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$$

Watch Video Solution

21. Answer any one question

(b) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3h of craft man's time in its making, while a cricket bat takes 3h of machine time and 1 h of craftman's time. In a day, the factory has the availability of not more than 42h of machine time and 24 h of craftman's time. If the profits on a racket and a bat are Rs. 20 and Rs. 10. respectively then find the number of tennis rackets and cricket bats that the factory must manufacture

to earn the maximum profit. Make an LPP and solve it graphically.

View Text Solution

22. Let f : N $\rightarrow$ N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

Show that f is many one and onto function.

Watch Video Solution

23. Answer any one question

(a) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ 7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the

following system of equations

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$

View Text Solution

24. Answer any one question

(b) Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that bath balls drawn are black?

View Text Solution

25. Answer any one question

(c) Find the inverse of the following matrix using elementary transformation $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

View Text Solution