



MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 1



1. If $y = x \sin^{-1} x + x \cos^{-1} x$, then whtat is $\frac{dy}{dx}$?



2. Evaluate
$$\int 10^x dx$$
.





3. Find IF of
$$x \frac{dy}{dx} + y = x^3$$
.

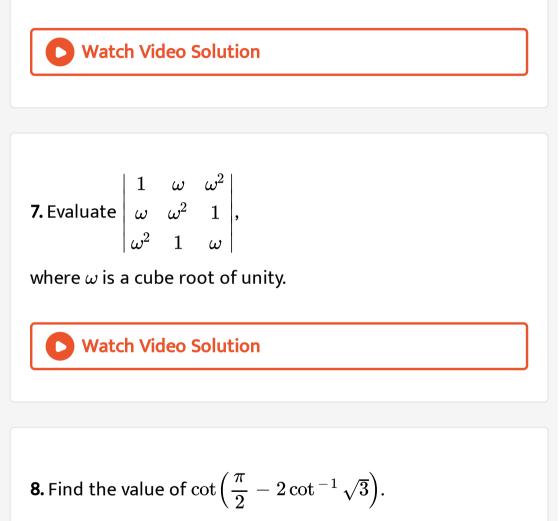
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4. Write the equation of YZ-plane.

5. Find the unit vector in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

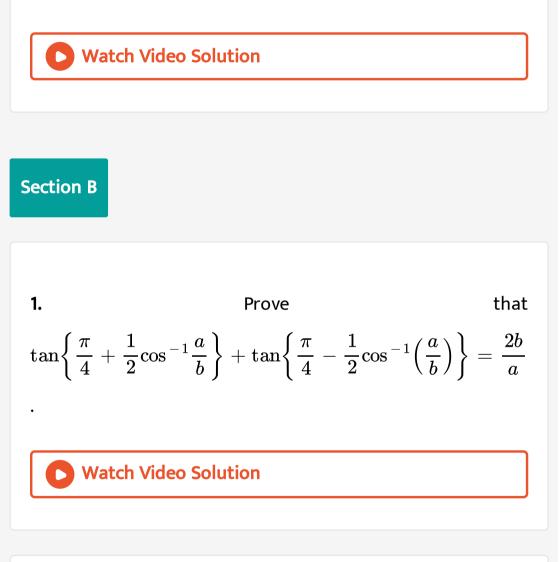
6. Find an angle heta where $0 < heta < rac{\pi}{2}$ which increase twice

as fast as its sine



9. A die is tossed thrice. Find the probability of getting an

even number atleast once.



2. Solve the following LPP.

Maximise: Z = 5x + 3y

Subject to: $5x+2y\leq 10, x,y\geq 0$



3. Let f ,g: R $\rightarrow R$ be two functions defined as f(x) = |x| +x

and $g(x) = |x| - x \quad \forall x \in R$.

Then find fog and gof.

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4. If $A = N \times N$ and * is a binary operation on A defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Also, find identity element for * on A, if any.



5. Solve for x,

$$an^{-1}(x-1) + an^{-1}x + an^{-1}(x+1) = an^{-1}3x.$$

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6. In bag A, there are 5 white and 8 red balls, in bag B, 7 white and 6 red balls and in bag C, 6 white and 5 red balls. One ball is taken out at random from each bag. Find the probability that all three balls are of the same colour.

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7. Prove the following
$$\begin{bmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{bmatrix} - (5x+4)(4-x)^2$$

:



8. If
$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$
, then find $A^2 - 5A + 4I$ and hence

find a matrix X, such that $A^2 - 5A + 4I + X = O$.



9. If A=
$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \text{ and } (A+B)^2 = A^2 + B^2,$$

then find the values of a and b .

10. A box contains 4 orange and 4 green balls, another box contains 3 orange and 5 groen balls, one of the two box is selected at random and a ball is drawn from the box, which is found to be orange. Find the probability that the ball is drawn from the first box.



11. If
$$y = \sin^{-1} \left(2x \sqrt{1-x^2} \right)$$
, then find $rac{dy}{dx}$ and determine

its value at x = 0.

12. If
$$y = \left(x + rac{1}{x + rac{1}{x + \dots \infty}}
ight)$$
 find $rac{dy}{dx}$, the rhs being a

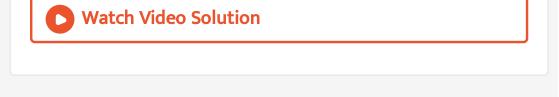
valid expression.



13. Find a point on the curve $y = (x - 2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

14. Find the intervals in which the following functions is increasing or decreasing.

$$f(x) = \ - \ 2x^3 - 9x^2 - 12x + 1$$



15. Examine the applicability of mean value theorem for the

function

$$f(x)=1-x^2$$
 for $x\in [1,2].$



16. Evaluate the integral
$$\int_{0}^{2\pi} rac{1}{1+e^{\sin x}} dx.$$

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17. Evaluate
$$\int\!\!\logig(1+x^2ig)dx$$

18. Find the area of the region enclosed by the curve $y = xe^{x^2}$, the X-axis and the ordinates x=1 and x=2.

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19. Solve
$$rac{dy}{dx}+2y=6e^x.$$

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20. Find the differential equation of the family of curves given by $x^2 + y^2 = 2ax$.

21. Prove by vector method that the diagonals of a parallelogram bisect each other.

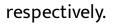
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22. The scalar product of the vector $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\overrightarrow{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\overrightarrow{b} + \overrightarrow{c}$.

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23. Find the acute angle between the lines passing through

$$(\,-3,\,-1,0),\,(2,\,-3,1)$$
 and $(1,2,3),\,(\,-1,4,\,-2)$





24. Find the equation of the plane passing through the intersection of the planes 3x-y+2z-4=0 and x+y+z-2=0 and the point (2, 2, 1).

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25. Find the coordinates of the point, where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane x+y+4z=6.



1. If
$$y = e^x(\sin x + \cos x)$$
, then find $\frac{dy}{dx}$.



2. A jet of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3, 2). What is the nearest distance between the soldier and the jet ?

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3. Evaluate the following integrals :

$$\int \cot^{-1} \Bigg[rac{\sqrt{1+\cos 2x}+\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}-\sqrt{1-\cos 2x}} \Bigg] dx.$$



straight lines y = x, x = 0 and x = 3.

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5. Solve the following differential equation,

$$\frac{dy}{dx} = e^{x - y}$$

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6. Find the distance of the point (-2,3,-4) from the line $rac{x+2}{3}=rac{2y+3}{4}=rac{3z+4}{5}$ measured parallel to the plane 4x + 12y - 3z + 1 = 0



7. Find the altitude of a parallelopiped determined by the vectors $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{b} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\overrightarrow{c} = \hat{i} + \hat{j} + 3\hat{k}$, if the base is taken to the parallelogram determined by \overrightarrow{a} and \overrightarrow{b} .

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8. If
$$(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$
, then find x.



9. Solve the following system of equations using matrices.

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1 \text{ and } \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$
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10. Using elementary transformation, find the inverse of the

following matrix if it exists.

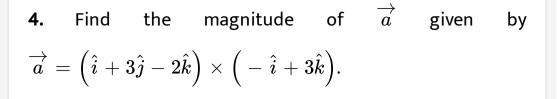
$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

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Section A 10 Marks

1. Show that
$$f(x) = \frac{\log x}{x}$$
 has minimum value at x=e
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2. Evaluate $\int_2^4 \frac{x}{x^2 + 1} dx$
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3. Write the order of the differential equation whose solution is given by $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ **Vatch Video Solution**



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5. Find the distance of the point (2, 1, 0) from the plane

2x + y + 2z + 5 = 0.

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6. Show that the function $f\colon N o N$ given by f(x)=2x, is

one-one but not onto.



7. If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{4}$, find the value of $\sin^{-1}x + \sin^{-1}y$.

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8. Show that
$$\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0.$$

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9. if P(A)=0.4, P(B)=P, $P(A\cup B)=0.6$ and A and B

are given to be independent events, find the value of P

10. Answer all questions

(j) A function f(x) is defined as $f(x) = \begin{cases} rac{x^2 - x - 6}{x - 3}, & ext{if} x
eq 3 \end{cases}$ Show that f(x) is continuous at x = 3.



11. Show that
$$f(x) = rac{\log x}{x}$$
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12. Evaluate
$$\int_2^4 rac{x}{x^2+1} dx$$

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14. Find the magnitude of
$$\overrightarrow{a}$$
 given by
 $\overrightarrow{a} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k}).$
(Vatch Video Solution

15. Find the distance of the point (2, 1, 0) from the plane

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, find the value of $\sin^{-1} x + \sin^{-1} y$.

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Section B 60 Marks

1. Prove that
$$\left(\cos^{-1}{3\over 5} + \sin^{-1}{5\over 13}\right) = \sin^{-1}\!\left({63\over 65}\right)$$

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2. Answer any three questions

(b) A house wife wishes to mix together two kinds of food X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of food are given below

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs Rs. 6 and 1 kg of food Y costs Rs. 10. Find

the least cost of the mixture will produce the diet.



3. Answer any three questions

(c) Show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one, find the inverse of the function $f: [-1, 1] \to$ Range (f).

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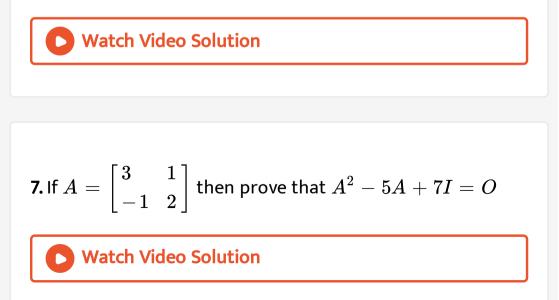
4. Show that the relation R in the set of real numbers, defined as $R = \{(a, b) : a \le b^2\}$ is neither reflexive nor symmetric nor transitive.



5. If
$$a > b > c > 0$$
, then prove that
 $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c.-a}\right) = 0$



6. A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that these two balls are of the same colours



8. Answer any three questions

(c) If
$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -1 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$
, find A.

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9. If
$$A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that $|AB| = |A||B|$.
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10. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7 to 5. Find the probability that the problem will be solved.

11. Find the equations of all the lines of slope 2 and that are

tangent to the curve
$$y=rac{1}{x-3}, x
eq 3.$$

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12. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\Big(-\frac{\pi}{3}, \frac{\pi}{3}\Big).$

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13. If
$$y = \cos^{-1} \Big\{ 2x \sqrt{1-x^2} \Big\}$$
, find $rac{dy}{dx}$

14. If
$$y = (\sin y)^x$$
, find $\frac{dy}{dx}$.
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15. Verify Rolle.s theorem for $f(x) = \frac{\sin x}{e^x}$ on $0 \le x \le \pi$.
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16. Show that the function $y = (A + Bx)e^{3x}$ is a solution

of the equation
$$rac{d^2y}{dx^2}-6rac{dy}{dx}+9y=0$$

17. Solve
$$(x+1)rac{dy}{dx}=2xy.$$



18. Evaluate
$$\int e^{x} \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$$

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19. Find the area of the region bounded by the curve $y=x^3$

and the lines y = x + 6 and y = 0.

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20. Show that the points whose position vectors are $5\hat{i} + 5\hat{k}, 2\hat{i} + \hat{j} + 3\hat{k}$ and $-4\hat{i} + 3\hat{j} - \hat{k}$ are collinear.

21. Find the vector equation of the plane passing through

the points $3\hat{i}+4\hat{j}+2\hat{k}, 2\hat{i}-2\hat{j}-\hat{k} ~~\mathrm{and}~~7\hat{i}+6\hat{k}.$

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22. Prove that if a plane has the intercepts a, b, c and is at a

distance of p units from the origin, then $rac{1}{a^2}+rac{1}{b^2}+rac{1}{c^2}=rac{1}{p^2}.$

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23. Find the equation in vector and Cartesian from of the plane passing through the point (3, -3, 1) and normal to

the line joining the points $(3,\,4,\,-1)$ and $(2,\,-1,\,5)$



24. Prove that
$$\sin\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{63}{65}$$

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25. Answer any three questions

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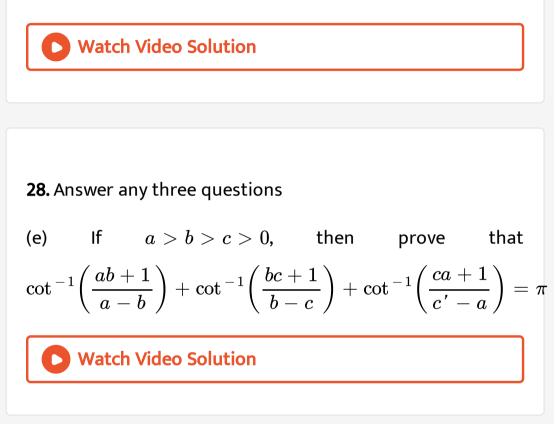


26. Answer any three questions

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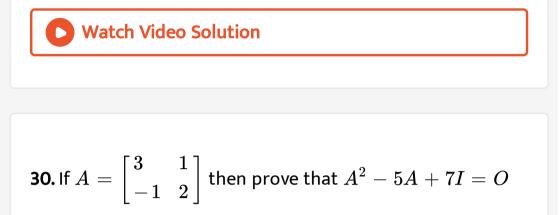
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each bag, find the probability that these two balls are of the

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31. Answer any three questions

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$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -1 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$
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33. The odds against A solving a certain problem are 4 to 3

and the odds in favour of B solving the same problem are 7

to 5. Find the probability that the problem will be solved.



34. Find the equations of all the lines of slope 2 and that are

tangent to the curve
$$y=rac{1}{x-3}, x
eq 3.$$

35. Prove that the function f(x) = an x - 4x is strictly

decreasing on $\left(-\frac{\pi}{3},\frac{\pi}{3}\right)$.

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36. If
$$y = \sin^{-1} \Big[x \sqrt{1-x} - \sqrt{x} \sqrt{1-x^2} \Big]$$
 then find $rac{dy}{dx}$

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37. If
$$\left(\cos x
ight)^y = \left(\cos y
ight)^x$$
, then find $rac{dy}{dx}$.

38. Verify Rolle.s theorem for $f(x) = rac{\sin x}{e^x}$ on $0 \le x \le \pi$.

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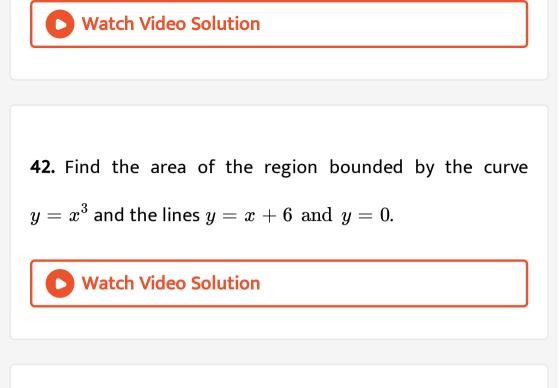
39. Show that the function $y = (A + Bx)e^{3x}$ is a solution

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$$rac{d^2y}{dx^2}-6rac{dy}{dx}+9y=0$$

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40. Solve
$$(x+1)rac{dy}{dx}=2xy.$$

41. Evaluate
$$\int e^{x} \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$$



43. Show that the points whose position vectors are $5\hat{i} + 5\hat{k}, 2\hat{i} + \hat{j} + 3\hat{k}$ and $-4\hat{i} + 3\hat{j} - \hat{k}$ are collinear.

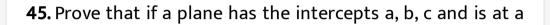
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44. Answer any three questions

(b) Find the vector equation of the plane passing through

the points $3\hat{i} + 4\hat{j} + 2\hat{k}$, $2\hat{i} - 2\hat{j} - \hat{k}$ and $7\hat{i} + 6\hat{k}$.





distance of p units from the origin, then $rac{1}{a^2}+rac{1}{b^2}+rac{1}{c^2}=rac{1}{p^2}.$

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46. Find the equation in vector and Cartesian from of the plane passing through the point (3, -3, 1) and norrmal to the line joining the points (3, 4, -1) and (2, -1, 5)

1. If
$$x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$$
 and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1$
prove that $\frac{dy}{dx} = 1$.



2. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$, which is

(i) parallel to the line 2x - y + 9 = 0.

(ii) perpendicular to the line 5y - 15x = 13.

3. Find the area of the region bounded by $y = -1, y = 2, x = y^3$ and x = 0.

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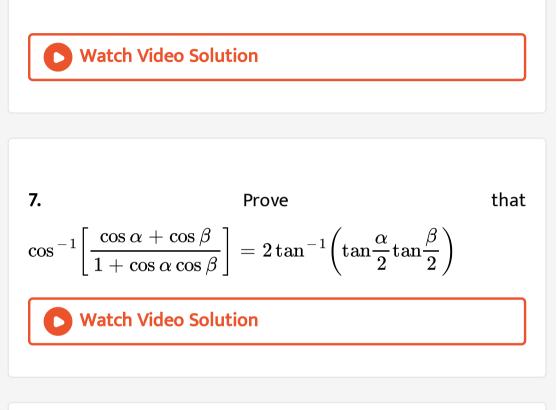
4. Solve
$$rac{dy}{dx}=e^{x+y}+e^{-x+y}.$$

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5. Evaluate
$$\int e^{x} \left(\frac{1 + \sin x \cos x}{\cos^{2} x} \right) dx.$$

6. Find the points on the line $rac{x+2}{3}=rac{y+1}{2}=rac{z-3}{2}$ at

a distance of 5 units from the point P(1, 3, 3).



8. Answer any one question

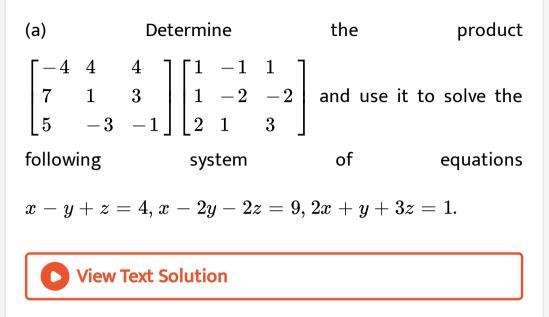
(b) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3h of craft man.s time in its making, while a cricket bat takes 3h of machine time and 1 h of craftman.s time. In a day, the factory has the availability of not more than 42h of machine time and 24 h of craftman.s time. If the profits on a racket and a bat are Rs. 20 and Rs. 10. respectively then find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make an LPP and solve it graphically.

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9. Let $f: N \rightarrow N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$

Show that f is many one and onto function.

10. Answer any one question



11. Three cards are drawn successively, without replacement

from a pack of 52 well shuffled cards. What is the probability

that bath balls drawn are black?



12. Find the inverse of the following matrix using elementary

transformation

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}.$$

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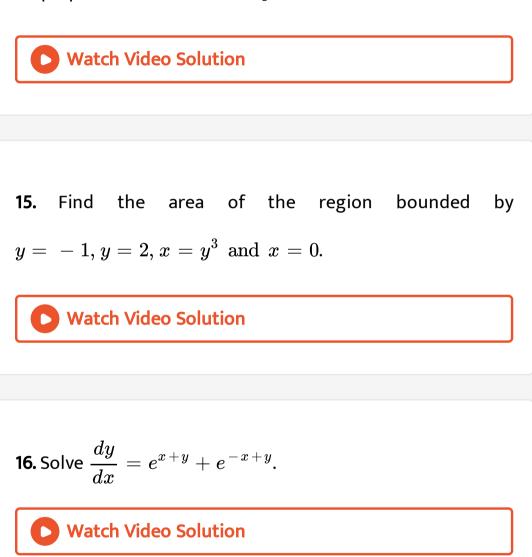
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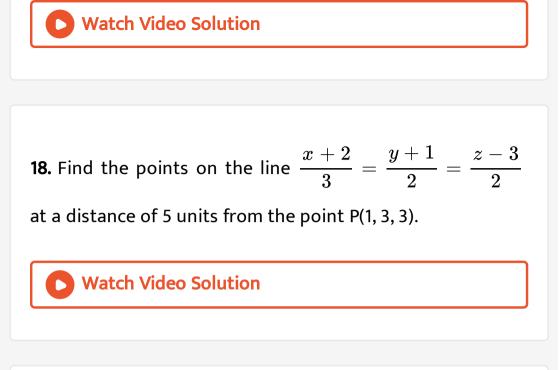
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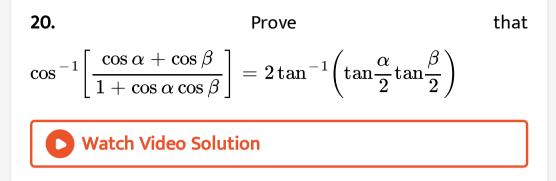


17. Evaluate
$$\int e^{x} \left(rac{1 + \sin x \cos x}{\cos^{2} x}
ight) dx.$$



19. A variable plane is at a constant distance 3r from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the ΔABC is $x^{-2} + y^{-2} + z^{-2} = r^{-2}$.





21. Answer any one question

(b) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3h of craft man's time in its making, while a cricket bat takes 3h of machine time and 1 h of craftman's time. In a day, the factory has the availability of not more than 42h of machine time and 24 h of craftman's time. If the profits on a racket and a bat are Rs. 20 and Rs. 10. respectively then find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make an LPP and solve it graphically.



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Show that f is many one and onto function.

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23. Answer any one question

(a) Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ 7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$
 and use it to solve the



x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.



24. Answer any one question

(b) Three cards are drawn successively, without replacement

from a pack of 52 well shuffled cards. What is the probability

that bath balls drawn are black?



25. Answer any one question

(c) Find the inverse of the following matrix using elementary

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