MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 10

Section A 10 Marks

1. Show that $f(x) = \frac{\log x}{x}$ has minimum value at x=e

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2. Evaluate $\int_2^4 \frac{x}{x^2+1} dx$

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3. Write the order of the differential equation whose solution is given by $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$

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4. Find the magnitude of $\vec{a}$ given by $\vec{a} = \left(\hat{i} + 3\hat{j} - 2\hat{k}\right) \times \left(-\hat{i} + 3\hat{k}\right)$.

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5. Find the distance of the point $(2, 1, 0)$ from the plane $2x + y + 2z + 5 = 0$.

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6. Show that the function $f: N \to N$ given by $f(x) = 2x$, is one-one but not onto.

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7. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$, find the value of $\sin^{-1} x + \sin^{-1} y$.

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8. Show that $\begin{vmatrix} b-c & c-a & a-b \\ c-a & a-b & b-c \\ a-b & b-c & c-a \end{vmatrix} = 0.$

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9. if $P(A) = 0.4, P(B) = P, P(A \cup B) = 0.6$ and A and B are given to be independent events, find the value of P

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10. Answer all questions

(j) A function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & \text{if} x \neq 3 \\ 5, & \text{if} x = 3 \end{cases}$ Show that $f(x)$ is continuous at $x = 3$.

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(j) A function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x^2-x-6}{x-3}, & \text{if} x \neq 3 \\ 5, & \text{if} x = 3 \end{cases}$ Show that $f(x)$ is continuous at $x = 3$.

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Section B 60 Marks

1. Prove that $\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \sin^{-1}\left(\frac{63}{65}\right)$

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2. Answer any three questions

(b) A house wife wishes to mix together two kinds of food X and Y, in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

The vitamin contents of 1 kg of food are given below

	Vitamin A	Vitamin B	Vitamin C
Food X	1	2	3
Food Y	2	2	1

1 kg of food X costs Rs. 6 and 1 kg of food Y costs Rs. 10. Find the least cost of the mixture will produce the diet.

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3. Answer any three questions

(c) Show that $f:[-1,1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one, find the inverse of the function $f:[-1,1] \to$ Range (f).

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4. Show that the relation R in the set of real numbers, defined as $R = \{(a,b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

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5. If $a > b > c > 0$, then prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c.-a}\right) = 0$

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6. A bag contains 6 black and 3 white balls. Another bag contains 5 black and 4 white balls. If one ball is drawn from each bag, find the probability that these two balls are of the same colours

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7. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = O$

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8. Answer any three questions

(c) If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -1 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

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9. If $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$, verify that $|AB| = |A||B|$.

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10. The odds against A solving a certain problem are 4 to 3 and the odds in favour of B solving the same problem are 7

to 5. Find the probability that the problem will be solved.

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11. Find the equations of all the lines of slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, x \neq 3.$

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12. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right).$

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13. If $y = \cos^{-1}\left\{2x\sqrt{1-x^2}\right\}$, find $\frac{dy}{dx}.$

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14. If $y = (\sin y)^x$, find $\frac{dy}{dx}$.

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15. Verify Rolle.s theorem for $f(x) = \frac{\sin x}{e^x}$ on $0 \leq x \leq \pi$.

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16. Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

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17. Solve $(x+1)\frac{dy}{dx} = 2xy.$

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18. Evaluate $\int e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$

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19. Find the area of the region bounded by the curve $y = x^3$ and the lines $y = x + 6$ and $y = 0.$

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20. Show that the points whose position vectors are $5\hat{i}+5\hat{k}, 2\hat{i}+\hat{j}+3\hat{k}$ and $-4\hat{i}+3\hat{j}-\hat{k}$ are collinear.

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21. Find the vector equation of the plane passing through the points $3\hat{i}+4\hat{j}+2\hat{k}, 2\hat{i}-2\hat{j}-\hat{k}$ and $7\hat{i}+6\hat{k}$.

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22. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

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23. Find the equation in vector and Cartesian from of the plane passing through the point $(3, -3, 1)$ and norrmal to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$

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24. Prove that $\sin\left(\cos^{-1}\frac{3}{5}+\sin^{-1}\frac{5}{13}\right)=\frac{63}{65}$

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25. Answer any three questions

(b) A house wife wishes to mix together two kinds of food X and Y, in such a way that the mixture contains at least 10

units of vitamin A, 12 units of vitamin B and 8 units of vitamin C.

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27. Check if the relation R on set of real numbers, defined as $R = \{(a, b) : a \leq b^3\}$ is reflexsive, symmetric or transitive.

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28. Answer any three questions

(e) If $a > b > c > 0$, then prove that $\cot^{-1}\left(\frac{ab+1}{a-b}\right) + \cot^{-1}\left(\frac{bc+1}{b-c}\right) + \cot^{-1}\left(\frac{ca+1}{c'-a}\right) = \pi$

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30. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = O$

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35. Prove that the function $f(x) = \tan x - 4x$ is strictly decreasing on $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right).$

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36. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ then find $\frac{dy}{dx}$

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37. If $(\cos x)^y = (\cos y)^x$, then find $\frac{dy}{dx}$.

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38. Verify Rolle.s theorem for $f(x) = \frac{\sin x}{e^x}$ on $0 \leq x \leq \pi$.

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39. Show that the function $y = (A + Bx)e^{3x}$ is a solution of the equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

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44. Answer any three questions

(b) Find the vector equation of the plane passing through the points $3\hat{i} + 4\hat{j} + 2\hat{k}$, $2\hat{i} - 2\hat{j} - \hat{k}$ and $7\hat{i} + 6\hat{k}$.

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45. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

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46. Find the equation in vector and Cartesian from of the plane passing through the point $(3, -3, 1)$ and norrmal to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$

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Section C 30 Marks

1. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1$ prove that $\frac{dy}{dx} = 1$.

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2. Find the equations of the tangent to the curve $y = x^2 - 2x + 7$, which is

(i) parallel to the line $2x - y + 9 = 0$.

(ii) perpendicular to the line $5y - 15x = 13$.

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3. Find the area of the region bounded by $y = -1, y = 2, x = y^3$ and $x = 0$.

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4. Solve $\frac{dy}{dx} = e^{x+y} + e^{-x+y}$.

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5. Evaluate $\int e^x \left(\frac{1 + \sin x \cos x}{\cos^2 x} \right) dx.$

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6. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

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7. Prove that $\cos^{-1}\left[\frac{\cos\alpha + \cos\beta}{1 + \cos\alpha\cos\beta}\right] = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$

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8. Answer any one question

(b) A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 h of machine time and 3h of craft man.s time in its making, while a cricket bat takes 3h of machine time and 1 h of craftman.s time. In a day, the factory has the availability of not more than 42h of machine time and 24 h of craftman.s time. If the profits on a racket and a bat are Rs. 20 and Rs. 10. respectively then find the number of tennis rackets and cricket bats that the factory must manufacture to earn the maximum profit. Make an LPP and solve it graphically.

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9. Let f : N $\rightarrow$ N be defined by

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

Show that f is many one and onto function.

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10. Answer any one question

(a) Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ 7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the following system of equations

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$

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11. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that bath balls drawn are black?

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12. Find the inverse of the following matrix using elementary transformation $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$.

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13. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right), t > 1$ prove that $\frac{dy}{dx} = 1$.

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19. A variable plane is at a constant distance 3r from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the ΔABC is $x^{-2}+y^{-2}+z^{-2}=r^{-2}$.

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20. Prove that $\cos^{-1}\left[\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right]=2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$

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