MATHS

BOOKS - ARIHANT PRAKASHAN

VERY SIMILAR TEST 5

Section A

1. What is the derivative of $\cos^{-1}\left(2x^2-1\right)$ if x $\in$ (-1,0).

Watch Video Solution

2. Show that the function $f(x) = \cos^2 x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$.

Watch Video Solution

3. Name of curve which is represented by the solution of differential equation $2x\frac{dy}{dx} - y = 3$

Watch Video Solution

4. if $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b}\right|$ the what is the relation between $\vec{a}$ and $\vec{b}$?

Watch Video Solution

5. Find the value of $\sin^{-1}\left[\cos\left(\frac{3\pi}{5}\right)\right]$.

Watch Video Solution

6. Consider f : (1,2,3) $\rightarrow$ {a,b,c) given by f(1) = a, f(2)=b, f(3) = c . Find f^{-1} . Show that $\left(f^{-1}\right)^{-1}$ = f.

Watch Video Solution

7. State true or False .The planes $2x + 4y - z + 1 = 0$ and $x - 2y - 6z + 3 = 0$ are perpendicular to each other.

Watch Video Solution

Watch Video Solution

8. Evaluate $\int_0^{\pi/4} \tan^2 x dx$.

Watch Video Solution

9. If $\begin{vmatrix} a & b & c \\ b & a & b \\ x & b & c \end{vmatrix} = 0$ then `x=____.

Watch Video Solution

10. Three events A, B and C have probabilities $\frac{2}{5}, \frac{1}{3}$ and $\frac{1}{2}$, respectively. If

$P(A \cap C) = \frac{1}{5}$ and $P(B \cap C) = \frac{1}{4}$, then find the values of $P(C/B)$ and $P(A' \cap C')$.

Watch Video Solution

Section B

1. A binary operation $\cdot$ on the set {0,1,2,3,4,5} is defined as

$$a \cdot b = \begin{cases} a+b & \text{if} a+b<6 \\ a+b-6 & \text{if} a+b \geq 6 \end{cases}$$

Find the composition table for $\cdot$ Also, show that zero is the identity for this operation and each non-zero element a of the set is invertible with 6-a, being the inverse of a.

Watch Video Solution

2. Discuss the commutativity and associativity of binary operation .*. defined on A = Q - {1} by the rule $a * b = a - b + ab$ for all $a, b \in A$. Also, find the identity element of * in A and hence find the invertible elements of A.

Watch Video Solution

3. Directions (Q. Nos. 27 and 28) Solve the following equations:

$$\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}, \sqrt{6} > x > 0$$

Watch Video Solution

4. Answer any three questions

(d) Solve the following LPP graphically Maximize and Minimize $Z = 3x + 5y$

Subject to $3x - 4y + 12 \geq 0$

$2x - y + 2 \geq 0$

$2x + 3y - 12 \geq 0$

$0 \leq x \leq 4$

$y \geq 2$

View Text Solution

5. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right\} = \frac{2b}{a}$

.

Watch Video Solution

6. A die is rolled until a 6 is obtained. What is the probability that

you end up in the third roll.

Watch Video Solution

7. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$.

Find a matrix D such that CD -AB =O .

Watch Video Solution

8. Prove the following : $\begin{bmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{bmatrix} = \left(1 - x^3\right)^2$

Watch Video Solution

9. The probability that A hits a target is $\frac{1}{3}$ and the probability that B hits it is $\frac{2}{5}$. If each one of A and B shoots at the target, what is the probability that

(i) the target is hit?

(ii) exactly one of them hits the target?

Watch Video Solution

10. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ then prove that $A^2 - 4A - 5I = 0.$

Watch Video Solution

11. Verify Rolle.s theorem for the function $f(x) = x^2 - 3x + 2$ in the interval $[1, 2]$

Watch Video Solution

12. If $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right), \; y = \sin^{-1}\left(\frac{1}{\sqrt{t^2+1}}\right)$ then $\frac{dy}{dx}$ is independent of t.

Watch Video Solution

13. Find the equation of the normal at a point on the curve $x^2 = 4y$, which passes through the point (1, 2). Also, find the equation of the corresponding tangent.

Watch Video Solution

14. Find $\frac{dy}{dx}$, if $y = k\tan^{-1}\left(\frac{y}{x}\right)$.

Watch Video Solution

15. Determine the interval in which the function f(x) = $x^3 - 5x^2 + 3x + 97$ is decreasing and that in which it is

increasing.

Watch Video Solution

16. Reduce $x\frac{dy}{dx} + y = xy^3$ to linear differential equation.

Watch Video Solution

17. Evaluate $\int\left[\frac{1}{\text{In}x} - \frac{1}{(\text{In}x)^2}\right]dx.$

View Text Solution

18. Find the differential equation for the family of curve $y = a\sin^{-1}x + b\cos^{-1}x$.

Watch Video Solution

19. Solve $\frac{dy}{dx} = e^{\sin^{-1}x}.\frac{\sin^{-1}x}{\sqrt{1-x^2}}$.

Watch Video Solution

20. Find the area bounded by the curve $y = 2x - x^2$ and the straight line $y = -x$.

Watch Video Solution

21. If the vectors $a\hat{i}+\hat{j}+\hat{k}, \hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar, then prove that $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$.

Watch Video Solution

22. If a is any vector, then find $\left|\hat{i}\times\vec{a}\right|^2+\left|\hat{j}\times\vec{a}\right|^2+\left|\hat{k}\times\vec{a}\right|^2$.

View Text Solution

23. Find the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{4}$ and the plane $2x+4y-z+1=0$.

View Text Solution

24. Answer any three questions

(e) Find the foot of perpendicular drawn from the point $(5, 7, 3)$ to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Also, find the length of the perpendicular.

View Text Solution

Section C

1. Answer any one question

(a) The radius of a wire as measured by a screw gauge is

found to be 1.26 mm. If the correct radius is 1.25 mm, then find the approximate error, relative error and percentage error in calculation of area of its cross section.

View Text Solution

2. If $x = \sin t$ and $y = \sin(pt)$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$

Watch Video Solution

3. Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x \cos ecx} dx.$

Watch Video Solution

4. Solve $\left(1+x^2\right)\frac{dy}{dx}+2xy-x^3=0.$

View Text Solution

5. Using integration find the area of the region $\left\{(x,y):25x^2+9y^2\leq 225 \text{ and } 5x+3y\geq 15\right\}.$

Watch Video Solution

6. Find the volume of parallelopiped, whose sides are given by vectors $\hat{i}+\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$, Also, find

λ, such that the vectors $\hat{i}+2\hat{j}+3\hat{k}, 4\hat{i}+\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ are coplanar.

View Text Solution

7. Answer any one question

(b) Find the equation of the line through the point $(1, 0, -1)$ and intersecting the lines $x = 2y = 2z$ and $3x + 4y - 1 = 0 = 4x + 5z - 2$

View Text Solution

8. Answer any one question

(a) Show that :

$$2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\}=\tan^{-1}\frac{\sin\alpha.\cos\beta}{\cos\alpha+\sin\beta}$$

View Text Solution

9. Answer any one question

(b) Solve the following LPP.

Maximise : $Z=10x+2y$

Subject to

: $-x+y\geq -1, x+y\leq 6, y\leq 5, x, y\geq 0$

View Text Solution

10. If the function $f: R\to R$ is given by $f(x)=\frac{x+3}{3}$ and $g: R\to R$ is given $g(x)=2x-3$, then find

(i) fog (ii) gof.

Is $f^{-1} = g$?

Watch Video Solution

11. Answer any one question

(b) Solve by matrix inversion method.

$x + 2y + 3z = 14$

$2x - y + 5z = 15$

$2y + 4z - 3x = 13$

View Text Solution

12. Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

Watch Video Solution